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Results on Optimal Tuning of Fuzzy Models of Magnetic Levitation Systems

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ABSTRACT

This paper presents some results related to the optimal tuning of fuzzy models of magnetic levitation systems as widely used nonlinear processes. A modeling approach is given. The Takagi-Sugeno fuzzy models of the process (which is first stabilized) are obtained by the modal equivalence principle, where the rule consequents contain the state-space models of the stabilized process linearized at important operating points. The optimization problems are focused on the minimization of objective functions defined as the mean square modeling error (i.e., the difference between the real-world process output and the fuzzy model output). The vector variables of the objective functions consist of a part of the parameters of the input membership functions of the fuzzy models. An operating point selection algorithm is inserted in the initial phase of the modeling approach. Several nature-inspired optimization algorithms are employed to solve the optimization problems that result in optimal fuzzy models, and Particle Swarm Optimization, Simulated Annealing and Gravitational Search Algorithms are exemplified.

Keywords: fuzzy models, Gravitational Search Algorithms, magnetic levitation systems, objective functions, Particle Swarm Optimization, Simulated Annealing.

Mathematics Subject Classification: 82C21, 93A30

Computing Classification System: I.2.3, I.2.9

1. INTRODUCTION

The good accuracy of fuzzy models of magnetic levitation systems is important in the context of model-based fuzzy control. Several approaches to the fuzzy modeling of magnetic levitation systems are reported in the literature, and considered in the general framework of fuzzy models of various nonlinear processes (Angelov, 2004), (Baranyi, 2004), (Škrjanc et al., 2005), (Johanyák, 2010), (Vaščák and Hirota, 2011), (Precup et al., 2012a), (Macías-Escrivá et al., 2013), (Barchinezhad and Eftekhari, 2014), (Jafarian, 2014), (Melin and Castillo, 2014), (Wang et al., 2014), (Zhang and Wang, 2015).

A systematic way to ensure the accuracy expressed by adequate performance indices is to carry out the optimal tuning of the parameters of fuzzy models. Some representative examples of fuzzy models of magnetic levitation systems are presented as follows. The optimal gains of fuzzy control systems that involve fuzzy models are computed in (Yu and Huang, 2009) using particle swarm optimization and quantum-inspired evolutionary algorithms. The fuzzy model proposed in (Yu et al., 2003) is obtained using a linear self-constructing neural fuzzy inference network applied to an optimal fuzzy controller. The fuzzy neural network modeling approach given in (Yongzhi et al., 2011) models the gap sensor in high-speed maglev trains. A design method of parallel distributed compensation controllers for magnetic bearing of high-speed motors is suggested in (Wang and Wang, 2010). Simulated Annealing is applied in (David et al., 2012) and (Dragos et al., 2013) to tune the parameters of input membership functions of fuzzy models. A gradient descent algorithm is applied in (Su et al., 2015) to tune the parameters of fuzzy models for maglev suspension systems. Once the fuzzy models are obtained, appropriate control schemes must be developed (Tomescu et al., 2007), (Ferreira and Ruano, 2009), (Filip and Leiviskä, 2009), (Hermann et al., 2009), (Horváth and Rudas, 2012), (Precup et al., 2012b), (Tang et al., 2012), (Formentin et al., 2013), (Lamár and Kocsis, 2013), (He and Ge, 2015).

This paper presents an approach to the fuzzy modeling of magnetic levitation systems. As shown in (David et al., 2013), it starts with the derivation of an initial T-S fuzzy model of the process obtained by the modal equivalence principle. The initial fuzzy model is characterized by a set of local linearized state-space models of the process which are placed in the rule consequents. The nonlinear process models are linearized at several important operating points. However, as applied in (Precup et al., 2015a) and (Precup et al., 2015b) but for another nonlinear process and also combined with evolving fuzzy systems and with an input selection algorithm, an operating point selection algorithm is inserted in the approach. This algorithm uses importance factors, correlation functions, importance and significance threshold to select the most important and independent operating points of the process. A part of the parameters of the input membership functions of the fuzzy models is next optimized by nature-inspired optimization algorithms that solve the optimization problems which aim the minimization of the sum of squared modeling errors.

The paper is focused on the magnetic levitation system built around the representative Magnetic Levitation System with Two Electromagnets (MLS2EM) (Inteco, 2008). The operation and aim of this system is to solve the magnetic levitation problem for a metallic sphere maintained in an electromagnetic field. The derivation of accurate fuzzy models is challenging for this classical nonlinear unstable application. Three algorithms are involved in the optimal tuning of the parameters of input membership functions, viz. Particle Swarm Optimization, Simulated Annealing and Gravitational Search Algorithms.

The paper is structured as follows: Section 2 is dedicated to the crisp mathematical modeling of the process. The fuzzy modeling approach is presented in Section 3. Some details on the implementation of the three nature-inspired optimization algorithms are given in Section 4. The results, which lead to optimal fuzzy models, are discussed in Section 5. The conclusions are outlined in Section 6.

