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ON THE ALGORITHMIC DESIGN OF A CLASS OF CONTROL SYSTEMS BASED ON PROVIDING THE SYMMETRY OF OPEN-LOOP BODE PLOTS

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ABSTRACT

The paper deals with a design method related to a class of systems having the open-loop transfer function $H_0(s) = k_0(1+\beta T_\Sigma s)/s^2(1+sT_\Sigma)$. The design - based on previous computed parameterized relations - has the following features:

- it permits the designer to choose a convenient solution from a great variety of possible solutions;
- the control system performance can be pointed out in the form of diagrams easy to be interpreted;
- for every adopted value of the free parameter depicted in the paper as β the values of controller tuning parameters ensure the maximum possible value of system phase reserve; these feature is based on the symmetry of open-loop Bode plots;
- for a particular value of the free parameter β the design relations specific to the symmetric optimum criterion after Kessler are found again.

The application domain of the proposed design method is (but it is not restricted to) control of electrical drives.

1. INTRODUCTION

It is well-known that the methods for algorithmic design of control systems (briefly, CS) are the more attractive for the user they provide him the possibility to choose a more advantageous solution from the point of view of the proposed goal, and the more simple design relations and previous computed, if possible.

This category includes the parameter tuning methods after Kessler, known under the names of "magnitude optimum" (Betragsoptimum) and "symmetric optimum" (symmetrische Optimum) [Kes58a], [Kes58b]. In the conditions when the parameters of the controlled plant are relatively constant and well-known the two criteria are frequently used to the parameter tuning of the controllers as part of (but not restricted to) speed control loops of electrical drives. The two criteria offer the "optimum" solutions frequently presenting examples for two classes of CS characterized by the open-loop transfer functions (t.f.), depicted as $H_0(s)$, in the form:

$$H_0(s) = \frac{k_0}{s(1 + s\,T_\Sigma)} \quad \text{- for the magnitude optimum criterion,} \tag{1.1}$$

$$H_0(s) = \frac{k_0(1 + s\,T_r)}{s^2(1 + s\,T_\Sigma)} \quad \text{- for the symmetric optimum criterion} \tag{1.2}$$

(the significance of the parameters will result in the sequel).

The "optimum" solutions are obtained if between the coefficients of the t.f. of the control loops, $H_w(s)$, in the two situations:

$$H_w(s) = \frac{b_0}{a_0 + a_1\,s + a_2\,s^2}, \tag{1.3}$$

respectively:

$$H_w(s) = \frac{b_0 + b_1\,s}{a_0 + a_1\,s + a_2\,s^2 + a_3\,s^3}, \tag{1.4}$$

the following optimum conditions are imposed:

$$2\,a_0\,a_2 = a_1^2, \tag{1.5}$$

respectively:

$$2\,a_0\,a_2 = a_1^2 \quad \text{and} \quad 2\,a_1\,a_3 = a_2^2 \tag{1.6}$$

(e.g., see the papers [Căl75], [Bab85] a.s.o. referring this case).

The performance of CS tuned after Kessler's relations can be considered as:

- very good in the case of magnitude optimum criterion, even "ideal",
- less acceptable in the case of symmetric optimum criterion.

The paper presents a design method for the controllers meant for the class of systems with t.f. of the form (1.2) presenting the following particular features:

- it is based on a parametrization of the design relations (1.6) and, therefore, it provides the designer with the possibility to choose a convenient solutions from a set of solutions having specific the fact that:
- it ensures for every solution the symmetry of open-loop Bode plots with respect to the croswsover frequency and, therefore, the maximum phase reserve.

So, the case of optimization relations (1.6) after Kessler becomes a particular case of some more general relations.

