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Takagi-Sugeno Fuzzy Control Solutions for Mechatronic Applications

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ABSTRACT

This paper treats several application-oriented fuzzy control solutions with Takagi-Sugeno fuzzy controllers (TS-FCs) developed for machatronic application. Low-cost fuzzy control solutions are offered with simple design approaches and easy implementation results. The solutions and organized such that to represent useful recommendations for specialists who apply artificial intelligence techniques in wide range of practical applications related to mechatronic systems. It is proved that our fuzzy control solutions can ensure good control system performance and compensation for plant nonlinearities in mechatronic systems as well. Therefore they enable the application and full utilization of such systems. Three case studies related to the speed and position control of three mechatronic applications are included: a vehicular power train system with continuously variable transmission, an electromagnetically actuated clutch and a magnetic levitation system. Plant models expressed as first principle nonlinear models and linearized models are offered. Simulations and real-time experimental results validate the low-cost TS-FCs.

Keywords: continuously variable transmission, electromagnetically actuated clutch, magnetic levitation system, mechatronic applications, Takagi-Sugeno fuzzy controllers.

Mathematics Subject Classification: 82C21, 93A30

Computing Classification System: I.2.3, I.2.9

1. INTRODUCTION

In practical applications the presence of nonlinearities in the structure of the controlled plants leads to the idea of introducing fuzzy control (Precup et al., 2011). In addition in the design of fuzzy controllers (FCs) can be considered in certain conditions as nonlinear but linearizable in the vicinity of operating points belonging to the input-output map also called control surface. The fuzzy control solutions given in this paper are designed by the extension of our previous design methods (Dragoş et al., 2010a), (Dragoş et al., 2010d), and of other popular fuzzy control and modeling approaches (Baranyi and Kóczy, 1996), (Precup and Preitl, 1999), (Albertos, 2002), (Baranyi et al., 2003), (Škrjanc et al., 2005), (Johanyák and Kovács, 2006), (Precup et al., 2008), (Johanyák, 2010), (Linda and Manic, 2011). Our low-cost fuzzy control solutions are widely used in mechatronic applications because of their simplicity in structure, design and implementation and because they ensure very good control system (CS) performance with respect to reference and disturbance inputs.

The controlled plants taken in consideration are three applications coming from mechatronics: a vehicle power train system with continuously variable transmission (CVT), an electromagnetically actuated clutch and a complete control laboratory system based on the Magnetic Levitation System with Two Electromagnets (MLS2EM). The mechatronic applications are used successfully in the field of industrial and non-industrial domains due to their simple and robust structures (Gillespie, 1992), (Isermann, 2005), (Kiencke and Nielsen, 2005), (Bishop, 2002), (Bishop, 2007).

The development of the automotive control systems represents one of the key elements as regards innovation in vehicle industry. Taking into account the more and more sophisticated vehicles' structures the efforts towards increasing the overall vehicle performance, the fuel economy and the safety are challenging (Isermann, 2005), (Kiencke and Nielsen, 2005). Many research results have been reported in the literature in this context. Therefore, the design of fundamental components, configuration and kinematics of vehicle power train system with spark-ignition engine and a continuously variable transmission, the dynamical modeling of the CVT system (according to different stages of clutch engaging) and the development of the adaptive nonlinear controllers have been discussed in (Guzzella and Schmid, 1995), (Setlur et al., 2003), (Zhou et al., 2006), (Modak and Sane, 2007) and (Song and Wang, 2008).

In the framework of automotive applications, the electromagnetic actuated clutch is an important system (Di Cairano et al., 2007). This servo-system application may itself be considered a control system involved into a more complex system (Isemann, 2005), (Kiencke and Nielsen, 2005). Very good control system performance indices should be ensured by the control systems of electromagnetic actuated clutches viewed as actuators in automotive control systems. The main objective of this area of applications is to exhibit a small settling time, an exact positioning without overshoot, friction and backlash (Deur et al., 2006a).

The problems that occur in the levitation of a sphere are related to the stability of the sphere position which must levitate and move at the same time. Several control solutions dedicated to the robust stabilization and disturbance rejection of a magnetic levitation systems are reported in the literature (Lee et al., 2007), (Wu and Hu, 2009).

This paper is organized as follows. The theoretical support of fuzzy controller design is presented in Section 2. In Section 3, the mathematical modeling and the position and speed control of three mechatronic applications are discussed. Due to the existence of complexities of the mechatronic applications, a set of local mathematical models of the nonlinear systems obtained by linearization are, also, presented in Section 3. The Takagi-Sugeno fuzzy controllers designed in this paper are tested and validated by simulation and by real-time experiments. The concluding remarks are highlighted in Section 4.

2. DESIGN APPROACHES OF TAKAGI-SUGENO FUZZY CONTROL SYSTEMS

Our approach starts with the theoretical support of the Takagi-Sugeno fuzzy controller (TS-FC) with the integration of the output variable. The design of a Takagi-Sugeno PI-fuzzy controller is based on the fuzzy control system structure presented in Figure 1, where r is the reference input, e is the control error, u is the control input and y is the controlled output (Precup and Preitl, 1999).

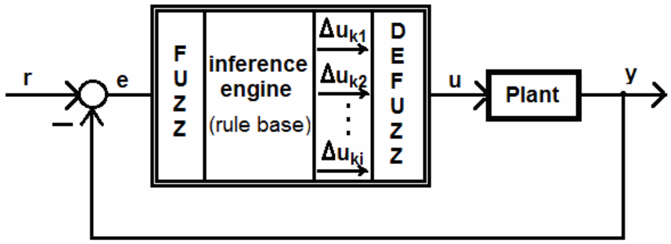

Figure 1. Structure of fuzzy control system with Takagi-Sugeno PI-fuzzy controller.

A low-cost design of the Takagi-Sugeno PI-fuzzy controller is based on the design of continuous-time PI controllers with the transfer function (t.f.) $C(s)$ for the plant with the transfer function $P(s)$:

$$C(s) = \frac{k_c}{s}(1 + T_c s) = k_C(1 + \frac{1}{sT_i}),\ T_c = T_i,\ k_c = \frac{k_C}{T_i}, \qquad (1)$$
$$P(s) = \frac{k_P}{s(1 + T_\Sigma s)},$$

where the tuning parameters of the PI controllers - the controller gain k_c and the integral time constant T_c - are obtained applying Modulus Optimum method (Åström and Hägglund, 1995):

$$T_c = T_1,\ k_c = \frac{1}{2k_P T_\Sigma}, \qquad (2)$$

k_P - the plant gain, and T_Σ - the small time constant or the algebraic sum of small time constants if the plant model in (1) is a simplified nonlinear plant model. The continuous PI controller is discretized using Tustin's method after setting the value of the sampling period T_s. The general recurrent equations of the discrete-time PI controller are expressed in (Precup and Preitl, 1999):

$$\Delta u_k^i = \gamma^i \left(K_P^i \Delta e_k^i + K_I^i e_k^i\right) = \gamma^i K_P^i \left(\Delta e_k^i + \alpha^i e_k^i\right), \qquad (3)$$
$$K_P^i = k_R^i (1 - \frac{T_s}{2T_i^i}), \quad K_I^i = \frac{k_R^i T_s}{T_i^i}, \quad \alpha^i = \frac{K_I^i}{K_P^i},$$

where i is the index of the controller used in the consequent of the rules of the TS-FC, Ts is the sampling period, e_k^i is the control error, Δe_k^i is the increment of control error, and Δu_k^i is the increment of control signal, K_P^i and K_I^i are the parameters of the PI controllers and the parameter γ^i introduce additional nonlinearities to adapt the performance control structure (Precup et al., 2008), (Sala, 2009). The TS-FC consists of *i* "IF-THEN" rules which can be expressed in terms of the following form:

$$\begin{array}{l} \text{IF} \quad (z_1 \text{ IS } TL_{z1}^i \text{ AND } z_2 \text{ IS } TL_{z2}^i \text{ AND} \ldots \text{AND } z_n \text{ IS } TL_{zn}^i) \\ \text{THEN} \quad u_k = \Delta \mathrm{u}_\mathrm{k}^\mathrm{i} \end{array}, \tag{4}$$

where TL_{zj}^i are the linguistic terms of the input linguistic variables (LVs) z_j, *j*=1..n, and u_k are the control signals produced by the rule consequents.