2. CRISP MODELS OF MAGNETIC LEVITATION SYSTEM

As shown in (David et al., 2012), the block diagram of the ML2SEM considered as a controlled process is presented in Figure 1. The following abbreviations are used in Figure 1: EM1, EM2 are the upper and lower electromagnet, respectively, *m* is the mass of the sphere, F_{em1} and F_{em2} are the electromagnetic forces, and F_g is the gravity force (Inteco, 2008).

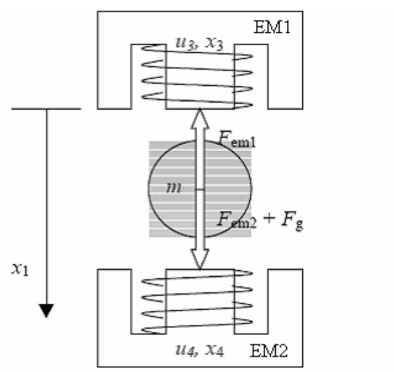

Figure 1. Block diagram of ML2SEM.

The nonlinear state-space model of MLS2EM is:

$$
\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -\frac{1}{m}\cdot\frac{F_{emP1}}{F_{emP2}}\cdot e^{-\frac{x_1}{F_{emP1}}}\cdot x_3^2 + g + \frac{1}{m}\cdot\frac{F_{emP1}}{F_{emP2}}\cdot e^{-\frac{x_d - x_1}{F_{emP2}}}\cdot x_4^2, \\
\dot{x}_3 &= \frac{1}{\frac{f_{iP1}}{f_{iP2}}\cdot e^{-\frac{x_1}{f_{iP2}}}}(k_i u_1 + c_i - x_3), \\
\dot{x}_4 &= \frac{1}{\frac{f_{iP1}}{f_{iP2}}\cdot e^{-\frac{x_d - x_1}{f_{iP2}}}}(k_i u_2 + c_i - x_4), \\
y &= x_1,
\end{aligned}
\tag{1}
$$

where the variables are:

- the control signal u_1, which is applied to the upper electromagnet (EM1),
- the disturbance input $d = u_2$, which is applied to the lower electromagnet (EM2), EM2 is not needed to stabilize the magnetic sphere, its role is just to disturb the controlled process and to make the control problem more challenging,
- the state variables: x_1 is the sphere position, x_2 is the sphere speed, x_3 and x_4 are the currents in the upper and lower electromagnetic coil, respectively;
- the controlled output $y = x_1$.

The numerical values of the parameters of the model (1) are given in (Inteco, 2008). The linearization of the nonlinear model (1) at nine operating points $A_j(x_{10}, x_{20}, x_{30}, x_{40})$ (the subscript j indicates the index of the operating point) results in the linearized state-space models (David et al., 2012), (Dragos et al., 2013):

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{b}\,\Delta u \\ \Delta y = \mathbf{c}^T \mathbf{x} \end{cases}, \mathbf{x} = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix},$$
$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{2,1} & 0 & a_{2,3} & a_{2,4} \\ a_{3,1} & 0 & a_{3,3} & 0 \\ a_{4,1} & 0 & 0 & a_{4,4} \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ b_3 \\ b_4 \end{bmatrix}, \mathbf{c}^T = [1 \quad 0 \quad 0 \quad 0], \tag{2}$$

with T – matrix transposition, and the expressions of the matrix elements:

$$\begin{aligned}
&a_{11} = 0, a_{12} = 1, a_{13} = 0, a_{14} = 0, \\
&a_{21} = \frac{x_{30}^2}{m} \frac{F_{emP1}}{F_{emP2}^2} e^{-\frac{x_{10}}{F_{emP2}}} + \frac{x_{40}^2}{m} \frac{F_{emP1}}{F_{emP2}^2} e^{-\frac{x_d - x_{10}}{F_{emP2}}}, a_{22} = 0, \\
&a_{23} = -\frac{2x_{30}}{m} \frac{F_{emP1}}{F_{emP2}} e^{-\frac{x_{10}}{F_{emP2}}}, a_{24} = \frac{2x_{40}}{m} \frac{F_{emP1}}{F_{emP2}} e^{-\frac{x_d - x_{10}}{F_{emP2}}}, \\
&a_{31} = \frac{f_{iP2}}{f_{iP1}} e^{\frac{x_{10}}{f_{iP2}}} (k_i u_1 + c_i - x_{30}), a_{32} = 0, a_{33} = -\frac{1}{f_{iP1}} \cdot e^{\frac{x_{10}}{f_{iP2}}}, a_{34} = 0, \\
&a_{41} = -\frac{1}{f_{iP1}} e^{\frac{x_d - x_{10}}{F_{emP2}}} (k_i u_2 + c_i - x_{40}), \\
&a_{42} = 0, a_{43} = 0, a_{44} = -\frac{f_{iP2}}{f_{iP1}} e^{\frac{x_d - x_{10}}{F_{emP2}}}, \\
&b_{11} = 0, b_{21} = 0, b_{31} = k_i \frac{f_{iP2}}{f_{iP1}} e^{\frac{x_{10}}{f_{iP2}}}, b_{41} = k_i \frac{f_{iP2}}{f_{iP1}} e^{\frac{x_d - x_{10}}{F_{emP2}}}.
\end{aligned} \tag{3}$$