2. PRESENTATION OF THE DESIGN METHOD

The CS structure taken into consideration - corresponding to the practical applications from the domain of electrical drives - leading to a t.f. of the form (1.2) is presented in Fig.1, in which:

F-w - signal filter, placed on the reference channel; it can be used for the correction of input-output properties (w_z --> w --> y);

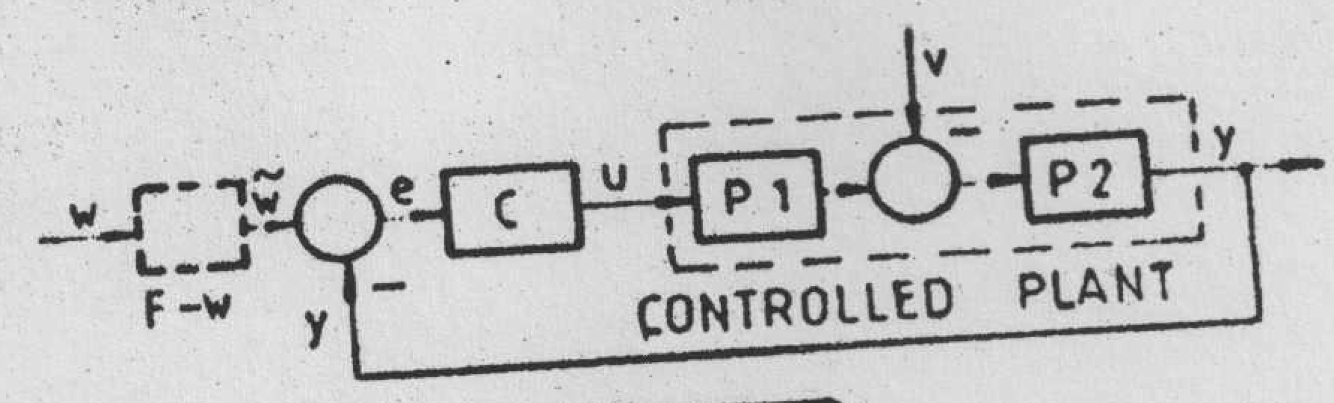

FIG. 1

P1, P2 - informational subsystems characterizing the controlled plant (CP), with the following forms of the t.f.:

$$H_{P1}(s) = \frac{k_{P1}}{1 + sT_\Sigma} \quad \text{(a)} \quad \text{or} \quad H_{P1}(s) = \frac{k_{P1}}{(1 + sT_\Sigma)(1 + sT_1)} \quad \text{(b)} \qquad (2.1)$$

$$H_{P2}(s) = \frac{k_{P2}}{s}, \quad \text{with} \quad T_\Sigma < T_1, \qquad (2.2)$$

k_P are transfer coefficients (gains), and T_1 and T_Σ are time constants characterizing the plant; T_Σ can characterize, in particular, the sum of small time constants of the controlled plant,

$$H_{CP}(s) = H_{P1}(s)\, H_{P2}(s), \quad \text{with} \quad k_{CP} = k_{P1}\, k_{P2}, \qquad (2.3)$$

C - controller (regulator). Depending on the form (a) or (b) of t.f. (2.1), the controller can be chosen of type PI or PID, respectively, with the t.f.:

$$\text{(a): } H_R(s) = \frac{k_r}{s}(1 + sT_r) \quad \text{or} \quad \text{(b): } H_R(s) = \frac{k_r}{s}(1 + sT_r)(1 + sT_r'). \qquad (2.4)$$

In the case (b), when a PID controller is used, the large time constant of controller plant, T_1, is canceled by the controller time constant $T_r' = T_1$.

Consequently, in both cases for the open-loop t.f., $H_0(s)$, the form (1.2) is obtained:

$$H_0(s) = H_R(s)\, H_{CP}(s) = \frac{k_0\,(1 + sT_r)}{s^2\,(1 + sT_\Sigma)}, \quad \text{with} \quad k_0 = k_r\, k_{CP}, \qquad (2.5)$$

which can be also rewritten down in the form:

$$H_0(s) = \frac{1 + \beta\, T_\Sigma\, s}{\alpha\, T_\Sigma^2\, s^2\,(1 + sT_\Sigma)}, \quad \text{with} \quad \beta = T_r/T_\Sigma, \quad \alpha = 1/(k_0 T_\Sigma^2). \qquad (2.6)$$

Accordingly, the closed-loop system t.f. $H_w(s)$ obtains the form (1.4); by taking into account the relation (2.5) or (2.6), $H_w(s)$ can be also expressed in one of the following forms:

$$H_w(s) = \begin{cases} \dfrac{1 + sT_r}{1 + s\,T_r + s^2/k_0 + s^3\,T_\Sigma/k_0}, & \text{(a)} \\[2ex] \dfrac{1 + \beta\, T_\Sigma\, s}{1 + \beta\, T_\Sigma\, s + \alpha\, T_\Sigma^2\, s^2 + \alpha\, T_\Sigma^3\, s^3}. & \text{(b)} \end{cases} \qquad (2.7)$$