In order to solve the inference, an odd number of linguistic terms with membership functions with uniform distribution are used. The controllers can use the MIN and MAX operators in the inference engine and the weighted average method for defuzzification (Precup and Preitl, 1999), (Precup et al., 2008).

The parameters of the fuzzy controllers with output integration, B_e and $B_{\Delta e}$, are tuned according to the modal equivalence principle (Precup and Preitl, 1999), (Precup et al., 2008):

$$B_e = chosen, \; B_{\Delta e} = \frac{K_P^i}{K_I^i} B_{\Delta e} = \alpha^i B_e, \tag{5}$$

3. CASE STUDIES

This section is dedicated to the mathematical modelling of three mechatronic applications: the vehicular power train system (in Section 3.1), the electromagnetic actuated clutch (in Section 3.2) and the MLS2EM (in Section 3.3).

3.1. Vehicular power trains system with continuously variable transmission

3.1.1. The modeling of the vehicular power train system

The first plant taken into consideration in order to test the Takagi-Sugeno fuzzy controller is a vehicular power train system (VPT-S) with continuously variable transmission (Mussaeus, 1997), (Lazăr, 2009). The schematic structure of such a system is illustrated in Figure 2 (Dragoş et al., 2010b), (Dragoş et al., 2011).

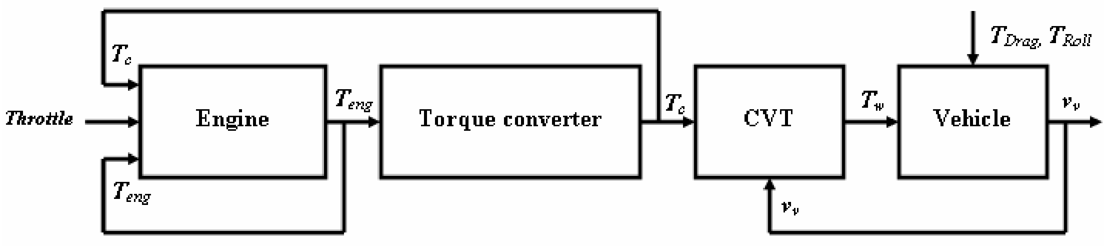

Figure 2. The schematic structure of a vehicular power train system.

The power train system's dynamics can be modeled in different ways depending on the purpose and on the first principle equations of the components of the system (Guzzella and Schmid, 1995), (Mussaeus, 1997), (Isermann, 2005), (Kiencke and Nielsen, 2005), (Modak and Sane, 2007):

- The electronic throttle:

$$\hat{throttle} = \frac{1}{1+T_{t_p} s} \cdot \hat{TP} . \tag{6}$$

where $T_{t_p} = 0.03$ is the relative electronic throttle position time constant and $\hat{TP}$ characterize the normalized values of electronic throttle position in percent.

- The Internal Combustion Engine:

$$\begin{aligned} &J_{eng} \cdot \dot{\omega}_{eng} = T_{eng} - T_c \\ &T_{eng} = \Gamma_1(throttle, \omega_{eng}) \\ &= \left[T_{\max} - \frac{T_{\max} - T_p}{(\omega_P - \omega_M)^2} \cdot (\omega_{eng} - \omega_M)^2 \right] \cdot throttle, \\ &0\% \le throttle \le 100\%. \end{aligned} . \tag{7}$$

- The Torque Converter:

$$\begin{aligned} &T_c = \omega_{eng}^2 / k^2, \\ &k = \Gamma_2(\omega_c, \omega_{eng}), \\ &i_{tq} = \Gamma_3(\omega_c, \omega_{eng}). \end{aligned} . \tag{8}$$

- The Continuously Variable Transmission:

$$\begin{aligned} &i_{CVT} = \Gamma_4(v_v), \quad T_{tr} = i_{CVT} \cdot i_{tq} \cdot T_c, \\ &T_w = i_{frg} \cdot T_{tr}, \quad \omega_c = i_{CVT} \cdot \omega_w. \end{aligned} . \tag{9}$$

- The Vehicle Dynamics:

$$\begin{aligned} &J_{vech}\cdot\dot{\omega}_w = T_w - T_{Drag} - T_{Roll} - T_{rez}, \\ &J_{veh} = r_w^2 m_v \\ &T_{Drag} = 0.5 * \mathrm{c} * \mathrm{A} * \rho * \mathrm{r}_\mathrm{w}^3 \cdot \mathrm{sgn}(\omega_w). \\ &T_{Roll} = \mathrm{c}_{\mathrm{Roll}} * \mathrm{m}_\mathrm{v} * \mathrm{g} * \mathrm{r}_\mathrm{W}, \\ &v_v = 3.6\cdot r_w \cdot \omega_w, \end{aligned} \tag{10}$$

The modification of the moment of inertia J_{vech} (as a parametric disturbance) is due to modification of vehicle mass, which can be expressed according to the additional weight (luggage and passengers):

$$m_v = m_0 + \Delta m_v \ , \tag{11}$$

where Δm_v varies between: $0 \le \Delta m_v \le 431$ kg.

The variation of the moment of inertia according to the vehicle mass affects directly the vehicle performances in terms of settling time to achieve a speed of 100 km/h, as it can be seen from Table 1.

Table 1: The influence of the variation of moment of inertia on the system performance.

Δm_v	J_{vech}	0-100 km/h
54	65.875	8.4 sec
200	78.375	9.88 sec
431	92.813	11.6 sec

So, on the basis of the primary equations of each subsystem of the vehicular power train system (Deur et al., 2006b), the new nonlinear mathematical model (NL-MM) results as follows:

$$\begin{aligned} &\dot{x}_1 = -\frac{1}{T_{t_p}}\cdot x_1 + \frac{1}{T_{t_p}}\cdot u, \\ &\dot{x}_2 = \frac{1}{J_{eng}}\cdot[T_{\max} - \frac{T_{\max}-T_p}{(\omega_p-\omega_M)^2}\cdot(x_2-\omega_M)^2]\cdot x_1 - \frac{1}{J_{eng}}\cdot\frac{x_2^4}{3.6^2\cdot k^2\cdot k_f^2\cdot x_3^2\cdot k_v^2\cdot r_w^2\cdot x_3^2}, \\ &\dot{x}_3 = \frac{1}{J_{vech}}\cdot\frac{i_{FRG}\cdot k_{tq}\cdot k_t\cdot x_2^3}{k^2\cdot k_f^2\cdot x_3} - \frac{1}{J_{vech}}(0.5\cdot\rho\cdot A\cdot c_{air}\cdot r_w^3\cdot x_3^2 - T_{Roll} - T_{rez}), \\ &y = 3.6\cdot 0.01\cdot r_w\cdot x_3, \end{aligned} \tag{12}$$

where the characteristic variables are: the control signal (input variable), $u=\hat{T}P$, the state variables: x_1=*throttle*, $x_2=\omega_{eng}$ and $x_3=\omega_w$, and the controlled output (output variable): $y=v_v$. Due to the nonlinearities of the systems, the new nonlinear model was linearized in the vicinity of nine operating points; they were chosen taking into account the position of the acceleration and the modification of the moment of inertia J_{vech} according to the additional weight (Dragoş, 2011). The linearized state space model is

$$\begin{aligned} &\Delta\underline{\dot{x}} = \underline{A}\Delta\underline{x} + \underline{b}\Delta u, \\ &\Delta y = \underline{c}^T \Delta\underline{x}, \end{aligned} \quad . \tag{13}$$

where the matrices *A*, *B* and c^T are detailed as follows:

$$\begin{aligned} &\underline{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} b_{11} \\ b_{12} \\ b_{13} \end{bmatrix}, \\ &\underline{c}^T = [c_{11} \quad c_{12} \quad c_{13}]. \end{aligned} \quad . \tag{14}$$

and their elements are detailed as follows:

$$\begin{aligned} &a_{11} = -33.3, \quad a_{12} = 0, \quad a_{13} = 0 \\ &a_{21} = -\frac{1}{J_{eng}} \cdot \left(T_{\max} - \frac{T_{\max} - T_p}{(\omega_p - \omega_M)^2} (x_{20} - \omega_M)^2 \right) \\ &a_{22} = -\frac{1}{J_{eng}} \cdot \left(\frac{T_{\max} - T_p}{(\omega_p - \omega_M)^2} \cdot (2 \cdot x_{20} - 2 \cdot \omega_M) \cdot x_{10} \right) - \frac{1}{J_{eng}} \cdot \left(\frac{4 \cdot x_{20}^3}{3.6^2 \cdot k^2 \cdot k_f^2 \cdot k_v^2 \cdot r_w^2 \cdot x_{30}^4} \right) \\ &a_{23} = \frac{1}{J_{eng}} \cdot \frac{4 \cdot x_{20}^4}{3.6^2 \cdot k^2 \cdot k_f^2 \cdot k_v^2 \cdot r_w^2 \cdot x_{30}^5} \\ &a31 = 0, \quad a32 = \frac{1}{J_{veh}} \cdot \left(\frac{i_{FRG} \cdot k_t \cdot k_{tq}}{k^2 \cdot k_f^2} \cdot \frac{3 \cdot x_{20}^2}{x_{30}} \right), \\ &a33 = -\frac{1}{J_{veh}} \cdot \left(\frac{i_{FRG} \cdot k_t \cdot k_{tq}}{k^2 \cdot k_f^2} \cdot \frac{x_{20}^3}{x_{30}^2} + c \cdot \rho \cdot A \cdot r_w^3 \cdot x_{30} \right) \\ &b_{11} = 33.3, \quad b_{21} = 0, \quad b_{31} = 0 \\ &c_{11} = 0, \quad c_{12} = 0, \quad c_{13} = 3.6 \cdot 0.01 \cdot r_w \end{aligned} \quad .$$

The nonlinear model of the system can be expressed after linearization as benchmark models, which are detailed in Table 2 as t.f.s. The t.fs only for three operating point are presented here, and they depend on the modification of the throttle position.

3.1.2. Design of the Takagi-Sugeno fuzzy controllers to control the vehicle speed. Simulation results

In mechatronic applications, both accuracy and robustness are key features in speed / position control, because load changes require a high degree of stability (Leva and Bascetta, 2006). In this context, two new TS-FCs with output integration have been developed for application (VPT-S) (Dragoş, 2011);

the two solutions are appropriated in terms of calculations, but different by fuzzy controller structure and by general conclusions. The developed TS-FCs are:

Table 2: The transfer functions for three operating points.

Operating points	Transfer functions $H_P(s)$
$\mathrm{u}_{01} = 5,\ J_{vech} = 65.875$	$\frac{1.47}{(1+15.13\cdot s)(1+0.3\cdot s)(1+0.03\cdot s)}$
$\mathrm{u}_{01} = 7,\ J_{vech} = 65.875$	$\frac{1.65}{(1+11.8\cdot s)(1+0.23\cdot s)(1+0.03\cdot s)}$
$\mathrm{u}_{01} = 9,\ J_{vech} = 65.875$	$\frac{1.75}{(1+9.82\cdot s)(1+0.3\cdot s)(1+0.03\cdot s)}$

A. TS-1-FC with output integration with non-homogeneous structure having three inputs – the control error, e_k^i, the increment of control error, Δe_k^i, and the angular vehicle speed, ω_w –, and one output – the increment of control signal, Δu_k^i, Figure 3.

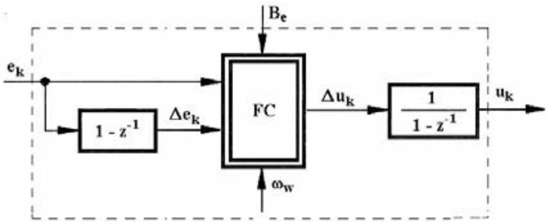

Figure 3. TS-1-FC with output integration with nonhomogeneous structure.

B. TS-2-FC with output integration with homogeneous structure having two inputs – the control error, e_k^i, and the increment of control error, Δe_k^i –, and one output – the increment of control signal, Δu_k^i, Figure 4.

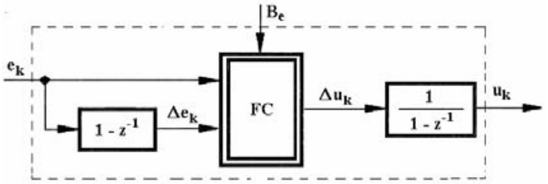

Figure 4. TS-2-FC with output integration with nonhomogeneous structure.

A. For the first case, three quasi-continuous digital PI controllers are obtained, which are used in the consequent rules. The numerical values of the parameters of the recurrent equations of the discrete-time PI controllers designed for the linear plants with the transfer functions detailed in Table 2 are presented in Table 3 (Dragoş et al., 2011).

For each input of the nonlinear TS fuzzy block, three linguistic terms with triangular and trapezoidal membership functions, $\mathrm{LTE}^i, \mathrm{LTDE}^i, \mathrm{LT}\omega\mathrm{w}^i \in \{\mathrm{N}, \mathrm{ZE}, \mathrm{P}\}$ are used. For the fuzzy block, the rule base is defined by 27 "IF-THEN" fuzzy rules as:

$$\begin{aligned}&\mathrm{IF}(e_k \text{ IS LTE}^i \text{ AND } \Delta e_k \text{ IS LTDE}^i \text{ AND } \omega_{wk} \text{ IS LT}\omega\mathrm{w}^i)\\&\text{THEN } \Delta u_k = \Delta u_k^i.\end{aligned} \quad . \tag{15}$$

Table 3: Numerical values of the parameters of recurrent equations.

Operating points	k_P^i	α^i	γ^i
$\mathrm{u}_{01} = 5, J_{vech} = 65.875$	14.54	0.0105	1
$\mathrm{u}_{01} = 7, J_{vech} = 65.875$	12.96	0.0105	0.1
$\mathrm{u}_{01} = 9, J_{vech} = 65.875$	11.92	0.0105	0.5

B. For the second case, nine quasi-continuous digital PI controllers are used, taking into account the linear PI controllers design for all nine operating points. The numerical values of the parameters of the recurrent equations of the discrete-time PI controllers are presented in Table 4 (Dragoş et al., 2011).

Table 4: Numerical values of the parameters of recurrent equations.

Operating points	K_P^i	α^i	γ^i
$\mathrm{u}_{01} = 5, J_{vech} = 65.875$	14.54	0.0105	1
$\mathrm{u}_{01} = 7, J_{vech} = 65.875$	12.96	0.0105	1
$\mathrm{u}_{01} = 9, J_{vech} = 65.875$	11.92	0.0105	1
$\mathrm{u}_{01} = 5, J_{vech} = 78.375$	1.0173	0.0105	1
$\mathrm{u}_{01} = 7, J_{vech} = 78.375$	1.1460	0.0105	1
$\mathrm{u}_{01} = 9, J_{vech} = 78.375$	1.2746	0.0105	1
$\mathrm{u}_{01} = 5, J_{vech} = 92.813$	1.0178	0.0105	1
$\mathrm{u}_{01} = 7, J_{vech} = 92.813$	1.1467	0.0105	1
$\mathrm{u}_{01} = 9, J_{vech} = 92.813$	1.2755	0.0105	1

The fuzzification problem was solved using three linguistic terms for each input variable with triangular and trapezoidal membership functions. The inference engine operates on the basis of the fuzzy rule base defined by 27 “IF-THEN” fuzzy rules expressed as:

$$\mathrm{IF}(e_k \text{ IS LTE}^i \text{ AND } \Delta e_k \text{ IS LTDE}^i) \text{ THEN } \Delta u_k = \Delta u_k^i. \tag{16}$$

In both cases, the TS fuzzy blocks use MAX and MIN operators in the inference engine and the weighted average method for the defuzzification. The modal equivalence principle leads to the parameters of the fuzzy controllers (Precup and Preitl, 1999), (Precup et al., 2008):

$$B_e = 7 \text{ (chosen) and } B_{\Delta e} = \alpha B_e. \tag{17}$$

In order to verify the two designed Takagi-Sugeno fuzzy control systems for the vehicular power train system with respect to the rectangular modifications (Cruise driving simulation scenario) of the reference input, two simulation scenarios were done (Dragoş, 2011):

a) the behavior of the fuzzy CS with TS-1-FC designed is illustrated in Figure 4;
b) the behavior of the fuzzy CS with TS-2-FC designed is illustrated in Figure 6.

