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Analytical Design of Robust Control Systems with Desirable Performances

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Abstract ***- In paper the analytical design method of the systems with control on output and impacts is offered. The resulting system has partially given structure, desirable performances, lowered dimension and increased robustness. Parameters of the system's controller are the solutions of the linear algebraic equations systems. Standard transfer functions are used for maintenance of the desirable quality parameters such as: the astatic orders to reference input and external disturbances; overshoot and settling time. Increase of the robustness is achieved by a reduction of the plant model and inclusion of a part of its poles and zero in roots of the closed system characteristic polynomial. The suggested method of analytical design can be applied in aggregate with the dynamic decomposition method for creation of the multivariable control systems. Efficiency of the analytical design method and the robustness increase method of the control systems by the reduction of the plant model are shown on the numerical examples. These methods can be used for creation of the systems with less complex, but more robust for plants of chemical, textile, food and other branches of production.***

Key words: ***plant, control system, design, performance, astatic, invariancy, reduction, robustness***

I. INTRODUCTION

The scope of control systems is very wide now [1-4]. Therefore they should be created with the minimal application of manual skills as it is the least effective. In this connection the computer aided design systems are the most effective way for qualitative of the control systems creation [1, 4]. Creation of such systems demands development of the analytical design methods, in particular, the systems with the direct quality parameters such as: the astatic order to reference input and external disturbances; overshoot, settling time and small fluctuation. These quality parameters full reflect of the engineering requirements to the control systems.

Now the simple laws of control *P*, *PI*, *PD*, etc. are applied more often [5-7]. Usually these laws get out a priori, but quality of a received control systems not always meets to requirements, because the possibility of these laws is bounded. Besides these simple control laws do not allow to use a possibility of the modern computer technologies to the full. Methods of a LQG control demand the formation of the quadratic criterion which parameters are connected with the direct quality parameters very uncertain. The method of modal control is analytical also [8-11]. Its parameters can be found or by the Ackerman formula, or by transformation of the system model to canonic controlled form [10, 12]. However modal control allows changing poles placement, leaving non change zeros of the plant's transfer function, therefore maintenance of the required values of quality parameters, similarly the optimal control is possible only by the iterative way. Frequently numbers of these iterations are big, that generate delays at creation of the effective control systems.

In report the rather new method of the analytical design of the control systems with partially given structure is considered. This method is focused on controllers which use a new control principle: «control on output and impacts» [12-17]. This principle differs from frequently used control principle on a deviation that the reference input and output variable are processed in the controller by different operators. Therefore even if the plant is one-dimensional and full the controller on output and impacts, generally, has several inputs and the closed control system is not full. The controllers on output and impacts are a little bit more complexity, than controllers on a deviation. But they allow to carry out independent control of the poles and zero of the closed system and to provide desirable values of the quality parameters in transitive and the steady-state mode. Control on output and impacts also allows taking into account conditions of a physical realizability of the controller. On the other hand, this control allows using more full the possibilities of the modern computer technologies at solution of the control problems. Parameters of the controller on output and impacts are determined analytically, i.e. by solution of the algebraic equations systems [13, 15, 16]. Considered method of the analytical design of the systems with control on output and impacts (ADS with COI) allows providing desirable performances in the transitive and the steady state mode [15-17]. Known standard transfer functions are used at design of control systems by this method [12, 19].

In practice factors of the plant model are known not precisely usually. Therefore control systems should be robust stable. This property is reduced with increase of the system's order [3, 10, 21-26]. It is offered to apply a reduction of the closed control systems to increase of their robust stability. One of the reduction methods is downturn order of the plant mathematical models that allows increasing in two-three times robustness of the control systems. In the report it is shown, that the most effective way increasing of the control systems robustness is a modal reduction method [27]. In aggregate with a method of dynamic decomposition method ADS with COI can be applied and in case of multi-variable plants. Efficiency of the considered analytical design of the control systems is illustrated by numerical examples. Higher robust stability of the reduced control systems also is shown on the numerical examples.

II. STATEMENT OF PROBLEM

It is supposed, that a plant is described by the equations:

$$\dot{x} = Ax + b_0 u + b_1 f_1 + b_2 f_2 , \quad (1)$$

$$y = c^T x + \beta_0 u + \beta_1 f_1 + \beta_2 f_2 , \quad (2)$$

where $x = [x_1 \dots x_n]^T$ is the vector of state variables; $f_1 = f_1(t)$ and $f_2 = f_2(t)$ are the measured and unmeasured disturbances; A and b_0, b_1, b_2, c are numerical matrix and vectors; β_0, β_1, β_2 are numbers [12, 13, 18, 20].

The equations of the controller, forming control on output and impacts (COI), look like:

$$\dot{z} = Rz + q_0 g - du - ly + q_1 f_1 , \quad (3)$$

$$u = k^T z + \vartheta_0 g - \theta u - \lambda y + \vartheta_1 f_1 , \quad (4)$$

where z is a state vector; g is reference input of the control system; f_1 is measured disturbance; R and q_0, d, l, q_1, k are constants matrix and vectors; ϑ_0, θ, λ and ϑ_1 are factors.

The closed system (1) – (4), Fig. 1, is described with $\gamma_0^{-1} = 1 + \theta + \lambda\beta_0 \neq 0$ by the equations:

$$\dot{w} = Dw + h_0 g + h_1 f_1 + h_2 f_2 , \quad (5)$$

$$y = a^T w + \eta_0 g + \eta_1 f_1 + \eta_2 f_2 , \quad (6)$$

where w is the state vector with dimension n_{sys}.