The optimization relations (1.6) after Kessler, generalized as design relations:

$$m\, a_0\, a_2 = a_1^2 \quad \text{and} \quad m\, a_1\, a_3 = a_2^2, \qquad (2.8)$$

with m - real parameter (m > 1 due to stability conditions), imposed to the relation (1.4) or to the equivalent formulae (2.7), lead to the following relations between the coefficients α, β and m:

$$\alpha = m^3 = \beta\sqrt{\beta}\ , \quad \beta = m^2\ , \quad \text{with } \alpha, \beta > 1. \qquad (2.9)$$

Consequently, the following "optimized" forms of the t.f. (2.6) and (2.7) are obtained:

$$H_0(s)_{opt} = \frac{1 + \beta T_\Sigma s}{\beta\sqrt{\beta}\, T_\Sigma^2 s^2 (1 + s T_\Sigma)}; \qquad (2.10)$$

respectively:

$$H_w(s)_{opt} = \begin{cases} \dfrac{1 + \beta T_\Sigma s}{1 + \beta T_\Sigma s + \beta\sqrt{\beta}\, T_\Sigma^2 s^2 + \beta\sqrt{\beta}\, T_\Sigma^3 s^3}, & \text{(a)} \\[2ex] \dfrac{1 + \beta T_\Sigma s}{(1 + \sqrt{\beta}\, T_\Sigma s)(1 + (\beta - \sqrt{\beta})\, T_\Sigma s + \beta T_\Sigma^2 s^2)}. & \text{(b)} \end{cases} \qquad (2.11)$$

It can be noticed that by imposing the conditions (2.8), CS performance will depend, for a given T_Σ (specific to the application), only on the value of parameter β (or m).

The controller tuning parameters are computed by means of some well specified relations in which, for the value of T_Σ only the value of β (or m) appears as parameter:

$$k_0 = 1/\alpha T_\Sigma^2 = 1/(\beta\sqrt{\beta}\, T_\Sigma^2), \quad k_r = 1(\beta\sqrt{\beta}\, k_{CP} T_\Sigma^2), \qquad (2.12)$$

respectively:

$$T_r = \beta T_\Sigma = m^2 T_\Sigma \quad \text{and} \quad T_r' = T_1. \qquad (2.13)$$

<u>Remark</u>: For $\beta = 4$ (m = 2) all the relations specific to symmetric optimum criterion after Kessler (SC-K) are found again, namely:

$$k_r = 1/(8 k_{CP} T_\Sigma^2), \quad T_r = 4\, T_\Sigma, \; T_r' = T_1. \qquad (2.14)$$

and:

$$H_0(s)_{opt} = \frac{1 + 4 T_\Sigma s}{8 T_\Sigma^2 (1 + T_\Sigma s)}, \qquad (2.15)$$

$$H_w(s)_{opt} = \frac{1 + 4 T_\Sigma s}{(1 + 2 T_\Sigma s)(1 + 2 T_\Sigma s + 4 T_\Sigma^2 s^2)} \qquad (2.16)$$

(see, for instance, [Bab85]).

CS performance with controller tuned according to the relations (2.12), (2.13) and (2.14) are determined by the pole-zero plot as follows:

- for the open-loop system:

$$p_{1,2} = 0, \quad p_3 = -1/T_\Sigma \quad \text{and} \quad z_1 = -1/\beta T_\Sigma; \qquad (2.17)$$

the double pole in origin $p_{1,2}$ guarantees that the CS will ensure zero steady-state error with respect to step and ramp variations of the reference input and zero static coefficient with respect to ramp variations, and zero static coefficient with respect to step variations of the disturbance input;

- for the closed-loop system:

$$p_{1,2}^* = -\frac{\beta - \sqrt{\beta}}{2\beta T_\Sigma} \pm \frac{\sqrt{\beta^2 - 2\beta\sqrt{\beta} - 3\beta}}{2\beta T_\Sigma},$$

$$p_3^* = -1/(\sqrt{\beta}\, T_\Sigma), \quad z_1^* = -1/(\beta T_\Sigma). \qquad (2.18)$$