In both cases, the responses of the following variables are plotted versus time: vehicle speed (a), iCVT (b) and angular engine speed (c).

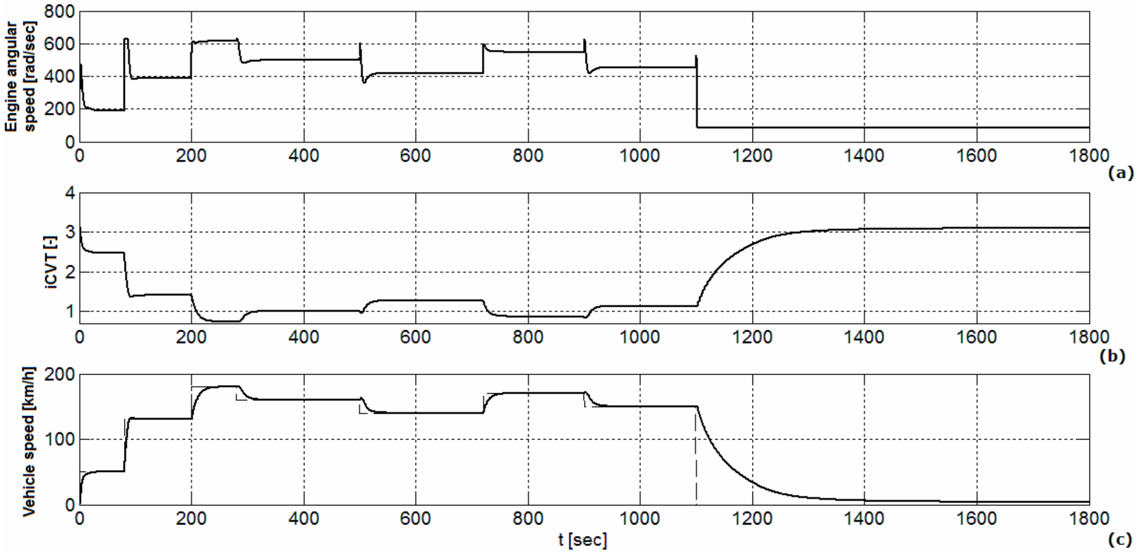

Figure 5. Simulation results for TS-1-FC structure developed for STPA in cruise operating scenario: (a) engine angular speed (b) iCVT and (c) vehicle speed.

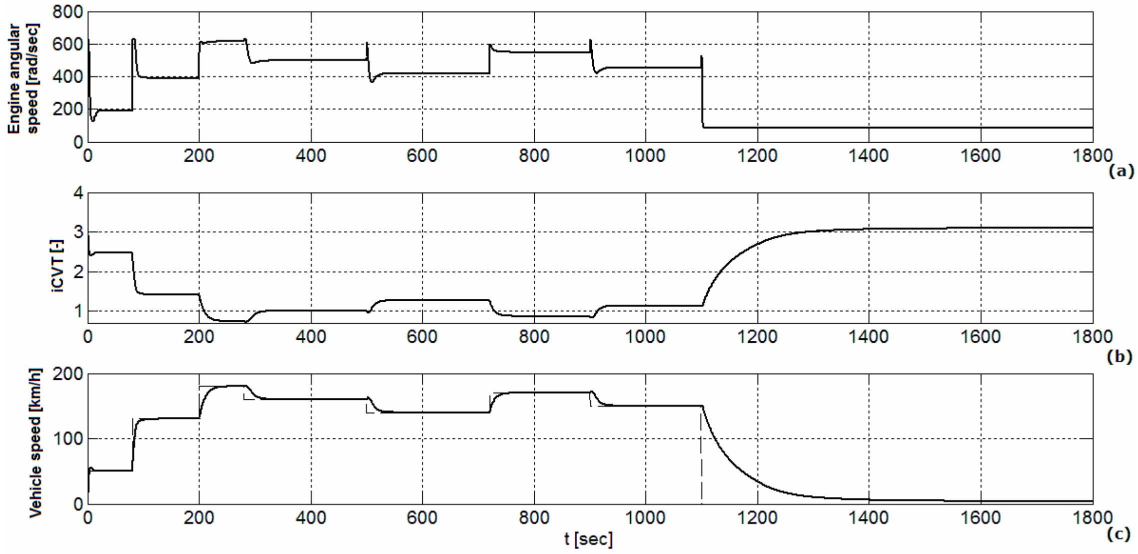

Figure 6. Simulation results for TS-2-FC structure developed for STPA in cruise operating scenario: (a) engine angular speed (b) iCVT and (c) vehicle speed.

Analyzing the simulation results presented in Figure 5 and Figure 6, can be noticed that the system response tracks very well the reference signal, which was defined in order to imitate a real case of a vehicle in traffic. Therefore, the TS-FC structure seems to be very useful to solve the tracking problem due to the flexibility of modifying the transfer properties of the TS-FC with respect to the operating point modifications.

3.2. Magnetically actuated clutch system

3.2.1. The modeling of the magnetically actuated clutch system

The second case analyzed in this paper is a magnetically actuated clutch system (MAC-S) (Lazăr, 2009), whose mathematical modeling is based on the dynamic information of a mechanical subsystem actuated by the electromagnetic subsystem according to Figure 7 (Di Cairano et al., 2007).

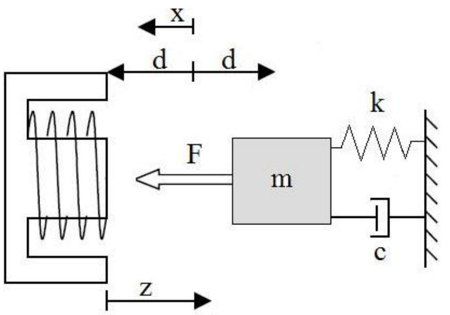

Figure 7. Schematic structure of magnetically actuated mass-spring-damper system.

Starting with the first principle equations which describe the physical plant, the nonlinear state-space mathematical model (MM) of the controlled plant can be expressed as (Di Cairano et al., 2007), (Dragoş, 2011), (Dragoş et al., 2011):

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -\dfrac{k}{m} x_1 - \dfrac{c}{m} x_2 + \dfrac{k_a x_3^2}{m\,(k_b + d - x_1)^2}, \\ \dot{x}_3 = -\dfrac{R\,(k_b + d - x_1)}{2k_a} x_3 - \dfrac{1}{k_b + d - x_1} x_2 x_3 + \dfrac{(k_b + d - x_1)}{2k_a} V, \\ y = 1000 x_1, \end{cases} \tag{18}$$

where $x_1 = x$ is the mass position, $x_2 = \dot{x}$ is the mass speed, $x_3 = i$ is the current, *V* – the control signal, *y* – the controlled output, *k* – the stiffness of the spring, *c* – the coefficient of the damper, *R* – the electromagnetic coil resistance, and *k_a*, *k_b* – the constants in the relation between the magnetic flux and the current. The numerical values of the plant parameters are given in (Dragoş, 2011), (Dragoş et al., 2011). The MAC-S is subject to constraints of the form (Di Cairano et al., 2007):

- the constraint $-0.004 \le x \le 0.004$ - to avoid the undesired instability movement mass;
- the current can not be negative, $i \ge 0$, so the electromagnetic force can only attract => $F \ge 0$.

The input-output static map of MAC-S is presented in Figure 8; this characteristic has a typical shape of such a servo-system. Analyzing the input-output static map can be noticed that it is therefore necessary the using of a Takagi-Sugeno fuzzy control structure (Bellomo et al., 2008), (Bei, 2009).