The fourth-order state-space model (2) is reduced to the following third-order state-space model of MLS2EM, which is obtained by neglecting the lower electromagnet, $u_2 = 0$:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{b}\,\Delta u \\ \Delta y = \mathbf{c}^T \mathbf{x} \end{cases}, \mathbf{x} = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix},$$
$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ a_{21} & 0 & a_{23} \\ a_{31} & 0 & a_{33} \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix}, \mathbf{c}^T = [1 \quad 0 \quad 0]. \tag{4}$$

The expressions of parameters specific to the matrices $\mathbf{A}$, $\mathbf{b}$ and $\mathbf{c}^T$ are (David et al., 2012), (Dragos et al., 2013):

$$a_{21} = \frac{x_{30}^2}{m} \frac{F_{emP1}}{F_{emP2}^2} e^{-\frac{x_{10}}{F_{emP2}}} + \frac{x_{40}^2}{m} \frac{F_{emP1}}{F_{emP2}^2} e^{-\frac{x_d - x_{10}}{F_{emP2}}},$$
$$a_{23} = -\frac{2x_{30}}{m} \frac{F_{emP1}}{F_{emP2}} e^{-\frac{x_{10}}{F_{emP2}}}, \tag{5}$$
$$a_{31} = -(k_i u + c_i - x_{30})(x_{10} / f_{iP2})\, f_i^{-1}(x_{10}),\, a_{33} = -f_i^{-1}(x_{10}),$$
$$b_3 = k_i\, f_i^{-1}(x_{10}).$$

A state feedback control structure is designed to stabilize the ML2SEM by pole placement using the state-feedback gain matrix (David et al., 2012):

$$\mathbf{k}_c^T = [36 \quad 5 \quad 0.0075]. \tag{6}$$

This leads to the following closed-loop system (i.e., stabilized process) model, where the notation y is used as follows instead of the difference with respect to the output of the operating point Δy:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}_x\,\mathbf{x} + \mathbf{b}_r\, r_x \\ y = \mathbf{c}^T \mathbf{x} \end{cases}, \mathbf{x} = [\Delta x_1 \quad \Delta x_2 \quad \Delta x_3]^T, \tag{7}$$

where the numerical values of the elements of the matrices $\mathbf{A}_x$, $\mathbf{b}_r$ and $\mathbf{c}^T$ are given in (David et al., 2012), and $r_x = u$ is the reference input applied to the state feedback control structure and also the input of the stabilized process that is actually modeled.

3. FUZZY MODELING APPROACH

The fuzzy modeling approach consists of the following steps, with a similar formulation for other nonlinear process given in (Precup et al., 2015a):

Step 1. The Takagi-Sugeno fuzzy model structure is set, that means the number of operating points, which is equal to the number of rules n_R of the fuzzy models, the number of input linguistic terms of the input variables x_1 and x_3, the shapes of the membership functions of the input linguistic terms, the operators in the inference engine, and the method for defuzzification. As shown in (Precup et al., 2015a), triangular, trapezoidal and Gaussian membership functions, the SUM and PROD operators in the inference engine, and the weighted average method in the defuzzification module are recommended.

Step 2. The modal equivalence principle is applied. The continuous-time state-space model of the process (7) is linearized at n_R important operating points leading to n_R linearized continuous-time local process models placed in the rule consequents of the continuous-time Takagi-Sugeno fuzzy models. These local models are related to the modal values of the input membership functions, i.e., the coordinates of the operating point. The rule base of the continuous-time Takagi-Sugeno fuzzy models is:

$$R^i : \text{IF}\, x_1(t)\, \text{IS}\, LT^i_{x_1,l}\, \text{AND}\, x_3(t)\, \text{IS}\, LT^i_{x_3,p}\, \text{THEN} \begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i u(t) \\ y_m(t) = \mathbf{C}_i \mathbf{x}(t) \end{cases}, i = \overline{1, n_R}, \tag{8}$$

where $\mathbf{A}_i$ corresponds to $\mathbf{A}_x$, $\mathbf{B}_i$ corresponds to $\mathbf{b}_r$, $\mathbf{C}_i$ corresponds to $\mathbf{c}^T$, *t* is the continuous time variable, $y_m(t)$ is the continuous-time Takagi-Sugeno fuzzy model output, the notations $LT^i_{v,l}$ and $LT^i_{v,p}$ are used for the linguistic terms $LT_{x_1,l}$ and $LT_{x_3,p}$ of the input variables $x_1(t)$ and $x_3(t)$, respectively, referred to also as scheduling variables. As shown in (David et al., 2012), the following numbers of linguistic terms and ranges of indices are recommended: $l = \overline{1,3}$, $p = \overline{1,3}$, and the notation $v \in \{x_1, x_3\}$.