Three categories of situations concerning the nature of poles $p_{1,2}^*$ occur depending on the value of β:

* $1 < \beta < 9$, for which:

$\delta(\beta) = \beta^2 - 2\,\beta\sqrt{\beta} - 3\,\beta < 0,$ (2.19-a)

when the poles $p_{1,2}^*$ are complex conjugated;

* $\beta = 9$, for which:

$\delta(\beta) = 0,$ (2.19-b)

and, accordingly, all the three system poles are real and equal:

$p_1^* = p_2^* = p_3^* = -1/3T_\Sigma;$ (2.19-c)

* $\beta > 9$, for which:

$\delta(\beta) > 0,$ (2.19-d)

when all the poles all real and distinct.

But, even in this last case for usual values of β, $9 < \beta \le 20$), the presence of the dominant zero z_1^* determines an oscillatory behaviour (the effect of such a zero is already well-known, [Căl75])

Remark: The shapes of root locus can be easily drawn for the three above mentioned categories of situations.

The expressions of open-loop Bode plots:

- the open-loop Bode magnitude plot (m.-p.):

$|H_0(j\omega)_{opt}|_{dB} = 20\ \lg|H_0(j\omega)_{opt}| = f_1(\lg\ \omega),$ (2.20-a)

- the open-loop Bode phase plot (p.-p.):

$\underline{/H_0(j\omega)}_{opt} = f_2(\lg\ \omega),$ (2.20-b)

where:

$$|H_0(j\omega)_{opt}| = \frac{|1 + \beta\, T_\Sigma\, j\, \omega|}{|\alpha\, T_\Sigma^2\, (j\omega)^2|\ |1 + T_\Sigma\, j\, \omega|}, \qquad (2.21)$$

$$\underline{/H_0(j\omega)}_{opt} = -\,\pi + \text{arctg}\left[\frac{\omega/\omega_0 - \omega/\omega_{0d}}{1 + \omega^2/(\omega_0\, \omega_{0d})}\right], \qquad (2.22)$$

with:

$\omega_0 = 1/T_\Sigma$ and $\omega_{0d} = 1/\beta T_\Sigma,$ (2.23)

point out the symmetry of these plots with respect to the crossover frequency (pulsation) ω_t:

$\omega_t = \sqrt{\omega_0\, \omega_{0d}} = 1/(\sqrt{\beta}\, T_\Sigma).$ (2.24)

For any value of β at this value ω_t the system phase reserve φ_r is maximum, and its value depends only on the value of β:

$\varphi_r(\beta) = \text{arctg}[(\beta - 1)/2\sqrt{\beta}\,].$ (2.25)

The situations are shown in Fig.2, as follows:

- the shapes of principle of the open-loop Bode plots $|H_0(j\omega)_{opt}|_{dB}$ and $\underline{/H_0(j\omega)}_{opt}$ are presented in Fig.2a;
- the dependence of the maximum value of phase reserve $\varphi_r(\beta)$ as function on the value of parameter β is shown in Fig.3b; this dependence can be efficiently used in the design by providing, for a desired value of the phase reserve φ_{rd}, the necessary value of β; then, on the basis of relations (2.12) and (2.13), the values of controller tuning parameters can be immediately computed.

The relation (2.25) and the notice that increasing of β leads to increasing of the phase reserve result in the following conclusions to be used in the design practice:

(1) Values such as $\beta < 4$ are not justified from the point of view of phase reserve φ_r; therefore, the designer should usually orient to the values $\beta \ge 4$;

(2) Increasing of φ_r is useful only up to the values $\varphi_{rm} \approx 60^0$, that can be translated in $\beta \le 16$, equivalent to $m \le 4$;

(3) For values of practical interest $4 \le \beta \le 16$ ($2 \le m \le 4$) increasing of phase reserve is accompanied by a translation to left of the crossover frequency ω_t; this crossover