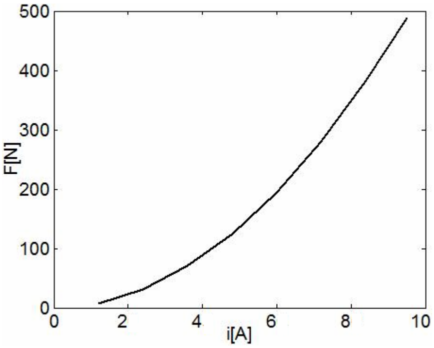

Figure 8. Input-output static map *F* versus *i*.

Due to the nonlinearity of the system, the MM of the plant was linearized around several operating points from the static input-output map and several state-space linearized models were obtained:

$$
\begin{aligned}
&\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{b}\,\Delta V(t),\\
&\Delta y(t) = \mathbf{c}^T \mathbf{x}(t),\\
&\mathbf{x} = [x_1 = x \quad x_2 = \dot{x} \quad x_3 = i]^T,\\
&\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} + \frac{2k_a x_{30}^2}{m(k_b + d - x_{10})^3} & -\frac{c}{m} & \frac{2k_a x_{30}}{m(k_b + d - x_{10})^2} \\ \frac{Rx_{30} - V_0}{2k_a} - \frac{x_{20}x_{30}}{(k_b + d - x_{10})^2} & -\frac{x_{30}}{k_b + d - x_{10}} & -\frac{x_{20}}{k_b + d - x_{10}} - \frac{R(k_b + d - x_{10})}{2k_a} \end{bmatrix},\\
&\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \frac{k_b + d - x_{10}}{2k_a} \end{bmatrix}, \mathbf{c}^T = [1000 \quad 0 \quad 0].
\end{aligned}
\tag{19}
$$

The linearized models can be detailed as t.f.s synthesized in Table 5 for the three operating points selected from the input-output static map (Dragoş, 2011), (Dragoş et al., 2011).

3.2.2. Design of the Takagi-Sugeno fuzzy controllers to control the position mass. Simulation results

The design of TS-FC starts with the using of linear PI controllers designed for the position control of MAC-S (Dragoş et al., 2010c), (Dragoş, 2011), (Dragoş et al., 2011), which were discretized using Tustin's method with the sampling period T_s.

Table 5: Transfer functions of plant.

Number of operating point	$H_P(s)$
(2)	$\dfrac{0.38}{(1+0.066s)(1+0.0164s)(1+0.0016s)}$
(3)	$\dfrac{0.47}{(1+0.07s)(1+0.0162s)(1+0.0016s)}$
(4)	$\dfrac{0.58}{(1+0.077s)(1+0.016s)(1+0.0016s)}$

Three quasi-continuous digital PI controllers are used in the consequent rule. The numerical values of the parameters of PI controllers, K_P^i and K_I^i, and the parameter γ^i are detailed in Table 6.

Table 6: Numerical values of the parameters of recurrent equations.

Operating points	K_P^i	K_I^i	γ^i
(2)	4.61	0.49	0.09
(3)	4.03	0.42	0.09
(4)	3.58	0.37	0.09

For each input of the nonlinear TS fuzzy block, three linguistic terms with triangular and trapezoidal membership functions, Figure 9, were used.

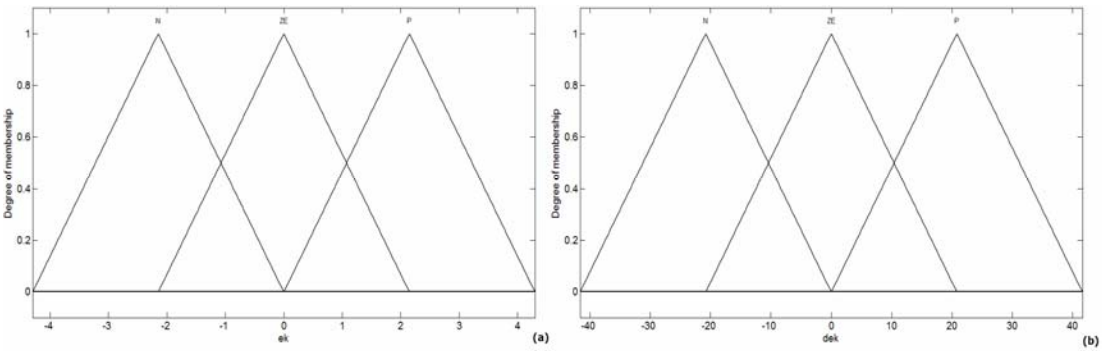

Figure 9. Input membership functions of TS-FC, e_k and Δe_k.

The parameters of the TS fuzzy controller with output integration, B_e and $B_{\Delta e}$, are tuned as follows:

$$B_e = 2.15\,(\text{chosen}),\ B_{\Delta e} = (K_P^{(4)} / K_I^{(4)}) B_e = 20.8\,. \tag{20}$$

The block TS-FC makes use of the weighted average method for defuzzification. The inference engine uses the MAX and MIN operators. The TS-FC consists of nine “IF-THEN” rules which can be expressed as:

$$
\begin{aligned}
&\text{Rule 1: IF } e(k) \text{ IS N AND } \Delta e(k) \text{ IS P THEN } \Delta u(k) = \gamma[K_P^1 \Delta e_k + \alpha K_P^1 e_k],\\
&\text{Rule 2: IF } e(k) \text{ IS ZE AND } \Delta e(k) \text{ IS P THEN } \Delta u(k) = \gamma[K_P^2 \Delta e_k + \alpha K_P^2 e_k],\\
&\text{Rule 3: IF } e(k) \text{ IS P AND } \Delta e(k) \text{ IS P THEN } \Delta u(k) = \gamma[K_P^3 \Delta e_k + \alpha K_P^3 e_k],\\
&\text{Rule 4: IF } e(k) \text{ IS N AND } \Delta e(k) \text{ IS ZE THEN } \Delta u(k) = \gamma[K_P^1 \Delta e_k + \alpha K_P^1 e_k],\\
&\text{Rule 5: IF } e(k) \text{ IS ZE AND } \Delta e(k) \text{ IS ZE THEN } \Delta u(k) = \gamma[K_P^2 \Delta e_k + \alpha K_P^2 e_k], \cdot\\
&\text{Rule 6: IF } e(k) \text{ IS P AND } \Delta e(k) \text{ IS ZE THEN } \Delta u(k) = \gamma[K_P^3 \Delta e_k + \alpha K_P^3 e_k],\\
&\text{Rule 7: IF } e(k) \text{ IS N AND } \Delta e(k) \text{ IS N THEN } \Delta u(k) = \gamma[K_P^1 \Delta e_k + \alpha K_P^1 e_k],\\
&\text{Rule 8: IF } e(k) \text{ IS ZE AND } \Delta e(k) \text{ IS N THEN } \Delta u(k) = \gamma[K_P^2 \Delta e_k + \alpha K_P^2 e_k],\\
&\text{Rule 9: IF } e(k) \text{ IS P AND } \Delta e(k) \text{ IS N THEN } \Delta u(k) = \gamma[K_P^3 \Delta e_k + \alpha K_P^3 e_k].
\end{aligned}
\tag{21}
$$

The block diagram illustrated in Figure 1 was used to verify the TS-FC designed for the position control of a magnetically actuated clutch. The behavior of the system response with respect to the rectangular modifications of the reference input *w* is detailed in Figure 10. Analyzing the simulation results, can be noticed that the reference input is tracked and the system response presents a small overshoot.