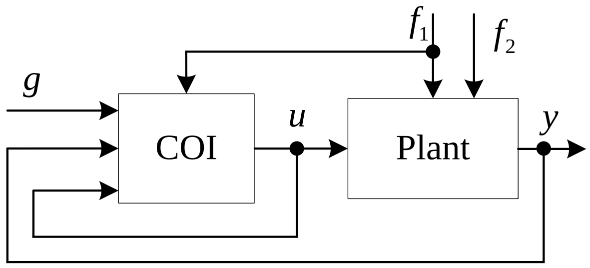

Fig. 1 The circuit of system with control on output and impacts

The matrix D, vectors h_0, h_1, h_2, a and numbers η_0, η_1, η_2 are determined in (5), (6) by the following expressions:

$$D = \begin{bmatrix} A - b_0\gamma_0\lambda c^T & b_0\gamma_0 k^T \\ [d\gamma_0\lambda - l(1-\beta_0\gamma_0\lambda)]c^T & R - (d + l\beta_0)\gamma_0 k^T \end{bmatrix},$$

$$h_0 = \begin{bmatrix} b_0\vartheta_0\gamma_0 \\ q_0 - (d + l\beta_0)\vartheta_0\gamma_0 \end{bmatrix},$$

$$h_1 = \begin{bmatrix} b_1 + b_0\gamma_0(\vartheta_1 - \lambda\beta_1) \\ q_1 - l\beta_1 - (d + l\beta_0)\gamma_0(\vartheta_1 - \lambda\beta_1) \end{bmatrix},$$

$$h_2 = \begin{bmatrix} b_2 - b_0\gamma_0\lambda\beta_2 \\ [(d + l\beta_0)\gamma_0\lambda - l]\beta_2 \end{bmatrix},$$

$$a^T = \begin{bmatrix} (1-\beta_0\gamma_0\lambda)c^T & \beta_0\gamma_0 k^T \end{bmatrix}, \ \eta_0 = \beta_0\gamma_0\vartheta_0 ,$$

$$\eta_1 = \beta_1 + \beta_0\gamma_0(\vartheta_1 - \lambda\beta_1) , \ \eta_2 = \beta_2(1 - \beta_0\gamma_0\lambda) . \quad (7)$$

$$n_{sys} = n + r . \quad (8)$$

Parameters of the of the closed system (5), (6), the plant (1), (2) and the controller (3), (4), contain in expressions (7), therefore these expressions can be used for definition of the structure and parameters of the controller. In this case for solution of the design problem it is necessary to form matrixes and vectors of the equations (5), (6) [8, 18]. However, first, the desirable kind of the matrixes and vectors (7) is difficult for substantiation. Second, the solution of the design problem as the equations (3), (4) is no unique since many equivalent systems are described by these equations.

With the purpose of narrowing set of the design problem solution it is expedient passing to the equations "input-output". The operational "input-output" equations of the plant (1), (2), the controller (3), (4) and the closed system (5), (6) look like:

$$A(p)y = B_0(p)u + B_1(p) f_1 + B_2(p) f_2 , \quad (9)$$

$$\bar{R}(p)u = Q_0(p)g - L(p)y + Q_1(p) f_1 , \quad (10)$$

$$D(p)y = H_0(p)g + H_1(p) f_1 + H_2(p) f_2 . \quad (11)$$

Polynomials in the equations (9)-(11) are determined by well-known expressions from the equations (1)-(7). The following equality can be received from the equations (9)-(11):

$$D(p) = A(p)\bar{R}(p) + B_0(p)L(p) , \quad (12)$$

$$H_0(p) = B_0(p)Q_0(p) , \quad (13)$$

$$H_1(p) = B_0(p)Q_1(p) + B_1(p)\bar{R}(p) , \quad (14)$$

$$H_2(p) = B_2(p)\bar{R}(p) . \quad (15)$$

These expressions contain also, on the one hand, the polynomials $D(p)$, $H_j(p)$ of the closed system, and, on the other hand, the polynomials $A(p)$, $B_0(p)$ and $B_i(p)$ of the plant and the polynomials $\bar{R}(p)$, $L(p)$ and $Q_j(p)$ of the controller. After replacement in equality (12)-(15) the polynomials $D(p)$, $H_j(p)$ by desirable polynomials

$D^*(p),\ H_j^*(p)$, these expressions become **the resolving equations** of the analytical design problem of the linear system with partially given structure. In these equations the polynomials $D^*(p),\ H_j^*(p)$ are denominators and numerators of the desirable transfer functions:

$$W_{yg}^*(p) = \frac{H_0^*(p)}{D^*(p)},\ W_{yf_j}^*(p) = \frac{H_j^*(p)}{D^*(p)},\ j = 1, 2, \quad (16)$$

which can be appointed on the base of requirements to performances of the designing closed system. In other words, expressions (12)-(15) are the equations concerning the polynomials $\bar{R}(p)$, $L(p)$ and $Q_j(p)$, which determine the equation "input-output" (10) of the controller. The found equation (10) of the controller must be realized as one dynamic block with one output and several inputs (see Fig. 1). This moment is very important as otherwise the order of the received controller will be equal 3*r* install *r*, i.e. is much higher.

The polynomial equations (12)-(15) are equivalent to the systems of the linear algebraic equations that provides analytical character of the considered analytical design control systems. In particular, the following systems

$$\begin{bmatrix} \beta_0 & 0 & 0 & \alpha_0 & 0 & \cdots & 0 \\ \beta_1 & \beta_0 & \ddots & \alpha_1 & \alpha_0 & 0 & \vdots \\ \vdots & \beta_1 & \ddots & \vdots & \alpha_1 & \ddots & 0 \\ \beta_{m_0} & \vdots & \ddots & \alpha_n & \vdots & \ddots & \alpha_0 \\ 0 & \beta_{m_0} & \ddots & 0 & \alpha_n & \ddots & \alpha_1 \\ \vdots & 0 & \ddots & \vdots & 0 & \ddots & \vdots \\ 0 & \vdots & & 0 & \vdots & \ddots & \alpha_n \end{bmatrix} \begin{bmatrix} \lambda_0 \\ \lambda_1 \\ \vdots \\ \lambda_{r_L} \\ \rho_0 \\ \vdots \\ \rho_r \end{bmatrix} = \begin{bmatrix} \delta_0^* \\ \delta_1^* \\ \delta_2^* \\ \vdots \\ \vdots \\ \vdots \\ \delta_{n_{D^*}}^* \end{bmatrix},$$