Step 3. The sampling period T_s is set. The n_R models in the rule consequents of the Takagi-Sugeno fuzzy model (8) are discretized in the presence of the zero-order hold. The rule base of the discrete-time Takagi-Sugeno fuzzy models is:

$$R^i : \text{IF}\, x_{1,k}\, \text{IS}\, LT^i_{x_1,l}\, \text{AND}\, x_{3,k}\, \text{IS}\, LT^i_{x_3,p}\, \text{THEN} \begin{cases} \mathbf{x}_{k+1} = \mathbf{A}_{d,i} \mathbf{x}_k + \mathbf{B}_{d,i} u_k \\ y_{k,m} = \mathbf{C}_{d,i} \mathbf{x}_k \end{cases}, i = \overline{1, n_R}, \tag{9}$$

where the subscript *k* indicates the discrete time variable, and $y_{k,m}$ is the discrete-time TSK fuzzy model output. The recommended value of the sampling period is $T_s = 0.005\ \text{s}$.

An operating point selection algorithm is proposed as follows in order to select the important n_R operating points from the set of all possible operating points $\{A_j \mid j = 1...n\}$, with *n* – the number of possible operating points. This algorithm is applied in (Precup et al.,. 2015a) to another nonlinear process, and it is similar to the input selection algorithms given in (Jang, 1996), (Linkens and Chen, 1999) and (Precup et al., 2015b). The parameters of the operating point selection algorithm are the importance threshold, and the significance threshold, with values within 0 and 1. Setting the importance threshold to 0.3 and the significance threshold to 0.5 leads to $n_R = 9$ operating points for this process.

Step 4. The optimization problem that leads to optimal Takagi-Sugeno fuzzy models is defined as:

$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}\in D} J(\boldsymbol{\rho}), \\ \text{subject to constraints,} \tag{10}$$

where $\boldsymbol{\rho}$ is the parameter vector of the Takagi-Sugeno fuzzy model, i.e., the vector variable of the objective function $J(\boldsymbol{\rho})$ (Precup et al., 2015a):

$$J(\boldsymbol{\rho}) = \frac{1}{N}\sum_{k=1}^{N}(y_k(\boldsymbol{\rho}) - y_{k,m}(\boldsymbol{\rho}))^2 = \frac{1}{N}\sum_{k=1}^{N}(e_{k,m}(\boldsymbol{\rho}))^2, \tag{11}$$

$\boldsymbol{\rho}^*$ is the optimal parameter vector of the Takagi-Sugeno fuzzy model, i.e., the solution to the optimization problem (10), $y_k(\boldsymbol{\rho})$ is the process output at k^{th} sampling interval, $y_{k,m}(\boldsymbol{\rho})$ is the fuzzy model output, $e_{k,m}(\boldsymbol{\rho})$ is the modeling error, *D* is the feasible domain of $\boldsymbol{\rho}$, and *N* is the length of the discrete time horizon.

Since the elements of the vector $\boldsymbol{\rho}$ are a part of the parameters of the input membership functions parameters, the optimization problem (10) should be viewed as a constrained optimization problem. Several inequality-type constraints are defined such that to ensure the convenient overlap of the membership functions.

Step 5. The nature-inspired optimization algorithms are implemented to solve the optimization problem defined in (11). The elements of the solution vector $\boldsymbol{\rho}^*$ are the optimal input membership function parameters.

4. IMPLEMENTATION OF NATURE-INSPIRED OPTIMIZATION ALGORITHMS

The application of the steps of the modeling approach is described as follows. The derivation of the initial Takagi-Sugeno fuzzy model starts with the setting of the largest domains of variation of the two input variables (David et al., 2012):

$$-0.2 \le x_1 \le 0.2, -8.757 \le x_3 \le 18.765. \tag{12}$$

As shown in (David et al., 2012), the fuzzification uses the linguistic terms assigned to the input variables and defined as follows. For the input variable x_1, three linguistic terms, $LT_{x_1,j}$, $j = \overline{1,3}$, with triangular membership functions are defined and referred to as $LT_{x_1,1}$, with the universe of discourse $[-0.2 \quad 0]$, $LT_{x_1,2}$, with the universe of discourse $[-0.1 \quad 0.1]$, and $LT_{x_1,3}$, with the universe of discourse $[0 \quad 0.2]$. The expressions of these triangular membership functions are:

$$\mu_{LT_{x_1,j}}(x) = \begin{cases} 0, & x < a_{x_1,j}, \\ 1 + \dfrac{x - b_{x_1,j}}{b_{x_1,j} - a_{x_1,j}}, & x \in [a_{x_1,j}, b_{x_1,j}), \\ 1 - \dfrac{x - b_{x_1,j}}{c_{x_1,j} - b_{x_1,j}}, & x \in [b_{x_1,j}, c_{x_1,j}), \\ 0, & x \geq c_{x_1,j}, \end{cases} \quad a_{x1,j} < b_{x1,j} < c_{x1,j},\ j = \overline{1,3}, \tag{13}$$

where the initial modal values of the membership functions are the parameters $a_{x1,j}$, $b_{x1,j}$, and $c_{x1,j}$, $j = \overline{1,3}$, given in Table I. The parameters $a_{x1,j}$, $j = \overline{1,3}$ and $c_{x1,j}$, $j = \overline{1,3}$ are fixed, and the parameters $b_{x1,j}$, $j = \overline{1,3}$ are variable.

TABLE I. MODAL VALUES OF LINGUISTIC TERMS

Linguistic terms, $LT_{x_1,j}$, $j = \overline{1,3}$	Triangular membership functions		
	$a_{x1,j}$	$b_{x1,j}$	$c_{x1,j}$
$LT_{x_1,1}$	-0.2	-0.1	0
$LT_{x_1,2}$	-0.1	0	0.1
$LT_{x_1,3}$	0	0.1	0.2

Three linguistic terms, i.e., $LT_{x_3,j}$, $j = \overline{1,3}$, are defined for the second input variable, x_3. The first and third one are modeled by trapezoidal membership functions, and the second one is modeled by a Gaussian membership function. The universes of discourse of the membership functions of these linguistic terms are: $[-8.757 \quad 4.3785]$ for $LT_{x_3,1}$, $[3.753 \quad 4.3785]$ for $LT_{x_3,2}$, and $[4.3785 \quad 18.765]$ for $LT_{x_3,3}$. The expressions of the trapezoidal membership functions are:

$$\mu_{LT_{x3,j}}(x) = \begin{cases} 0, & x < a_{x3,j}, \\ 1 + \dfrac{x - b_{x3,j}}{b_{x3,j} - a_{x3,j}}, & x \in [a_{x3,j}, b_{x3,j}), \\ 1, & x \in [b_{x3,j}, c_{x3,j}), \\ 1 - \dfrac{x - c_{x3,j}}{d_{x3,j} - c_{x3,j}}, & x \in [c_{x3,j}, d_{x3,j}), \\ 0, & x \geq d_{x3,j}, \end{cases} \quad a_{x3,j} < b_{x3,j} \leq c_{x3,j} < d_{x3,j},\ j \in \{1,3\}. \tag{14}$$

The initial modal values of the membership functions are the parameters $a_{x3,j}$, $j \in \{1,3\}$, $b_{x3,j}$, $j \in \{1,3\}$, $c_{x3,j}$, $j \in \{1,3\}$, and $d_{x3,j}$, $j \in \{1,3\}$, given in Table II. The parameters $a_{x3,j}$, $j \in \{1,3\}$, $b_{x3,1}$, $c_{x3,3}$ and $d_{x3,j}$, $j \in \{1,3\}$ are fixed, and the parameters $c_{x3,1}$ and $b_{x3,3}$ are variable.

TABLE II. PARAMETERS OF TRAPEZOIDAL LINGUISTIC TERMS

Linguistic terms, $LT_{x_3,j}$, $j = \{1,3\}$	Trapezoidal membership functions			
	$a_{x3,j}$, $j = \{1,3\}$	$b_{x3,j}$, $j = \{1,3\}$	$c_{x3,j}$, $j = \{1,3\}$	$d_{x3,j}$, $j = \{1,3\}$
$LT_{x_3,1}$	-8.757	-8.757	-1.251	4.3785
$LT_{x_3,3}$	4.3785	11.259	18.765	18.765

The expression of the Gaussian membership function is:

$$\mu_{LT_{x_3,2}}(x) = e^{-\frac{(x-a_{x3,2})^2}{2b_{x3,2}^{\;2}}}. \quad (15)$$

The parameter $b_{x3,2}$ is fixed, and the parameter $a_{x3,2}$ is variable. The initial values of these two parameters are $a_{x3,2} = 4.3785$ and $b_{x3,2} = 3.753$. The initial membership functions of x_1 and x_3 are illustrated in Figure 2.