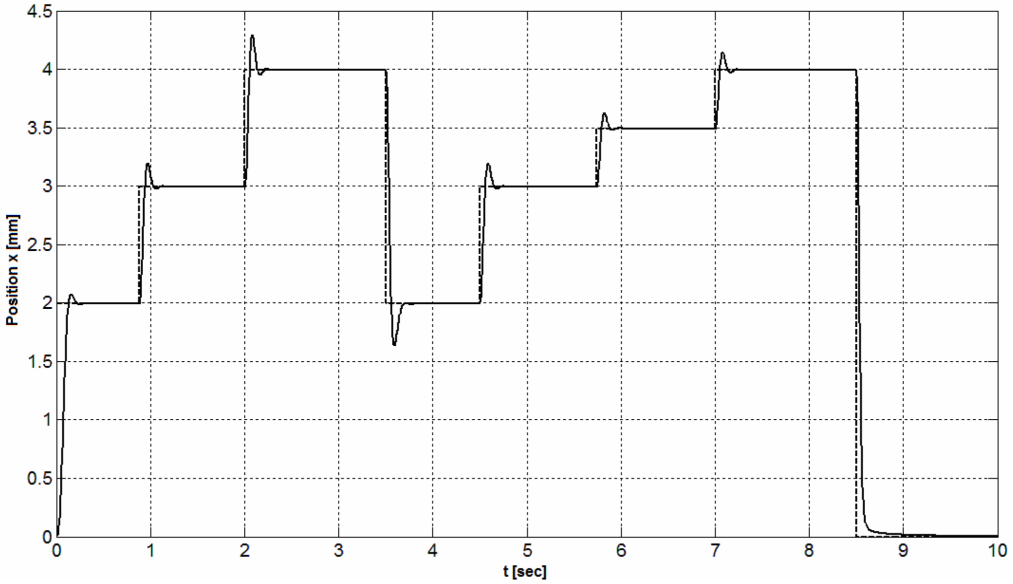

Figure 10. Simulation results of CS with TS-FC designed for the magnetically actuated clutch.

3.3. Magnetic levitation system with two electromagnets

3.3.1. The modeling of magnetic levitation system with two electromagnets

The third approached plant is the nonlinear and unstable laboratory equipment, the MLS2EM. This magnetic levitation problem for a metallic ball maintained in an electromagnetic field is attractive because it is a classical nonlinear and unstable application (Shameli et al., 2007). The nonlinear mathematical model of the MLS2EM can be obtained from the first principle equations and can be expressed as follows (Inteco, 2008):

$$
\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = -\frac{1}{m}\cdot\frac{F_{emP1}}{F_{emP2}}\cdot e^{-\frac{x_1}{F_{emP1}}}\cdot x_3^2 + g + \frac{1}{m}\cdot\frac{F_{emP1}}{F_{emP2}}\cdot e^{-\frac{x_d - x_1}{F_{emP2}}}\cdot x_4^2 \\
\dot{x}_3 = \frac{1}{\frac{f_{iP1}}{f_{iP2}}\cdot e^{-\frac{x_1}{f_{iP2}}}}(k_i u_1 + c_i - x_3) \\
\dot{x}_4 = \frac{1}{\frac{f_{iP1}}{f_{iP2}}\cdot e^{-\frac{x_d - x_1}{f_{iP2}}}}(k_i u_2 + c_i - x_4)
\end{cases} . \tag{22}
$$

The parameters of the MLS2EM used in the analysis and the design of control structures are presented in (Dragoş et al., 2010e), (Dragoş, 2011). Due to the nonlinearity and to instability of the plant, the nonlinear model was linearized around three operating points in order to design a fuzzy control structure (Škrjanc et al., 2003), (Shameli et al., 2007); the following state-space linearized model was obtained:

$$
\begin{cases}
\underline{\dot{x}} = \underline{A}\,\underline{x} + \underline{B}\,\Delta u \\
\Delta y = \underline{c}^T\,\underline{x}
\end{cases}
,\ \underline{x} = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix}
$$

$$
\underline{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix},\ \underline{b} = \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \\ b_{41} \end{bmatrix},\ \underline{c}^T = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \end{bmatrix} . \tag{23}
$$

where its parameters are detailed in (Dragoş et al., 2010d) and (Dragoş, 2011).

3.3.2. Design of the Takagi-Sugeno fuzzy controllers to control the position sphere. Simulation results.

In this section a cascade control system structure, with a state feedback control system (SFCS) as the inner control loop and a conventional control system structure with TS-FC in the outer control loop, is designed for the MLS2EM, Figure 11.

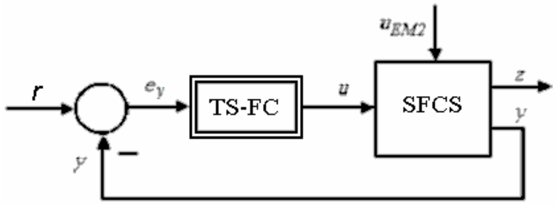

Figure 11. Block diagram of cascade control system structure.

The design is based on reducing the order of the linear system model from a fourth-order system to a third-order system with the following state variables: the position; the speed and the control error. The transfer functions obtained for three operating points are detailed in Table 7.

Table 7: Numerical values of the parameters of fuzzy controller.

Operating points $(x_{10}, x_{20}, x_{30}, x_{40})$	$H_P(s)$
0.007; 0; 0.3; 0;	$\frac{0.0336}{(1+0.0064s)(1-0.00873s+0.00146s^2)}$
0.008; 0; 0.285; 0;	$\frac{0.0346}{(1+0.0053s)(1-0.0071s+0.0018s^2)}$
0.009; 0; 0.6; 0;	$\frac{0.02}{(1+0.0041s)(1-0.0059s+0.00062s^2)}$

Therefore, in order to stabilize the sphere in the MLS2EM, a state feedback control structure (SFCS) (Lee et al., 2007) was designed and the state feedback gain matrix $\underline{k}_c^T = [36 \quad 5 \quad 0.0075]$ was obtained by applying the pole placement method to the linearized state-space models.

In the next step, to ensure the condition of zero steady-state control error the TS-FC (Wu and Hu, 2009) in designed for the outer control loop. The TS-FC design is based on the parameters of conventional PI controllers, which were discretized using Tustin's method with the sampling period T_s. Three quasi-continuous digital PI controllers are obtained. The numerical values of the parameters K_P^i, α^i and γ^i, are detailed in Table 8.

Table 8: Numerical values of the parameters of fuzzy controller.

Operating points $(x_{10}, x_{20}, x_{30}, x_{40})$	Parameters of TS-FC		
	K_P^i	α^i	γ^i
0.007; 0; 0.3; 0;	134.74	0.0038	0.217
0.008; 0; 0.285; 0;	132.47	0.0038	0.44
0.009; 0; 0.6; 0;	216.47	0.0038	0.6

Three linguistic terms with triangular membership functions (N, ZE, P) are used for each input variable of the nonlinear TS fuzzy block, Figure 12. The modal equivalence principle leads to the parameters of the fuzzy controller (Dragoş et al., 2010e), (Dragoş, 2011):

$$B_e = 0.05\ (chosen),\ B_{\Delta e} = \frac{k_P}{k_I} B_{\Delta e} = \alpha B_e\,. \tag{24}$$

The controller with this structure produces the nonlinear input-output map presented in Figure 13.

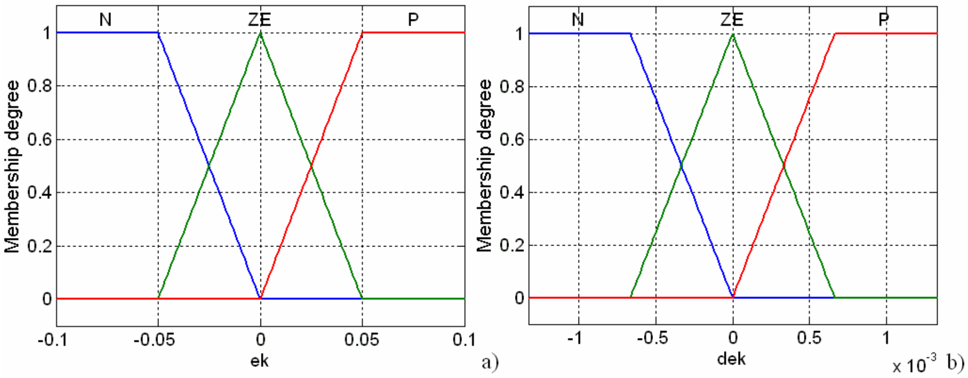

Figure 12. Input membership functions of fuzzy sets corresponding to linguistic terms of input LVs e_k and Δe_k.