$$\begin{bmatrix} \beta_0 & 0 & 0 \\ \beta_1 & \beta_0 & 0 \\ \vdots & \beta_1 & \ddots \\ \beta_{m_0} & \vdots & \ddots \\ 0 & \beta_{m_0} & \ddots \\ \vdots & \vdots & \beta_{m_0} \end{bmatrix} \begin{bmatrix} \vartheta_{00} \\ \vartheta_{01} \\ \vdots \\ \vartheta_{0r_0} \end{bmatrix} = \begin{bmatrix} \eta_{00}^* \\ \eta_{01}^* \\ \vdots \\ \eta_{0m_{H_0}}^* \end{bmatrix} \quad (17)$$

correspond to the equations (12) and (13). Here β_{0i}, α_i, λ_i, ρ_i, δ_i^*, ϑ_{0i} and η_{0i}^* are factors of the polynomials $B_0(p)$, $A(p)$, $L(p)$, $\bar{R}(p)$, $D^*(p)$, $Q_0(p)$ and $H_0^*(p)$, accordingly [12, 13].

Linearity of the algebraic equations systems equivalent to the ratio (12)-(15) is the major feature of the given approach. Condition, at which these algebraic systems have solutions concerning the polynomials of the controller (10) are resolvability conditions of the analytical design problem. The condition of the controller's physical realizability is

$$\mu_{cd} \geq \mu_{cd}^*, \quad (18)$$

where μ_{cd} is the controller's relative order determined by equality

$$\mu_{cd} = \min\{r - r_L,\ r - r_j \mid j = 0,\ 1\}. \quad (19)$$

μ_{cd}^* is the value of the controller's relative order, admissible on the realizability conditions; $r = \deg \bar{R}(p)$, $r_L = \deg L(p)$, $r_j = \deg Q_j(p)$.

In practice is accepted $\mu_{cd}^* = 0$ or $\mu_{cd}^* \geq 1$ more often. In the first case the controller can have direct uninertial the "input-output" channels. In the second case such channels are not supposed. According to (18) - (19) the relative order of the closed system (11) is determined by next expression

$$\mu_{sys}^* = \deg D^*(p) - \deg H_0^*(p). \quad (20)$$

Thus, for design of a control system, first of all, it is necessary to form polynomials $D^*(p)$ and $H_j^*(p)$, $j = 0, 1, 2$ by which the close control system with partially given structure (11), (16) have desired performances and corresponding equation of the controller (10) satisfies conditions (18), (19). This task has solution, if next conditions are satisfied [17]:

i) all coefficients (roots) of the polynomial $D^*(p)$ and partially of the polynomials $H_j^*(p)$, $j = 0, 1, 2$ can be appointed according to desirable performance of the control system, with the transfer function (16);

ii) the control system (9), (10) or (11) has desirable performance, if $D(p) = D^*(p)$ and $H_j(p) = H_j^*(p)$, $j = 0, 1, 2$;

iii) the equations system (12)-(15) has the mathematical solution relative to the polynomials $\bar{R}(p)$, $L(p)$, $Q_0(p)$ and $Q_1(p)$;

iv) the polynomials $\bar{R}(p)$, $L(p)$, $Q_0(p)$ and $Q_1(p)$ sat isfy to the conditions (18), (19).

III. CONTROL SYSTEMS ANALYTICAL DESIGN

The considered design method essentially depends from the assignment way of the roots of the system's characteristic polynomial (further they refer to as system's poles). If the system's poles are appointed without taking into account properties of the plant it refers as «*system with independent poles*». If the system's poles are appointed so, that the part from them coincides with zero or poles of the plant transfer function, the system refers as «*system with coordinated poles*». In the given report we shall be bounded to consideration of the systems with the coordinated poles, as in this case the realizability conditions of a transfer functions least rigid [12, 17].

Poles of the qualitative control systems usually settle down in some area $\Omega_п$ of the left part of the complex plane [18, 19], therefore polynomials $A(p)$ and $B_0(p)$ from the equation (9) are factorized as follows:

$$A(p) = A_{\Omega}(p)A_{\bar{\Omega}}(p)\,,\; B_0(p) = \beta_{m_0} B_{\Omega}(p)B_{\bar{\Omega}}(p), \quad (21)$$

where $A_{\Omega}(p)$, $A_{\bar{\Omega}}(p)$ and $B_{\Omega}(p)$, $B_{\bar{\Omega}}(p)$ are the polynomials normalized on the senior degree of *p*; β_{m_0} is factor of the polynomial $B_0(p)$ at the senior degree *p*. Here $A_{\Omega}(p)$ and $B_{\Omega}(p)$ are polynomials, which roots are equal to the roots of the polynomials $A(p)$ and $B_0(p)$ located in area Ω_{π}. All roots of the polynomials $A_{\Omega}(p)$ also $B_{\Omega}(p)$ are included in number of roots of the closed system characteristic polynomials. Generally each of the polynomials $A_{\Omega}(p)$, $A_{\bar{\Omega}}(p)$, $B_{\Omega}(p)$ and $B_{\bar{\Omega}}(p)$ can be equal 1.

In case of the systems with coordinated poles the polynomials from the controller's equation (10) undertake as:

$$\bar{R}(p) = B_{\Omega}(p)\tilde{R}(p)\,,\; L(p) = A_{\Omega}(p)\tilde{L}(p)\,,$$
$$Q_j(p) = A_{\Omega}(p)M_{\Omega}(p)\tilde{Q}_j(p)\,, \quad (22)$$

where $\tilde{R}(p)$, $\tilde{L}(p)$, $\tilde{Q}_j(p)$, j = 0, 1 and $M_{\Omega}(p)$ are the auxiliary polynomials determined during the solution of a design task.