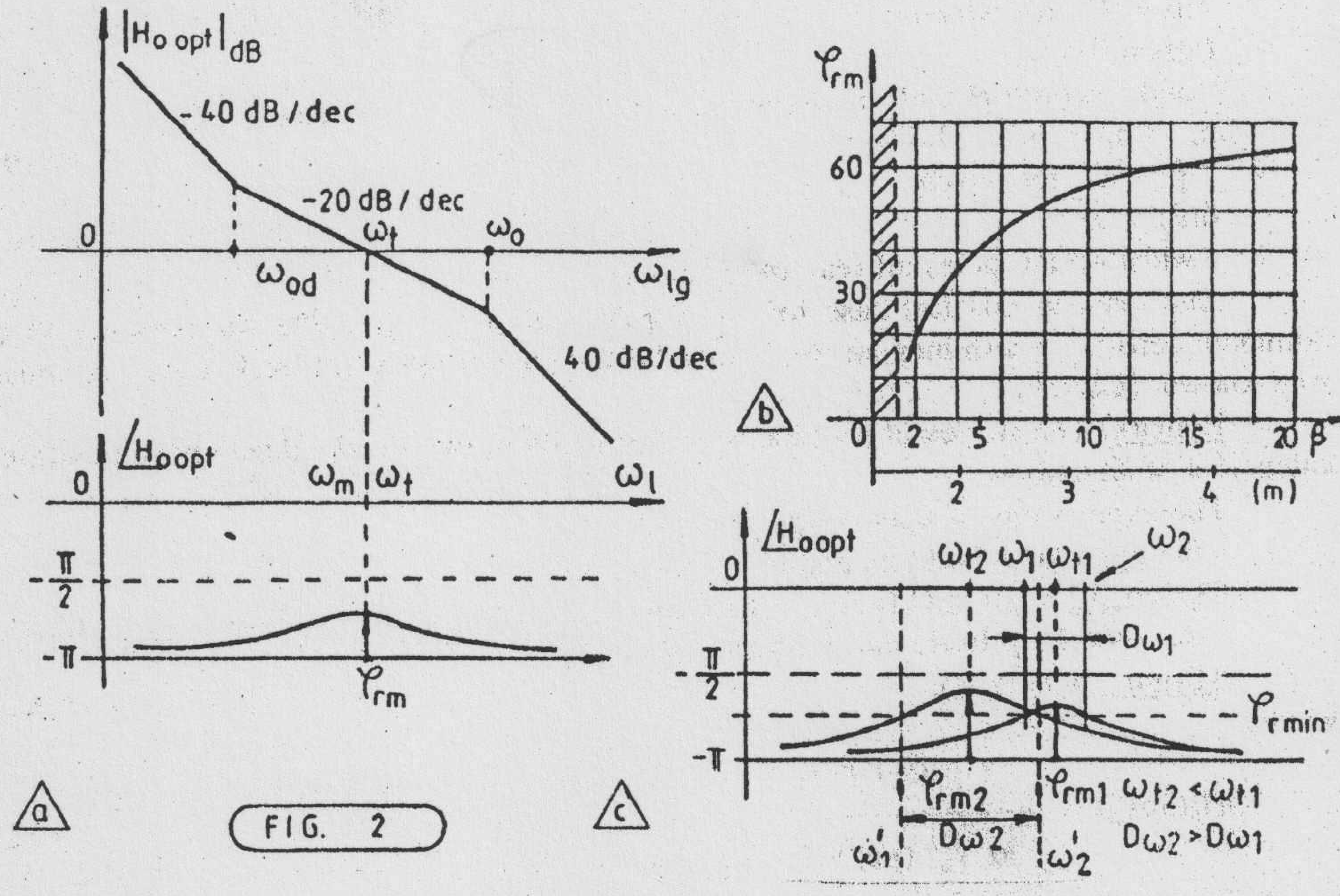

FIG. 2

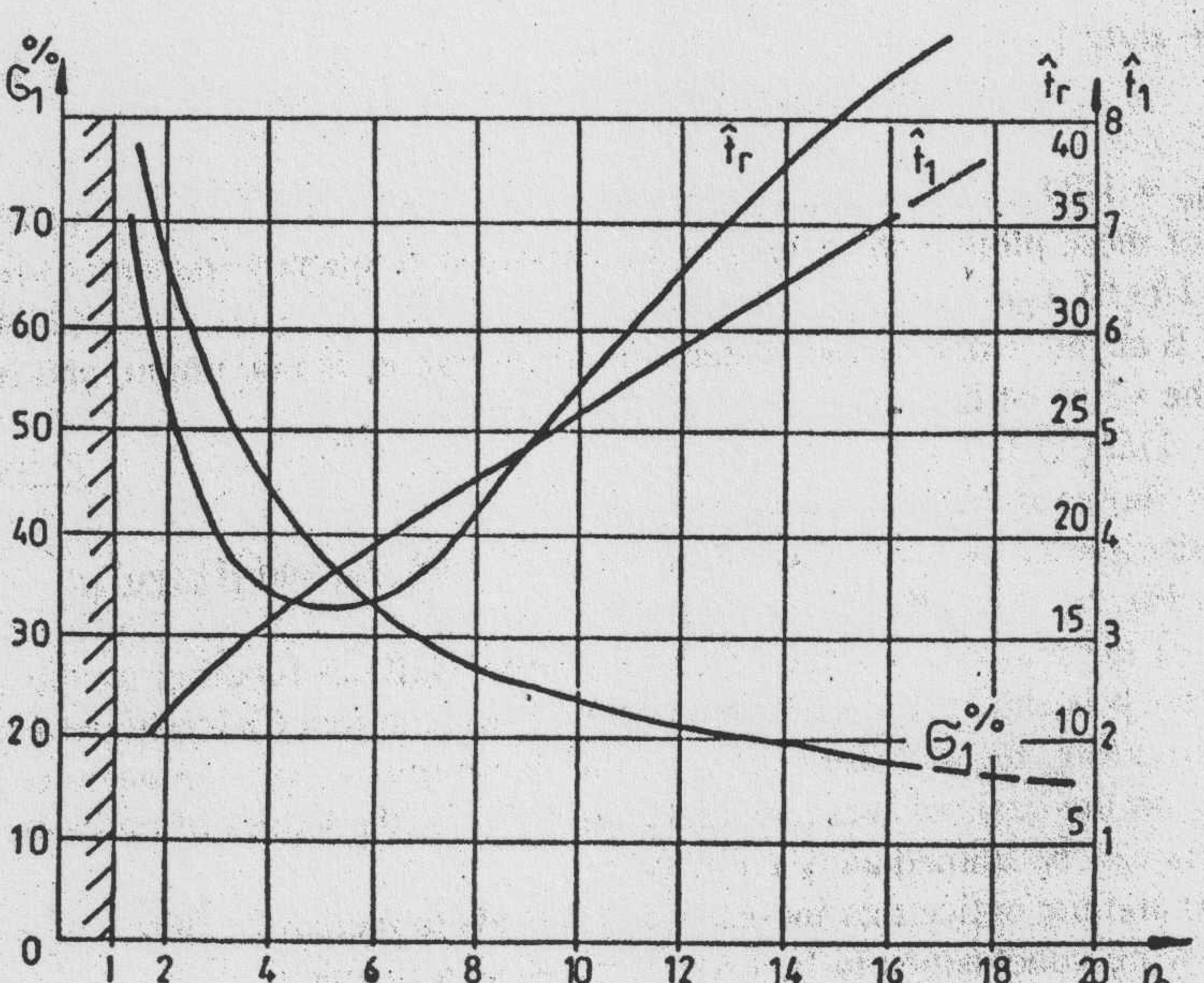

FIG. 3

frequency shift brings the following two well-known effects:

* decreasing of closed-loop system bandwidth;
* increasing of settling time t_r.

(4) By accepting that the parameter k_{CP} of CP can be subject to modifications within a known (or estimated) domain and that a minimum phase reserve φ_{rmin} is imposed to the system, it is relatively easy to determine the value of β for which the imposed condition is ensured, Fig.2c.

Performance of CS with the controller tuned on the basis of relations (2.12) and (2.13) can be appreciated by means of the empirical quality indexes defined in system response with respect to the step variation of reference input w, depicted as $y_{w\sigma}(t)$; Fig.3 presents diagrams with the values of quality indexes σ_1 - the overshoot, t_r - the settling time and t_1 - the first settling time as function of the value of parameter β. The last two indexes are expressed in per unit values (p.u.) with respect to the value of T_Σ, namely: $\hat{t}_r = t_r/T_\Sigma$ and $\hat{t}_1 = t_1/T_\Sigma$; the values $\hat{t}_r$ and $\hat{t}_1$ represent medium values which cannot point out the real existent discontinuities (e.g., see [Bab85]).