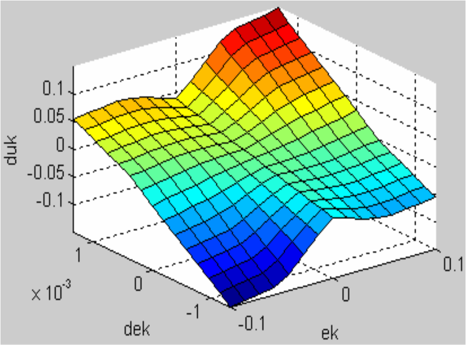

Figure 13. Nonlinear input-output map of TS-FC block.

Real-time experimental results for the TS fuzzy CSs are presented in Figure 14. The experimental scenario is characterized by the application of some disturbance signals to the bottom electromagnet as: pulse with modulator, sinusoidal signal and pseudo-random binary signal. The experimental results include the evolutions of the (a) sphere position, (b) currents in EM1 and EM2 and (c) control signals applied to EM1 and EM2 versus time. The reference input *r* is tracked.

4. CONCLUSIONS

This paper has offered a Takagi-Sugeno fuzzy control solution dedicated to the speed and position control of three mechatronic applications: a vehicle power train system with continuously variable transmission, an electromagnetic actuated clutch system and a magnetic levitation system with two electromagnets laboratory equipment. The models of the controlled plants were linearized in the vicinity of several operating points starting with the nonlinear models and with its identified parameters.

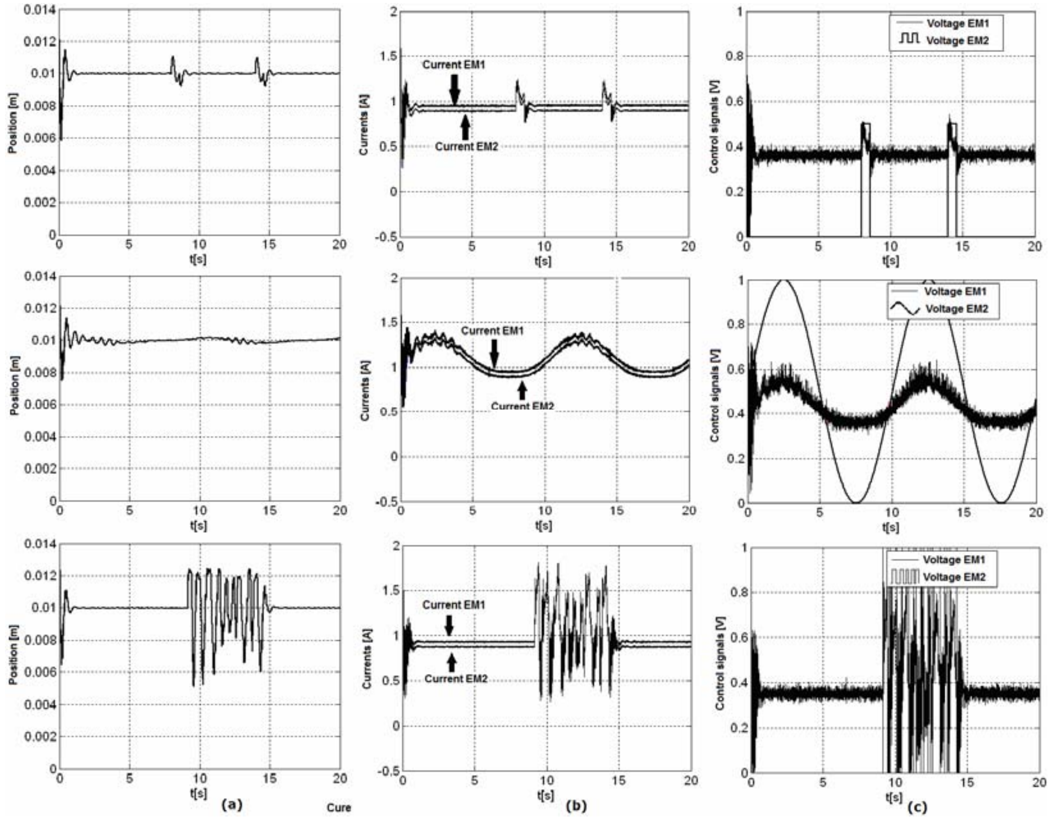

Figure 14. Experimental results for the TS fuzzy CS: (a) position of sphere, (b) currents and (c) voltages versus time.

The simulation and real-time experimental results validate the control solutions and their design method. The fuzzy controllers design and tuning guarantee the improvement of the control system performance regarding the modifications of reference input and the rejection of disturbances signal as pulse with modulator, sinusoidal signal and pseudo-random binary signal. They also ensure zero steady-state control error, small settling time and small overshoot.

The future research will be focused on the stability analysis, robustness and sensitivity analysis and on the extension of the control structures by inserting additional functionalities and on the improvement of the performance indices. Other applications will be handled as the implementation of fuzzy control solutions presented here should be accomplished after the careful analysis of those applications (Angelov et al., 2008), (Yu and Kaynak, 2009), (Vaščák and Madarász, 2010), (Derr and Manic, 2011), (Milojković et al., 2010), (Cotton and Wilamowski, 2011), (Joelianto and Wiranto, 2011), (Vaščák and Hirota, 2011), (Zhou and Tan, 2011).

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REFERENCES

Albertos, P., 2002, Fuzzy logic control: light and shadow. *IFAC Newsletter* **26**, 1–2.

Angelov, P., Lughofer, E., Zhou ,X., 2008, Evolving fuzzy classifiers with different architectures. *Fuzzy Sets and Systems* **159**, 3160–3182.

Åström, K.J., Hägglund, T., 1995, *PID Controllers Theory: Design and Tuning.* Research Triangle Park, NC.

Baranyi, P., Kóczy, L.T., 1996, A general and specialised solid cutting method for fuzzy rule interpolation. *Journal of BUSEFAL* **66**, 13–22.

Baranyi, P., Yam, Y., Várkonyi-Kóczy, A., Patton, R.J., 2003, SVD based reduction to MISO TS fuzzy models. *IEEE Transactions on Industrial Electronics* **50**, 232–242.

Bei, S.-Y., 2009, Fuzzy controller for automotive semi-active suspension based on damping control. *Proceedings of 2009 ISECS International Colloquium on Computing, Communication, Control, and Management (CCCM 2009)*, Sanya, China, 296–299.

Bellomo, D., Naso, D., Babuška, R., 2008, Adaptive fuzzy control of a non-linear servo-drive: theory and experimental results. *Engineering Applications of Artificial Intelligence* **21**, 846–857.

Bishop, R.H., 2002, *The Mechatronics Handbook*. CRC Press, Boca Raton, FL.

Bishop, R.H., 2007, *Mechatronic Systems, Sensor, and Actuators: Fundamentals and Modeling*. CRC Press, Boca Raton, FL.

Cotton, N.J., Wilamowski, B.M., 2011, Compensation of nonlinearities using neural networks implemented on inexpensive microcontrollers. *IEEE Transactions on Industrial Electronics* **58**, 733–740.

Deur, J., Asgari, J., Hrovat, D., 2006a, Modeling and analysis of automatic transmission engagement dynamics - Nonlinear case including validation. *Transactions ASME, Journal of Dynamic Systems, Measurement, and Control* **128**, 251–262.

Deur, J., Petric, J., Asgari, J., Hrovat, D., 2006b, Recent advances in control-oriented modeling of automotive power train dynamics. *IEEE/ASME Transactions on Mechatronics* **11**, 513–523.

Di Cairano, S., Bemporad, A., Kolmanovsky, I.V., Hrovat, D., 2007, Model predictive control of magnetically actuated mass spring dampers for automotive applications. *International Journal of Control*, **80**, 1701–1716.

Derr, K., Manic, M., 2011, Extended Virtual Spring Mesh (EVSM): The distributed self-organizing mobile ad hoc network for area exploration. *IEEE Transactions on Industrial Electronics* **58**, 5424–5437.