From expressions (9) (10) and (22) is follows, that polynomials $D(p)$ and $H_0(p)$ of the system (11) with coordinated poles and transfer function $W^*_{yg}(p) = H^*_0(p)/D^*(p)$ look like

$$D(p) = A_{\Omega}(p)B_{\Omega}(p)D^*(p)M_{\Omega}(p)\,, \quad (23)$$
$$H_0(p) = A_{\Omega}(p)B_{\Omega}(p)H^*_0(p)M_{\Omega}(p)\,. \quad (24)$$

The polynomials $\tilde{R}(p)$, $\tilde{L}(p)$ and $\tilde{Q}_j(p)$ in equality (22) are determined by the solution of the algebraic systems, corresponding to the polynomials equations (12) – (15) in view of the equality (21) – (22) [12, 16].

In [17] it has been shown the conditions i) – iv) can be executed, if polynomials of the desirable transfer function $W^*_{yg}(p) = H^*_0(p)/D^*(p)$ (16) satisfy to the conditions:

$$\mu^*_{yg} = \deg D^*(p) - \deg H^*_0(p) \geq \mu_{pl} + \mu^*_{cd}\,,$$
$$H^*_0(p) = B_{\bar{\Omega}}(p)\bar{H}^*_0(p)\,, \quad (25)$$
$$n_{sys} \geq 2n + \mu^*_{cd} - 1\,, \quad (26)$$

and the degree of the polynomial $M_{\Omega}(p)$ in the equality (22) – (24) is determined by expression

$$\deg M_{\Omega}(p) = \max\{0;\; 2n + \mu^*_{cd} - 1 - n_D\}\,, \quad (27)$$

where $\mu_{pl} = n - m_0$; $n_D = \deg[A_{\Omega}(p)B_{\Omega}(p)D^*(p)]$. In expressions (25) $\bar{H}^*_0(p)$ is a multiplier of the polynomial $H^*_0(p)$.

With purpose of maintenance some order astatic or selective invariancy in relation to the reference input and external disturbance, the polynomial $\tilde{R}(p)$ in expressions (22) it is possible to take as $\tilde{R}(p) = \Phi(p)\vec{R}(p)$, and polynomials $\tilde{L}(p)$, $\tilde{Q}(p)$ so that $\tilde{L}(p) - \tilde{Q}_0(p) = G(p)\vec{L}(p)$, where $\vec{R}(p)$ and $\vec{L}(p)$ are some polynomials. Here the polynomial $\Phi(p) = \mathrm{LCM}\{G(p), F_1(p), F_2(p)\}$ and polynomials $G(p)$, $F_1(p)$, $F_2(p)$ are *K*(*p*)-image of the reference input *g*(*t*) and the disturbances $f_1(t)$, $f_2(t)$ [12, 17]. At such choice of the polynomial $\tilde{R}(p)$, the condition (26) is replaced by inequality $n_{sys} \geq 2n + \deg\Phi(p) + \mu^*_{yy} - 1$. However, if $f_1(t) \equiv 0$ and (or) $f_2(t) \equiv 0$, then $F_1(p) = 1$ and (or) $F_2(p) = 1$.

Under these conditions from the expressions (11) – (13), (16) and (21) – (24) is follows the transfers function on error from the reference input looks likes:

$$W_{\varepsilon g}(p) = \frac{\vec{H}_0(p)G(p)}{D^*(p)}\,, \quad (28)$$

where $\vec{H}_0(p) = A_{\bar{\Omega}}(p)\Phi_0(p)\vec{R}(p) + \beta_{m_0}B_{\bar{\Omega}}(p)\vec{L}(p)$ there is the polynomial. Part of its factors can be appointed with purpose to give desirable properties to the closed system on the channel $g \to y$; the polynomial $\Phi_0(p) = G^{-1}(p)\Phi(p)$.

If the polynomial $Q_1(p) = F_1(p)\vec{Q}_1(p)$, then the transfer functions $W^*_{yf_j}(p)$, j = 1, 2 (16) on the measured and unmeasured disturbances, are realized by the system (9), (10) or (11) look like:

$$W^*_{yf_1}(p) = \frac{\tilde{H}_1(p)F_1(p)}{A(p)D^*(p)M_{\Omega}(p)}\,,$$
$$W^*_{yf_2}(p) = \frac{\vec{R}_2(p)B_2(p)F_2(p)}{A(p)D^*(p)M_{\Omega}(p)}\,. \quad (29)$$

Here $\tilde{H}_1(p) = \beta_{m_0}B^+(p)\vec{Q}_1(p) + B_1(p)\Phi_1(p)\vec{R}(p)$ there is the polynomial; part of its factors also can be appointed with the purpose to give desirable properties to the closed system on the channel $f_1 \to y$; $\Phi_1(p) = F_1^{-1}(p)\Phi(p)$, and $\vec{R}_2(p) = F_2^{-1}(p)\tilde{R}(p)$. On the channel $f_2 \to y$, i.e. on the unmeasured disturbances, by a choice of the polynomial $F_2(p)$ it is possible to provide or the some order of astatic, or the selective invariancy [12]. We shall note that known difficulties of the systems stability maintenance of the high astatic order do not arise here, owing to application of the principle of control on output and impacts.

Expressions (25) and (26), (27) represent the realizability conditions of the transfer function $W^*_{yg}(p)$ (16) by system with partially given structure. We shall note that conditions (25) are well-known and are resulted in works of J.Z. Tsypkin, C.T Chen and other authors [28, 18]. Conditions (26), (27) are received in works [13, 15] and in earlier works did not meet.

The additional conditions (26), (27), actually, provide, first, resolvability of the polynomial equations (12) concerning the polynomials $\bar{R}(p)$, $L(p)$ under condition (18), (19); and second, an opportunity to give to all factors of the polynomial $D^*(p)$ any values, proceeding from desirable performances of the designing system. The polynomial $M_\Omega(p)$ is entered in the equality (22) – (24) for increase up to necessary size the order of the system, which realized of the transfer function $W^*_{yg}(p)$, if this function satisfies to the conditions (25), and $\deg D^*(p)$ satisfies to the next condition

$$\deg D^*(p) < \mu_{pl} + \mu^*_{cd} + m_{\bar{\Omega}} + n_{\bar{\Omega}} - 1,$$

where $m_\Omega = \deg B_\Omega(p)$, $m_{\bar{\Omega}} = \deg B_{\bar{\Omega}}(p)$, $n_{\bar{\Omega}} = \deg A_{\bar{\Omega}}(p)$. It is possible to show, that if the conditions (25) are satisfied and $\deg D^*(p) > \mu_{pl} + \mu^*_{cd} + \deg[B_{\bar{\Omega}}(p) A_{\bar{\Omega}}(p)] - 1$, then according to (27) $\deg M_\Omega(p) = 0$, i.e. in these cases it is possible to believe $M_\Omega(p) = 1$.