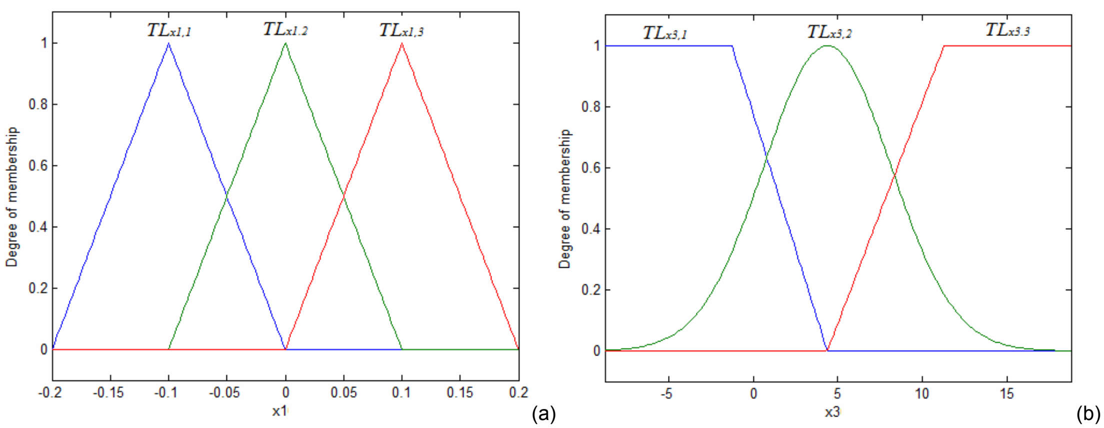

Figure 2. Initial membership functions of input variables x_1 and x_3 (with TL = LT).

The expression of the parameter vector of the fuzzy model is:

$$\boldsymbol{\rho} = [b_{x1,1} \quad b_{x1,2} \quad b_{x1,3} \quad c_{x3,1} \quad a_{x3,2} \quad b_{x3,3}]^T. \quad (16)$$

The operating mechanism of the Particle Swarm Optimization (PSO) algorithms uses swarm particles, which are characterized by two vectors, namely the particle position vector $\mathbf{X}_i$ and the particle velocity vector $\mathbf{V}_i$:

$$\mathbf{X}_i = [x_i^1 \quad \ldots \quad x_i^d \quad \ldots \quad x_i^q]^T,\ \mathbf{V}_i = [v_i^1 \quad \ldots \quad v_i^d \quad \ldots \quad v_i^q]^T,\ i = \overline{1, N_p}, \tag{17}$$

where $i,\ i = \overline{1, N_p}$, is the index of the current particle in the swarm, and N_p is the number of particles in the swarm. Using the notations $\mathbf{P}_{i,Best}$ for the best particle position vector of a specific particle with the index $i,\ i = \overline{1, N_p}$, and $\mathbf{P}_{g,Best}$ for the best swarm position vector:

$$\mathbf{P}_{i,Best} = [p_i^1 \quad \ldots \quad p_i^d \quad \ldots \quad p_i^q]^T,\ \mathbf{P}_{g,Best} = [p_g^1 \quad \ldots \quad p_g^d \quad \ldots \quad p_g^q]^T,\ i = \overline{1, N_p}, \tag{18}$$

the next particle velocity $v_i^d(\mu + 1)$ and the next particle position $x_i^d(\mu + 1)$ are obtained by the state-space equations (Kennedy and Eberhart, 1995a), (Kennedy and Eberhart, 1995b):

$$\begin{aligned} v_i^d(\mu+1) &= w(\mu)\, v_i^d(\mu) + c_1\, r_1\, [p_g^d(\mu) - x_i^d(\mu)] + c_2\, r_2\, [p_i^d(\mu) - x_i^d(\mu)], \\ x_i^d(\mu+1) &= x_i^d(\mu) + v_i^d(\mu+1),\ d = \overline{1, q},\ i = \overline{1, N_p}, \end{aligned} \tag{19}$$

where μ is the current iteration index, and the values of the PSO algorithm parameters in (19) are taken from (Precup et al., 2013).

The PSO algorithm is mapped onto the optimization problem using the following relations:

- between the agents' position vector $\mathbf{X}_i$ in the PSO algorithm and the parameter vector $\boldsymbol{\rho}$ in the optimization problem:

$$\mathbf{X}_i = \boldsymbol{\rho},\ i = \overline{1, N_p}, \tag{20}$$

- between the fitness function *g* in the PSO algorithm and the objective function *J* in the optimization problem:

$$g(\mathbf{X}_i) = J(\boldsymbol{\rho}),\ i = \overline{1, N_p}. \tag{21}$$

The PSO algorithm stops when the maximum number of iterations $\mu_{\max}$ is reached. The vector solution to the optimization problem (10) is:

$$\boldsymbol{\rho}^* = \mathbf{P}_{g,Best}, \tag{22}$$

where $\mathbf{P}_{g,Best}$ is the best swarm position vector obtained so far.