Dragoş, C.-A., 2011, Modern model-based control solutions applied to mechatronics systems. PhD Thesis, “Politehnica” Univ. Timisoara, Timisoara, Romania, 172 pp.

Dragoş ,C.-A., Preitl, S., Precup, R.-E., Bulzan, R.-G., Pozna, C., Tar, J. K., 2010a, Takagi-Sugeno fuzzy controller for a magnetic levitation system laboratory equipment. *Proceedings of International Joint Conferences on Computational Cybernetics and Technical Informatics (ICCC-CONTI 2010)*, Timisoara, Romania, 55–60.

Dragoş, C.-A., Preitl, S., Precup, R.-E., Pirlea, D., Neş, C.-S., Petriu, E.M., Pozna, C., 2010b, Modeling of a vehicle with continuously variable transmission. *Proceedings of 19th International Workshop on Robotics în Alpe-Adria-Danube Region (RAAD 2010)*, Budapest, Hungary, 441–446.

Dragoş, C.-A., Precup, R.-E., Preitl, S., Rădac, M.-B., 2010c, Low-cost fuzzy control solutions for electromechanical applications. *Proceedings of 2nd International Scientific and Expert Conference (TEAM 2010)*, Kecskemet, Hungary, 10–23.

Dragoş, C.-A., Preitl, S., Precup, R.-E., Neş, C.-S., Pirlea, D., Paul, A.S., 2010d, Control solutions for vehicles with continuously variable transmission. *Proceedings of 11th IEEE International Symposium on Computational Intelligence and Informatics (CINTI 2010)*, Budapest, Hungary, 157–162.

Dragoş, C.-A., Preitl, S., Precup, R.-E., Creţiu, M., Fodor J., 2010e, Modern control solutions with applications in mechatronic systems, *Computational Intelligence in Engineering*, Eds. Rudas, I. J., Fodor, J. and Kacprzyk, J. (Springer-Verlag), 87–102.

Dragoş, C.-A., Preitl, S., Precup, R.-E., Petriu, E.M., Stînean, A.-I., 2011, A comparative case study of position control solutions for a mechatronics application. *Proceedings of 2011 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM 2011)*, Budapest, Hungary, 814–819.

Gillespie, T.-D., 1992, *Fundamentals of Vehicle Dynamics.* Society of Automotive Engineers, Inc, 226–230.

Guzzella, L., Schmid, A.M., 1995, Feedback linearization of spark-ignition engines with continuously variable transmissions. *IEEE Transactions on Control Systems Technology* **3**, 54–60.

Inteco Ltd, 2008, *Magnetic Levitation System 2EM (MLS2EM)*, User's Manual (Laboratory Set), Krakow, Poland: Inteco Ltd.

Isermann, R., 2005, *Mechatronic Systems: Fundamentals*. Springer-Verlag, Berlin, Heidelberg, New York.

Joelianto, E., Wiranto, I., 2011, An application of ant colony optimization, Kalman filter and artificial neural network for multiple target tracking problems. *International Journal of Artificial Intelligence* **7**, 384–400.

Johanyák, Z.C., 2010, Student evaluation based on fuzzy rule interpolation. *International Journal of Artificial Intelligence* **5**, 37–55.

Johanyák, Z.C., Kovács, S., 2006, Fuzzy rule interpolation based on polar cuts. In *Computational Intelligence, Theory and Applications*, B. Reusch (Ed.), Springer Verlag, Berlin, Heidelberg, New York, 499–511.

Kiencke, U., Nielsen, L., 2005, *Automotive Control Systems for Engine, Driveline and Vehicle*, 2nd ed. Springer-Verlag, Berlin, Heidelberg, New York.

Lazăr, C. et al., 2010, *Real-time informatics technologies for embedded-system-control of power-train in automotive design and applications*, Research Report 2 of the SICONA CNMP Grant, "Gh. Asachi" Tech. Univ. Iasi, Iasi, Romania (in Romanian).

Lee T.-E., Su J.-P., Yu K.-W., 2007, Implementation of the state feedback control scheme for a magnetic levitation system. *Proceedings of 2nd IEEE Conference on Industrial Electronics and Applications (ICIEA 2007)*, Harbin, China, 548–553.

Leva, A., Bascetta, L., 2006, On the design of the feedforward compensator in two-degree-of-freedom controllers. *Mechatronics* **16**, 533–546.

Linda, O., Manic, M., 2011, Interval Type-2 fuzzy voter design for fault tolerant systems. *Information Sciences* **181**, 2933–2950.

Milojković, M., Nikolić, S., Danković, B., Antić, D., Jovanović, Z., 2010, Modelling of dynamical systems based on almost orthogonal polynomials. *Mathematical and Computer Modelling of Dynamical Systems* **16**, 133–144.

Modak G.S., Sane S.S., 2007, Mechanical continuously variable transmission (CVT) for parallel hybrid vehicle. *Proceedings of IEEE Conference on Electric and Hybrid Vehicles (ICEHV '06)*, Pune, India, 4 pp.

Mussaeus, M., 1997, *Control issues of hybrid and conventional drive lines*, M.Sc. Thesis, Dept. Mechanical Engineering, Section Systems and Control, Eindhoven Univ. Technology, Eindhoven, The Netherlands.

Precup, R.-E., Preitl, S., 1999, *Fuzzy Controllers*. Orizonturi Universitare Publishers, Timisoara.

Precup, R.-E., Preitl, S., Rudas, I.J., Tomescu, M.L., Tar, J.K., 2008, Design and experiments for a class of fuzzy controlled servo systems. *IEEE/ASME Transactions on Mechatronics* **13**, 22–35.

Precup, R.-E., Preitl, S., Rădac, M.-B., Petriu, E.M., Dragoş, C.-A., Tar, J.K., 2011, Experiment-based teaching in advanced control engineering. *IEEE Transactions on Education* **54**, 345–355.

Sala, A., 2009, On the conservativeness of fuzzy and fuzzy-polynomial control of nonlinear systems. *Annual Reviews in Control* **33**, 48–58.

Setlur, P., Wagner, J.R., Dawson, D.M., Samuels, B., 2003, Nonlinear control of a continuously variable transmission (CVT). *IEEE Transactions on Control Systems Technology* **11**, 101–108.

Shameli, E., Khamesee, M.B., Huissoon, J.P., 2007, Nonlinear controller design for a magnetic levitation device. *Journal of Microsystem Technologies* **13**, 831–835.

Škrjanc, I., Blažič, S., Agamennoni, O.E., 2005, Identification of dynamical systems with a robust interval fuzzy model. *Automatica* **41**, 327–332.

Škrjanc, I., Blažič, S., Matko, D., 2003, Model-reference fuzzy adaptive control as a framework for nonlinear system control. *Journal of Intelligent and Robotic Systems* **36**, 331–347.

Song, J., Wang, C., 2008, Modeling and simulation of hydraulic control system for vehicle continuously variable transmission. *Proceedings of 3rd IEEE Conference on Industrial Electronics and Applications (ICIEA 2008)*, Singapore, 799–803.

Vaščák, J., Hirota, K., 2011, Integrated decision-making system for robot soccer. *Journal of Advanced Computational Intelligence and Intelligent Informatics* **15**, 156–163.

Vaščák, J., Madarász, L., 2010, Adaptation of fuzzy cognitive maps - a comparison study. *Acta Polytechnica Hungarica* **7**, 109–122.

Wu, H., Hu, Y., 2009, Study on fuzzy control algorithm for magnetic levitated platform. *Proceedings of 2009 International Conference on Measuring Technology and Mechatronics Automation (ICMTMA 2009),* Hunan, China, 598–601.

Yu, X., Kaynak, O., 2009, Sliding-mode control with soft computing: A survey. *IEEE Transactions on Industrial Electronics* **56**, 3275–3285.

Zhou, M., Wang, X., Zhou, Y., 2006, Modeling and simulation of continuously variable transmission for passenger car. *Proceedings of 1st International Forum on Strategic Technology*, Ulsan, Korea, 100–103.

Zhou, Y., Tan, Y., 2011, GPU-based parallel multi-objective particle swarm optimization. *International Journal of Artificial Intelligence* **7**, 125–141.