The expressions (12) – (29) represent the algorithmic base of the analytical design systems with control on output and impacts (ADS with COI). In aggregate with a method of dynamic decomposition this approach can be applied and in case of the multivariable plants [12]. Examples of their application are resulted below.

IV. EXAMPLES OF CONTROL SYSTEMS DESIGN

For an illustration of opportunities of the submitted method ADS with COI we shall consider two examples.

Example 1. For the plant is described by the equation

$$(p^3 + 0{,}8p^2 - p)y = 75u + (0{,}1p - 2{,}5) f_2$$

to design a control system with 5-th astatic order to reference input and 4-th order to unmeasured disturbance f_2. The controller's relative order $\mu^*_{yy} = 0$. Desired settling time $t_s \le t^*_s = 5$ s and overshoot $\sigma \le \sigma^* = 25\,\%$. The deviation $\varepsilon = g - y$ and output y are measured.

Solution. In this case under expressions (25) – (28) we find: the polynomial $\Phi(p) = p^4$, $\deg M_\Omega(p) = 0$, $r = 6$, $n_{sys} = 9$. Transfer function of a 9-th order system with 5-th astatic order in the literature is absent. Therefore desirable transfer function it will be generated from a third astatic order standard transfer function of the sixth order systems. Resulting desirable transfer function looks like:

$$W_{yg}(p) = \frac{C(p)}{p^9 + 92p^8 + 3503p^7 + 57060p^6 + 420005p^5 + C(p)},$$

where

$$C(p) = 785861p^4 + 435992p^3 + 80401p^2 + 5134p + 300.$$

Corresponding transient response is resulted on Fig. 2. The system has required t_s and σ % evidently.

The result of corresponding calculations under formulas (12) – (16) and (28), (29) is the following "input-output" equation of the controller:

$$(p^6 + 91{,}2p^5 + 3431p^4)u = (4 + 68{,}45p + 1072p^2 + \\ +5813p^3 + 10478p^4)\varepsilon - (5646p^5 + 725{,}4p^6)y. \quad (30)$$

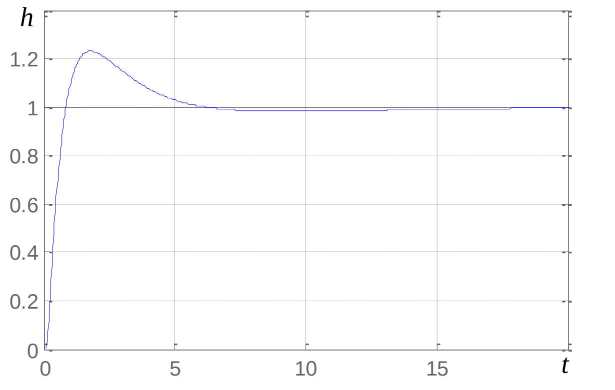

Fig. 2. Transient response of astatic system

The circuit corresponding to the equation (30) is resulted on Fig. 3.

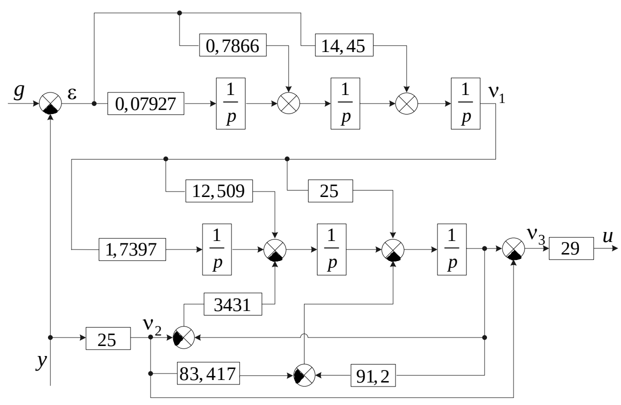

Fig. 3. Controller's circuit of the system with 5-th astatic order

The circuit on Fig. 3 is received by transformation of the equation (30) to the equations in state variables with application of the canonical observable form [12].

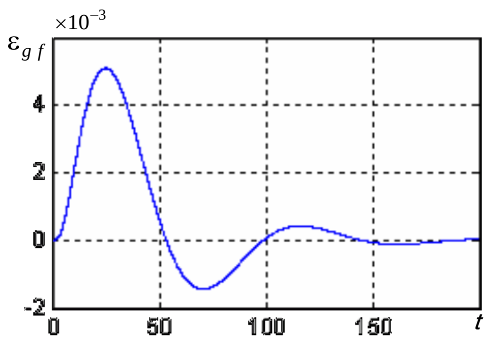

Fig. 4. System's deviations with impacts: $g(t) = 2{,}5 \cdot 10^{-6} t^4$, $f(t) = 2 \cdot 10^{-4} t^3$

For an estimation of the systems real astatic order its simulation carried out in MATLAB. The received schedules of deviations are resulted on Fig. 4 and Fig. 5.