The steps of the simulated annealing algorithm, obtained from (Kirkpatrick et al., 1983), (Geman and Geman, 1984), and extended according to (David et al., 2012), are:

- *Step 1*. Set $\mu = 0$, $s_r = 0$ and the minimum temperature $\theta_{\min}$. Choose the initial temperature θ_0.

- *Step 2*. Generate a random initial solution φ and compute its fitness value $g(\varphi)$.

- *Step 3*. Generate a probable solution $\boldsymbol{\psi}$ by disturbing φ, and evaluate the fitness value $g(\boldsymbol{\psi})$.
- *Step 4*. Compute $\Delta g_{\varphi\psi} = g(\varphi) - g(\boldsymbol{\psi})$. If $\Delta g_{\varphi\psi} \leq 0$, then accept $\boldsymbol{\psi}$ as the new solution. Otherwise, set the value of the random parameter r_n, $0 \leq r_n \leq 1$, and compute the probability of $\boldsymbol{\psi}$ to be the next solution: $p_\psi = \begin{cases} 1 & \text{if } \Delta g_{\varphi\psi} > 0, \\ \exp(\Delta g_{\varphi\psi} / \theta) & \text{otherwise,} \end{cases}$. If $p_\psi > r_n$, then $\boldsymbol{\psi}$ is the new solution.
- *Step 5*. If the new solution is accepted, then update the new solution, increment s_r and set $r_r = 0$. Otherwise, increment r_r. If r_r has reached its maximum value $r_{r\max}$, the algorithm is stopped; otherwise, continue with step 6.
- *Step 6*. Increment s_r. If s_r has reached its maximum value s_r, go to step 7; otherwise increment μ. If μ has reached its maximum value $\mu_{\max}$, go to step 7; otherwise, go to step 2.
- *Step 7.* Alleviate the temperature according to the temperature decrement rule:

$$\theta_{\mu+1} = \alpha_{cs}\theta_\mu, \ \alpha_{cs} = \text{const}, \ \alpha_{cs} \approx 1. \tag{23}$$

- *Step 8.* If $\theta_\mu > \theta_{\min}$ then go to step 3, otherwise the algorithm is stopped.

The Simulated Annealing algorithm is mapped onto the optimization problem using the following relations:

- between the parameter vectors:

$$\boldsymbol{\rho} = \boldsymbol{\psi}, \ \boldsymbol{\rho} = \varphi, \tag{24}$$

- between the fitness function *g* in the Simulated Annealing algorithm and the objective function *J* in the optimization problem:

$$J(\boldsymbol{\rho}) = g(\boldsymbol{\psi}), \ J(\boldsymbol{\rho}) = g(\varphi). \tag{25}$$

The operating mechanism of Gravitational Search Algorithm (GSA) makes use of *N* agents and a *q*-dimensional search space, and the position of i^{th} agent is defined by the vector:

$$\mathbf{X}_i = [x_i^1 \ \ \dots \ \ x_i^d \ \ \dots \ \ x_i^q]^T, \ i = \overline{1, N_p}, \tag{26}$$

The force acting on i^{th} agent from j^{th} agent is defined as follows at the current iteration index μ:

$$F_{ij}^d(\mu) = g(k)\frac{m_{Pi}(\mu) m_{Aj}(\mu)}{r_{ij}(\mu) + \varepsilon\, x_j^d(\mu)}[x_j^d(\mu) - x_i^d(\mu)], \tag{27}$$

where $m_{Pi}(\mu)$ is the active gravitational mass related to i^{th} agent, $m_{Aj}(\mu)$ is the passive gravitational mass related to j^{th} agent, $\varepsilon > 0$ is a small constant, and $r_{ij}(\mu)$ is the Euclidian distance between i^{th} and j^{th} agents. The position and velocity of an agent are updated in terms of the following state-space equations (Rashedi et al., 2009), (Rashedi et al., 2010):

$$\begin{aligned} v_i^d(\mu+1) &= \rho_i v_i^d(\mu) + a_i^d(\mu), \\ x_i^d(\mu+1) &= x_i^d(\mu) + v_i^d(\mu+1), \end{aligned} \tag{28}$$

where $\rho_i, 0 \le \rho_i \le 1$, is a uniform random variable, $a_i^d(\mu)$ is the acceleration of i^{th} agent in d^{th} dimension, and the values of the GSA parameters are taken from (David et al., 2013).

The GSA is mapped onto the optimization problem (10) using (20) and (21). The formulation of GSA is closer to PSO algorithms than to Simulated Annealing algorithms.

5. EXPERIMENTAL RESULTS

The results are exemplified as follows for the Simulated Annealing algorithm embedded in the modeling approach out of the three tested algorithms. These results are extracted from (David et al., 2012).

