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Lease Contracts with Servicing Strategy Model for Used Product Considering Crisp and Fuzzy Usage Rates

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ABSTRACT

Leasing activity is becoming popular in recent years for several reasons. One among the reasons is in regards to extremely high cost of purchasing for some products or equipment, e.g. heavy equipment used in a mining industry and other manufactures using sophisticated equipment. In these areas, due to the expensive production equipment, many operators adopt a leasing contract, rather than purchasing the equipment directly. In this paper, we study lease contracts for used product involving servicing strategy. The lessor (agent) will give servicing strategies which allows more than one imperfect repair to the lessee. A penalty cost incurred when the time required to perform a repair action exceeds a predetermined target. The servicing strategy proposed reduces equipment failures and hence decreases total maintenance cost during the lease contract. We develop a mathematical model to find the optimal maintenance strategy, such as the cost structure and the maintenance degree during the lease period. We find the optimal maintenance level and the threshold values for the servicing strategy such that to minimize the expected total maintenance cost. Numerical examples are presented to illustrate the optimal solution, firstly by considering crisp usage rates level and secondly by considering fuzzy usage rates level.

Keywords: Servicing strategy, imperfect repair, expected cost, lease contract, fuzzy usage rate.

Mathematics Subject Classification: 90B25, 60K10, 91B70

Computing Classification System: G.3

1. INTRODUCTION

Lease is a contract where one party grants the right to use a particular equipment for a period of time to the other party. The party who gives the right is the lessor, while the other party is the lessee (or tenant), who will pay for the transfer of the right to use that particular equipment an agreed amount regularly to the lessor. Leasing activity is becoming popular nowadays for several reasons. One among the reasons is especially in regards to the extremely high cost of purchasing for some product or equipment, *e.g.* heavy equipment used in a mining industry. In this area, due to the expensive production equipment, many operators adopt a leasing contract, rather than purchasing them. Hajej *et al.* (2015) argued that a lease contract contains the possibility of obtaining an extended warranty for a given additional cost. During the contract period, the lessee is free from some maintenance problem, since it is covered in the contract. This is among other benefits comes up from the leasing contract.

On the other hand, for heavy, sophisticated, and expensive equipment, used equipment is becoming more often available in the market. As an example, used heavy equipment such as wheel loader, hydraulic excavator, bulldozer, valmet, and articulated dump trucks are commonly advertised. In the advertisement, not only the price but other details of the products, such as the age and usage data are available to the prospective costumer. National vendor such as the United Tractor has put an advertisement in web regarding the used product together with the aforementioned details in a webpage (http://www.unitedtractors.com/id/used-equipment). Used product has a significantly lower price compared to the new one. As a rule of thumb, the price of used equipment is lower compared to the new one depending on some factors. Even well-known or reputed branded equipment are far below normal price. Moreover, evidences suggest that used products are customary priced at 45% - 65% of comparable new product. There are eight factors affecting the price of used product. Two related ones are age and conditions. Other factors are listed in a commercial web such as https://www.rbauction.com/blog/8-factors-influencing-the-price-of-used-heavy-equipment.

Beside price, there are other reasons why used equipment are becoming popular and sought after in the market. It is an environmental concern, which often called as green technology or green engineering. Anastas and Zimmerman (2003) pointed out that there are 12 principles of green technology, *i.e.* Inherent Rather Than Circumstantial; Prevention Instead of Treatment; Design for Separation; Maximize Efficiency Output-Pulled Versus Input-Pushed; Conserve Complexity; Durability Rather Than Immortality; Meet Need, Minimize Excess; Minimize Material Diversity; Integrate Material and Energy Flows; Design for Commercial "Afterlife"; Renewable Rather Than Depleting. Other party rising The San Destin Declaration with 9 Principles of Green Engineering, in which all engineering activities should target to strive to prevent waste and create engineering adhere to environmentally friendly activities (see Abraham and Nguyen, 2004). The action such as marketing refurbished articulated trucks has been considered as offering price and environmental benefits (http://www.hsssearch.co.uk/page_298340.asp). Hence buying refurbished equipment is budget-friendly as well as eco-friendly.