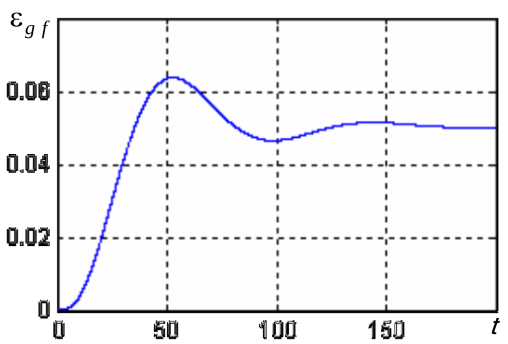

Fig. 5. System's deviations with impacts $g(t)=10^{-5}t^3+3\cdot10^{-7}t^5$, $f(t)=2\cdot10^{-4}t^3$

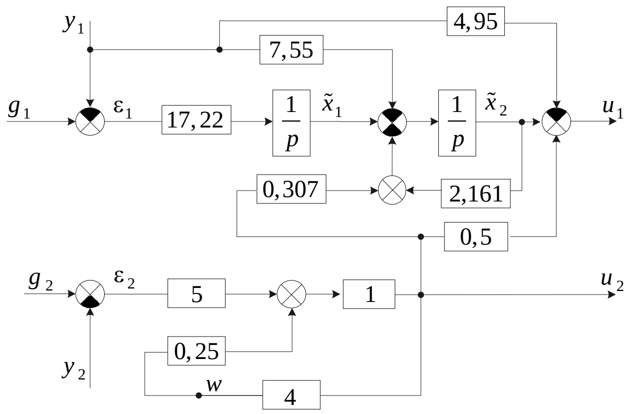

Fig. 6. Circuit of the invariant system's controller

Results of simulation testify that the designed system has required astatic orders, really.

Let's show, that suggested method ADS with COI can be applied to design of multivariable control systems.

Example 2. To design absolutely invariant control system for an aircraft power-plant describing by following equations:

$$A(p)y_1=(0,62p+1,34)u_1-(0,31p+0,48)u_2\,;$$
$$A(p)y_2=(0,14p^2+0,41p+0,29)u_1-$$
$$-(0,18p^2+0,51p+0,32)u_2\,, \qquad (31)$$

where $A(p)=0,6074p^2+1,6671p+1$; y_1 and y_2 are controlled variables; u_1 and u_2 are controls of the aircraft power-plant [21]; the controlled output y_i and the deviations $\varepsilon_i=g_i-y_i$, $i=1,2$ are measured. The controller's relative order $\mu^*_{yy}\geq 0$.

Solution. The controls u_1 and u_2 can be considered as disturbances for the channels $g_2\to y_2$ and $g_1\to y_1$ accordingly. In this case in relation to the variable y_2 the approachibility condition of the absolute invariancy on G.V. Shchipanov's criterion, and in relation to the variable y_1 – on the B.N. Petrov's two channel criterion are satisfied [12].

As a result of the considered method application at according to the expressions (12) – (29) and (31) the following controller's equations are received:

$$\dot{\tilde{x}}_1=17,2223\varepsilon_1\,;$$
$$\dot{\tilde{x}}_2=\tilde{x}_1-2,1613\tilde{x}_2-7,5508y_1-0,30646u_2\,;$$
$$u_1=\tilde{x}_2-4,9525y_1+0,5u_2,\quad u_2=5\varepsilon_2+0,25w,\quad w=4u_2. \; (32)$$

The circuit corresponding to the equations (32) is resulted on Fig. 6.

Simulation of the system (31), (32) was carried out also in MATLAB; on Fig. 7 – Fig. 9 the schedules received at zero initially conditions and reference inputs $g_1=1+0,2t$ and $g_2=2\sin t$ are resulted. The controlled variable y_2 completely coincides with reference input g_2 (see Fig. 7,*b*), i.e. the channel $g_2\to y_2$ is absolutely invariant to unmeasured disturbance u_1, really.

The deviation ε_2 actives on the second input of the controller, is practically equal to zero at all t (see Fig. 8,*b*). Despite of this, boundary oscillatory control u_2 opposite to the reference input g_2 is observed on the second output u_2 of the controller (see Fig. 9,*b*).

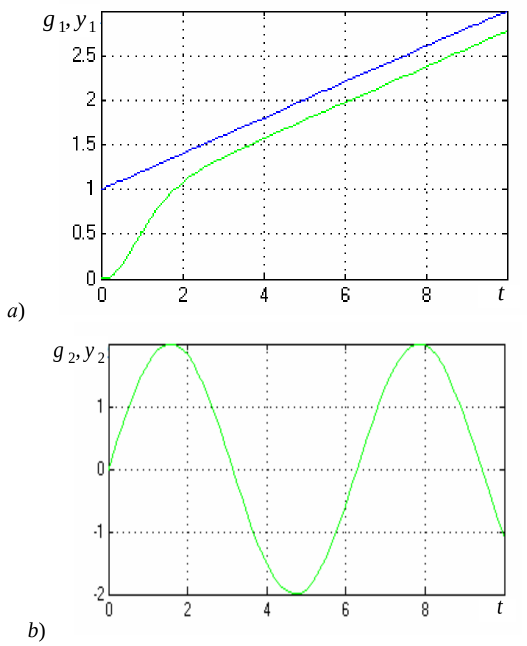

Fig. 7. Reference inputs and controlled variables

It follows from Fig. 8,*a* and Fig. 9,*a* that the channel $g_1\to y_1$ has the astatic first order to reference input g_1, and its deviation ε_1 is absolutely invariant to control u_2 which is entered in this channel of the controller (32).