The values of the maximum consecutive rejections and the maximum success were set to $r_{r\max} = 100$ and $s_{r\max} = 50$, respectively. The initial temperature was chosen as $\theta_0 = 1$. The algorithm has stopped after 84 iterations, when the temperature value was $\theta_{84} = 9.04626 \cdot 10^{-0.009}$. The initial solution is represented by the vector φ:

$$\boldsymbol{\rho} = \varphi = [-0.1 \quad 0 \quad 0.1 \quad -1.251 \quad 4.3785 \quad 11.259]^T, \tag{29}$$

and the final solution is represented by the vector $\boldsymbol{\psi}$:

$$\boldsymbol{\rho}^* = \boldsymbol{\psi} = [-0.075 \quad 0.0547 \quad 0.13 \quad -0.63 \quad 4.68 \quad 11.424]^T. \tag{30}$$

Figure 3 illustrates the performance improvement offered by the optimized Takagi-Sugeno fuzzy model. The modeling error converges to zero in both cases, before and after optimization, but it converges faster to zero after optimization versus the situation before optimization.

The evolution of the objective function versus the iteration index illustrated in Figure 4 shows that the solution to the optimization problem (10) obtained by the Simulated Annealing algorithm ensures a strong decrease of the objective function. Although the minimum of the objective function cannot be guaranteed, Figure 4 highlights that the improvement can continue by considering a larger number of iterations.

These conclusions cannot be generalized for other processes. The evolution of the objective function and the dynamics of the modeling error will be different.

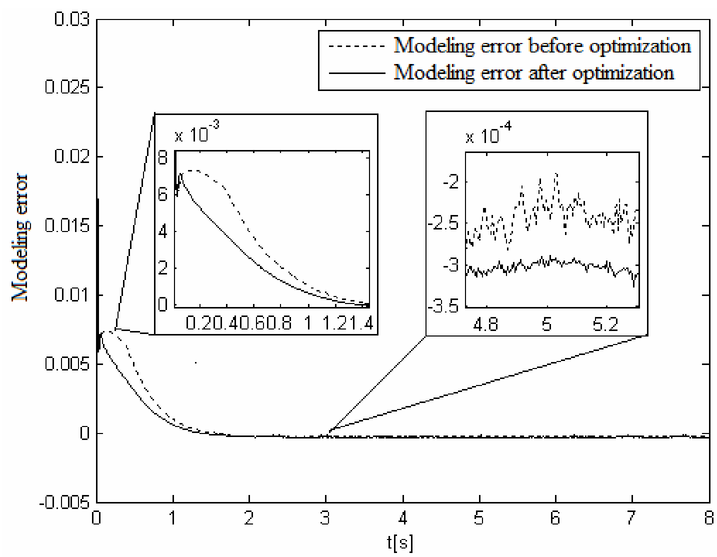

Figure 3. Modeling error versus time before and after optimization by Simulated Annealing algorithm.

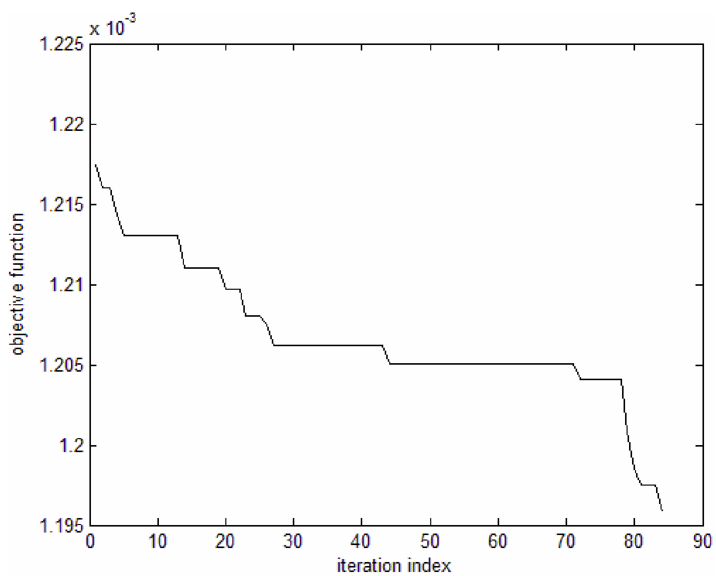

Figure 4. Objective function versus iteration index obtained by Simulated Annealing algorithm.

6. CONCLUSIONS

The paper has presented some results concerning the optimal tuning of fuzzy models of magnetic levitation systems. The modeling approach is based on the implementation of nature-inspired optimization algorithms that solve optimization problems with focus on the reduction of the modeling error. A part of the parameters of the input membership functions is optimally tuned.

The results are important in the context of model-based fuzzy control with recent results given in (He et al., 2015), (Lin and Chen, 2015), (Su et al., 2015), (Wai et al., 2015). Future research will lead to testing and implementing other optimization algorithms on this application and on other illustrative industrial processes.

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