3. CORRECTION OF PRIMARY PERFORMANCE

There are situations when even improved performance achieved by the CS with the controller designed according to (2.14) ... (2.16) give no satisfaction with respect to the step variation of reference input: it is the case of overshoot σ_1 having acceptable values of 15 ... 25 % only for $\beta \geq 9$.

The most frequently used method for performance correction (that does not affect system phase reserve) is to fed the step reference input to a signal filter F-w (Fig.4) accordingly computed; this method is used also in the case of SC-K.

By analyzing the expression of t.f. $H_w(s)_{opt}$, relation (2.17), it can be noticed that the correction filter is able to cancel from one case to another:

- <u>in the first version</u> - the effect of the zero z_1^* and of the pole pair $p_{1,2}^*$; this version is justified only for values $\beta \leq 9$, when the poles are complex conjugated;
- <u>in the second version</u> - the effect of the zero z_1^*; this version is general, leading to overshoot σ_1 alleviation and to settling time t_r modification.

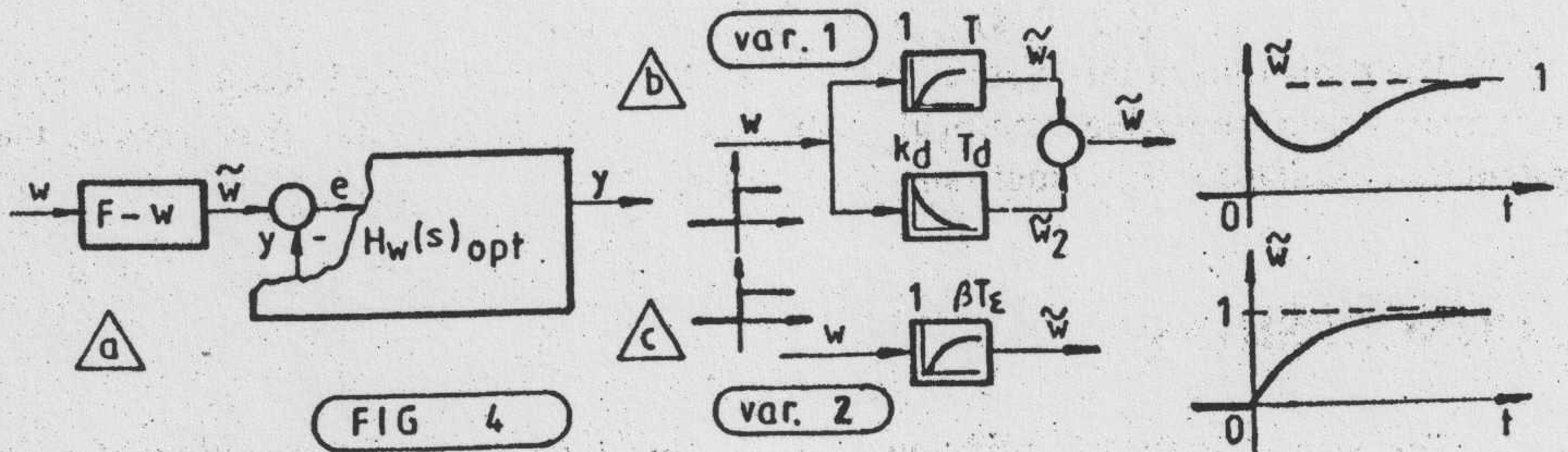

FIG 4

<u>The first version of filter F-w.</u> The t.f. of filter $H_{Fw}(s)$ is as follows according to the proposed goal:

$$H_{Fw}(s) = \frac{|p_f^*|\,|z_1^*|}{|p_1^*|\,|p_2^*|} \cdot \frac{(s - p_1^*)\,(s - p_2^*)}{(s - z_1^*)\,(s - p_f^*)}, \tag{3.1}$$

where the pole p_f^* guarantees the physical possibilty to construct the filter. The combination of elementary modules that make up the form (3.1) is a parallel connection of PT1 and DT1 filters, Fig.4b, with the t.f. and parameter expressions given by relation (3.2):

$$H_{PT1}(s) = \frac{1}{1 + \beta T_\Sigma s}, \quad H_{DT1}(s) = \frac{k_d \lambda T_\Sigma s}{1 + \lambda T_\Sigma s},$$

$$\text{with} \quad \lambda = 1/k_d = \beta - \sqrt{\beta} - 1 \quad \text{or} \quad T_d = \lambda T_\Sigma. \tag{3.2}$$