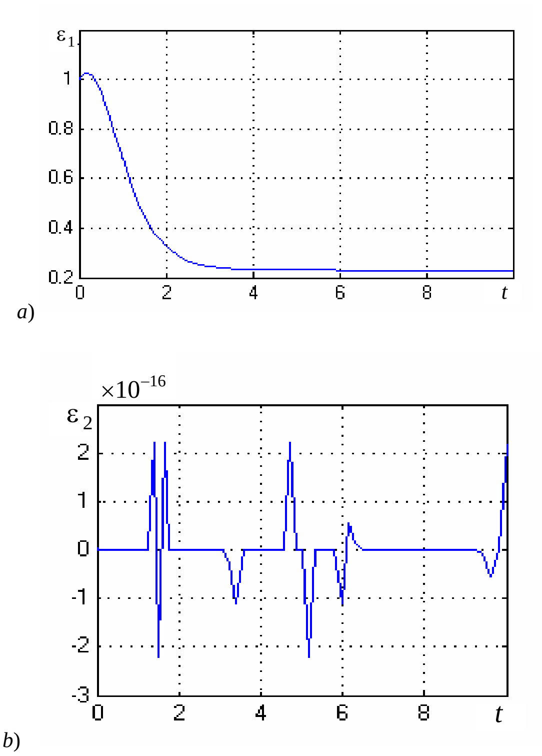

Fig. 8. Deviations of the first and second channels

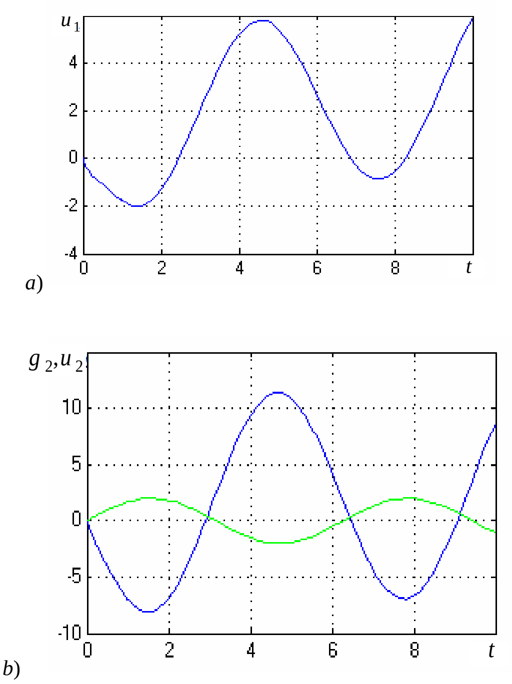

Fig. 9. Controls u_1, u_2 and the reference input g_2

The received results testify, that the considered method of analytical design allows creating control systems with the desired quality parameters in transitive and in the steady-state mode, but rather high order.

V. ROBUSTNESS INCREASE BY REDUCTION METHOD

Control systems of the high order have low robustness that reduces their quality, since the system's parameters are always known is inexact. The reduction of the plant dynamic model is applied to elimination of this lack, frequently. The exception method of the poorly influencing connections or the small time constants and also a balancing method are applied for reduction of the plant model, more often [10, 12, 24 and 25]. The new reduction method by cutting of the plant fast dynamics is suggested recently [27]. The reduced models allow designing more simple controllers and more robust systems. The design algorithm (12) – (29) of the reduced systems also becomes simpler.

However, the cutting out dynamics influences properties of the closed systems, actually. The influence of the reduction methods on the robust stability of the reduced control systems is investigated also in this paper.

Research is carried out on the numerical examples in view of the consideration complexity in a general view. It is supposed, that the initial model of the plant looks like

$$\dot{x} = \begin{bmatrix} -0,6358 & 0,7254 & 0,5709 \\ 13,4134 & 8,0531 & 9,5669 \\ -16,0835 & -15,7079 & -18,4173 \end{bmatrix} x + \begin{bmatrix} 0,689 \\ 0,5118 \\ 0,2205 \end{bmatrix} u,$$

$$y = [5,6 \quad 5,5 \quad 3,75]x. \qquad (33)$$

Transfer function of this plant is equal

$$W_{yu}(p) = \frac{7,5p^2 + 81p + 188}{(p+10)(p+2)(p-1)}. \qquad (34)$$

Let's agree the method exception of the small time constants to name as «time reduction» (*TR*) [10, 19, 22]; the reduction method on the basis of balancing models – «balancing reduction» (*BR*) [24, 25] and the method by a cutting of fast dynamics – «modal reduction» (*MR*) [26, 27]. Three reduced models (35) – (37) of the plant (33), (34) are received as a result of application of these methods:

$$W_{TR}(p) = \frac{0,75p^2 + 8,1p + 18,8}{(p+2)(p-1)}, \qquad (35)$$

$$W_{BR}(p) = \frac{6,771(p + 2,0617)}{p^2 + 0,474p - 1}, \qquad (36)$$

$$W_{MR}(p) = \frac{2,16p^2 + 5,636p + 18,8}{p^2 + p - 2}. \qquad (37)$$

To execute the modal reduction preliminary the spectral decomposition of the plant model (33) is carried out by replacement the vector x under similarity transformation: $x = P_i z_i$. Here the new vector z_i should consist from two component: fast z_i^{**} and dominating z_i^{*}. The fast component $z_i^{**} = z_i$ should depend only from fast mods (in considered case $c_1 e^{-10t}$) and the dominating component z_i^{*}

from other mods (in this case $c_2 e^{-2t}$ и $c_3 e^{t}$) of the plant. The modal reduction consists in replacement the fast components $z_i^{**} = z_i^{**}(t)$ by its steady state value $z_i^{\circ} = \lim_{t\to\infty} z_i^{**}(t)$ at which $\dot{z}_i^{**} = 0$ [27]. The matrix of the spectral decomposition P_i is not unique; therefore several modal reduced models can be constructed [26].

Fig. 10 represents the unite-step responses 1 – 4 of the initial (34) and the reduced models (35) – (37). The unite-step responses 1 and 3 of the initial model (34) and the balancing reduction (36) model practically coincide. This is one of the distinctive features of the balancing models of dynamic systems.