Accordingly, the t.f. of filter F-w will have the form:

$$H_{F\text{-}w}(s) = \frac{1 + (\beta - \sqrt{\beta})\, T_\Sigma\, s + \beta\, T_\Sigma^2\, s^2}{(1 + \beta\, T_\Sigma\, s)\, [1 + (\beta - \sqrt{\beta} - 1)\, T_\Sigma\, s]} \qquad (3.3)$$

Remark: The proposed filter can be used only for values $\beta > \sqrt{\beta} + 1$, i.e.

$\beta > \sqrt{(1 + \sqrt{5})/2}$; for $\beta = 4$ the well-known relations specific to SC-K are found again.

The dynamic properties of the equivalent system {filter + basic CS} in the relation w --> $\tilde{w}$ --> y, with the t.f.:

$$\tilde{H}_w(s)_{opt} = H_{Fw}(s)\, H_w(s)_{opt} = \frac{1}{(1 + \sqrt{\beta}\, T_\Sigma\, s)\, [1 + (\beta - \sqrt{\beta} - 1)\, T_\Sigma\, s]}, \qquad (3.4)$$

are easy to be computed, the equivalent system being aperiodic $\sigma_1 = 0$. The value of t_r can be appreciated by means of the approximate relation (3.5):

$$t_r \approx (3 \ldots 4) \sum_{v=1}^{2} T_v = (3 \ldots 4)\, (\beta - 1)\, T_\Sigma, \qquad (3.5)$$

where T_v are the two time constants belonging to the denominator of t.f. (3.4).

The second version of filter F-w. The t.f. of filter is as follows according to the proposed goal (Fig.4c):

$$H_{Fw}(s) = 1/(1 + \beta\, T_\Sigma\, s). \qquad (3.6)$$

So, the overall system in the relation w --> $\tilde{w}$ --> y will be characterized by the t.f.:

$$\tilde{H}_w(s)_{opt} = \frac{1}{(1 + \sqrt{\beta}\, T_\Sigma\, s)\, [1 + (\beta - \sqrt{\beta})\, T_\Sigma\, s + \beta\, T_\Sigma^2\, s^2]}. \qquad (3.7)$$

The properties of this class of systems are frequently presented in references by means of diagrams concerning the empirical quality indexes σ_1, t_r, t_1. It has to be noticed that the system has oscillatory behaviour only for values $\beta < 9$.

4. CONCLUSIONS

The large variety of conditions that can be imposed to the operation of a control system make the design methods of type "recipe" to become rigid and to give no fully satisfaction. The generalization of some design relations agreed in references, e.g. the relations given by the symmetric optimum criterion after Kessler, even for a particular class of systems provides the designer additional degrees of freedom to be used when a convenient control solution is chosen.

From this point of view the method presented in the paper has the following advantages:

- the possibility to use some previous computed relations for controller parameter tuning, relations (2.12) and (2.13), where the value of parameter β can be chosen by a compromise between the set of imposed and reachable performance for different values of β;

- the control system performance, with β in the role of parameter, can be pointed out in the form of some diagrams concerning the empirical quality indexes defined in the time domain σ_1, t_r, t_1, or in the frequency domain φ_r;

- for every adopted value of β the resulted tuning parameters ensure the maximum possible system phase reserve, relation (2.25); this property is based just on the symmetry of open-loop Bode plots due to the general conditions (2.8).

The main application domain of the proposed design method remains the domain specific also to the application of SC-K - electrical drives - with the remark that the latter becomes only a particular case corresponding to the value $\beta = 4$ or $m = 2$.

ABBREVIATIONS

SC-K - symmetric optimum criterion in Kessler's version;
t.f. - transfer function;
CS - control system;
C - controller (regulator);
CP - controlled plant;
PT1 - proportional with first order lag block;
PDT1 - lead-lag or lag-lead block;
PI - proportional-integral (controller);
PID - proportional-integral-derivative (controller).

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