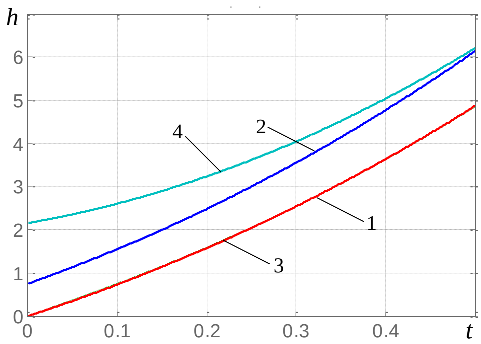

Fig. 10. Unite-step responses of the initial and the reduced models:

For comparison, the control systems designing by the analytical considered above method were carried out for initial, unreduced (*NR*) model (34) and for reduced models (35) – (37). The received controllers are described by the following equations:

$$(p^4 + 17{,}08p^3 + 92{,}891p^2 + 157{,}42076p)u_{UR} = 1431{,}66g - \\ -(16{,}4827p^3 + 184{,}712p^2 + 777{,}313p + 1431{,}66)y, \quad (38)$$

$$(p^3 + 10{,}8p^2 + 25{,}067p)u_{TR} = 166{,}67g - \\ -11{,}333p^2 + 76p + 166{,}67)y, \quad (39)$$

$$(p^3 + 8{,}762p^2 + 13{,}813p)u_{BR} = 22{,}153g - \\ -(5{,}638p^2 + 19{,}716p + 22{,}153)y, \quad (40)$$

$$(p^3 + 2{,}61p^2 + 8{,}704p)u_{MR} = 57{,}87g - \\ -(3{,}9352p^2 + 26{,}389p + 57{,}87)y. \quad (41)$$

Obviously, the order and the parameter's values of the controllers, designed for the reduced models of the plant, less, than for the unreduced model with the same quality of the designed systems.

V.L. Haritonov's criterion is applied to research the robust stability of the designed systems. With this purpose the factors β_i and α_i of the plant's models are replaced with the expressions $\beta_i = (1 \pm \tilde{\Delta})\beta_i^{\circ}$ and $\alpha_i = (1 \pm \tilde{\Delta})\alpha_i^{\circ}$, where β_i°, α_i° are nominal values of the factors. It is supposed, parameters of the real controllers are equal to the calculated values with very small errors. The characteristic polynomial $D_{rs}(s,T)$ of the reduced closed systems undertakes as

$$D_{rs}(p,T,\Delta) = (Tp+1)A_{red}(p,\Delta)R(p) + B_{red}(p,\Delta)L(p) \,. (42)$$

Here $A_{red}(p,\Delta)$, $B_{red}(p,\Delta)$ are the polynomials equal to a denominator and a numerator of the corresponding model's transfers function of the plant in view of the parametrical deviations $\Delta = 100\tilde{\Delta}$ %; the multiplier *Tp* + 1 represents the cut out fast dynamic of the plant model.

With the help of the V.L. Haritonov's criterion it is established, the unreduced system (34), (38) is robust stable with $\Delta \le 5{,}8$ %. The admissible on robust stability of the plant parameters deviation depend both from the method of a reduction and from value of the time constant *T*. At research, the time constant *T* changes in the equality (42) from zero up to such value $T_{m,red}$, at which the reduced closed systems lose stability already with $\Delta = 0$, i.e. with calculated values of the reduced model's parameters.

On Fig. 11 the schedules reflecting dependence of the deviations $\Delta = \Delta_{m,st}$, % from values of the time constant *T* at balancing (*BR*), time (*TR*) and modal (*MR*) reduction are submitted. Here $\Delta_{m,st} = \Delta_{m,st}(T^*)$ are deviations critical on the robust stability of the closed reduced system with $T = T^*$. In other words, if in expression (42) $\Delta = \Delta_{m,st}(T^*)$ and $T = T^*$, the closed system appears on border of robust stability.

Critical values of deviations $\Delta_{m,st}$ in the %, corresponding to the various reductions methods and values of *T* equal: 0,0 s; 0,01 s; 0,05 s and 0,10 s are submitted in columns from 2-th to 5-th table 1.

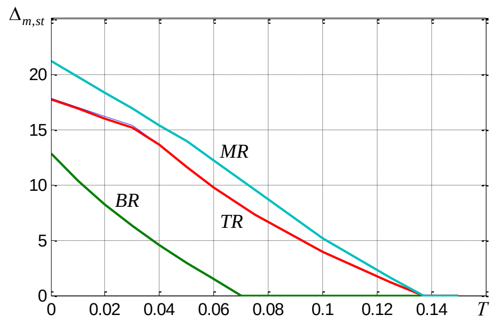

Fig. 11. Robust stability borders of the reduced systems

Values $\Delta_{m,st}$ on Fig. 11 with *T* = 0 and the second column of the table 1 correspond to critical deviations of the reduced model's parameters. Values of the time constant $T = T_{m,red}$ at which the reduced system appears on the stability border already with the calculated parameters are resulted in the last column of table 1.

TABLE I

Critical value of the deviations

T, s	0,0	0,01	0,05	0,10		$T_{m,red}$, s
TR	17,80	17,0	11,58	3,92		0,137
BR	12,78	10,40	2,92	0,00		0,0705
MR	21,22	19,78	13,93	5,15		0,137

Obviously, the time and modal reduction are characterized by the same boundary value $T_{m,red} = 0{,}137$ s. In case of the balancing reduction $T_{m,red} = 0{,}0705$ s, i.e. are much less. Under $T < T_{m,red}$ critical values of deviations $\Delta_{m,st}$ for balancing reduction also less in comparison with the time and modal reductions.

VI. CONCLUSION

On the basis of the received results it is possible to draw a conclusion. The considered analytical design method gives possibility to create the control systems with partially given structure and the desired quality parameters in transitive and in the steady-state modes. Parameters of the minimal dimension system's controller are the solutions of the linear algebraic equations systems. Use of the standard transfer functions provide such the desirable performances parameters as the astatic orders to the reference input and to external disturbances; overshoot and settling time.

The reduction of the plant model in three – four times raises robustness of the closed control systems, if the fast dynamics on two order is faster than the dominating dynamics. The robustness of control systems essentially raises, if the enough left poles or zero of the plant are made as roots of the closed system characteristic polynomial.

Parameter's deviations of the plant, critical on robust stability of the closed system, can serve as a «degree estimation of the system's robust stability» both reduced and unreduced control systems.

The method of the modal reduction allows receiving the most robustness control systems by a choice of the suitable matrix of the spectral decomposition of the plant's model. Considered analytical design method and the model plant reduction can be used for creation of the less complex control systems, but more robust for plants of chemical, textile, food and other branches of production.

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