Examples for other products are also abound, *e.g.* IT products. In 2013, an environmentally responsible networking, such as Curvature (https://www.curvature.com/GreenIT), argued that 501,875 items they shipped were pre-owned/used. This resulted in 1,651,361 kgs of potential e-waste that was saved from landfills. They said this amount is equivalent to 1,820 tons and represents nearly 3,150 dump trucks lined up for almost 13 kilometers. Hence buying a refurbished is significantly saving our natural environment. With this concept in mind, now more and more used material are in trade (https://www.tradeplantequipment.com.au/search/type-cranes/subtype-cranes+and+lifting/class-hydraulic+truck+crane), even flood the market to fill the need of environmentally concern manufacturers (http://www.cbsnews.com/news/off-lease-used-cars-flood-the-market-pushing-prices-down/). It is reported that during 2000–2010 the ratio of used-to-new car sale in the USA, which has increased from 2.4 to 3.2. In 2010, approximately 11.58 million new cars were sold in the USA, compared with the sale of 36.88 million used cars from both dealers and end users.

The above mentioned condition has led to the emerging new kind of economy, *i.e.* the circular economy. It is a regenerative system in which resource input and waste, emission, and energy leakage are minimized. The process of minimizing can be done either by slowing, closing, or narrowing material and energy loops. This can be achieved through long-lasting design, maintenance, repairing, reusing, remanufacturing, refurbishing, and recycling (Geissdoerfer *et al*., 2017). It is related to the Blue Economy, initiated by Gunter Pauly, in which "the waste of one product becomes the input to create a new cash flow". Some literatures influenced by this consideration are Cayseele (1993), Murayama *et al.* (2004), Vorasayan and Ryan (2006), Saidi-Mehrabad *et al.* (2010), Amin dan Zhang (2012), Chari *et al.* (2013), Hosseini *et al.* (2014); Hsueh (2015), Kim *et al.* (2015), and Qian and Dong (2017). To date, analytical investigation on the utilization of used equipment in mining industry is still scarce. Not to mention a comprehendship analysis combining the utilization of used equipment, maintenance strategies, and the purchase or leasing strategies. Only a few references are found in this area, *e.g.* Pongpech and Murthy (20016) and Pongpech *et al.* (2006).

Heavy equipment such as dump trucks, excavators, bull dozers, etc. are among the vital equipment in mining industry, without which there will be no mining products available. These equipment is now are available in refurbished or used condition in the market, either advertised for sale of lease. In the last five years, prices of ore, coal and other mining materials have fallen and this in turn decreases the revenue of mining companies. Due to the decrease of the revenue, many mining companies tend to cut back on capital expenses. As a result, leasing the equipment to an external agent or Original Equipment Manufacturer (OEM) is a viable option to meet the need of the equipment.

In general, the agent (or OEM) as a lessor gives a lease contract with a full coverage of the maintenance action (PM or/and CM). Study of leased products has received much attention. Yeh and Chang (2007) studied a lease contract in which PM is taken when the failure rate of the leased product reaches a certain threshold value. The failure rate reduction method is also used to obtain the optimal periodical maintenance policy for the leased equipment (Pongpech and Murthy, 2006). Other authors studied the optimal number and degrees of PM (Jaturenon *et al.*, 2006) and most of studies of leased products focus on determining the optimal PM policy in a specified contract period.

Recently, OEM tends to offer a longer lease contract – more than one year, in order to attract consumers. For the OEM, offering a leased product with a long period of contract will result in a greater maintenance costs for servicing the lease contract – as more failures are likely to occur.

Hence, the lessor is of interest for reducing the maintenance costs. For a repairable product such as heavy equipment, a proper combination of PM and servicing strategy can reduce the maintenance cost significantly. A variety of servicing strategies has been studied by several authors in the context of warranted product and they have shown that an appropriate servicing strategy gives a significant reduction to the warranty cost (Iskandar and Jack, 2011; Varnosafaderani and Chukova, 2012; Yun *et al.* 2008).

For economy and safety considerations, the agent (or OEM) often carries some upgrading and inspection actions of the used equipment before sale (pre-sale upgrade and inspection actions) and this will restore the equipment to a better condition. A prudent calculation in planning to invest those expensive, yet vital, equipment often done before a company buy the equipment. The procurement of heavy equipment usually taking into account several important consideration, such as the price of the equipment, type of equipment *i.e.* used or new and maintenance services as well as other after sales services offered by the OEM to the owner.

In selecting maintenance services some difficulties may arise. This may due to several reasons, e.g. Niraj and Kumar (2011) pointed out that the lack of a systematic, focused and adaptable approach in setting preventive maintenance instructions are among the major problems in maintenance practices. Some stages in maintenance process requires personnel with a good and thorough knowledge of the process or equipment under observation, but often this requirement slightly fail to be satisfied and may result in bias, subjective and vague data. Fuzzy logic and analysis are often used to overcome these difficulties and others that may arise (Niraj and Kumar, 2011; Qi *et al.*, 2012; Zhao *et al.*, 2019).

There are numerous fuzzy methods available in literatures that treat different models with successful applications in various fields. As examples, Tomescu *et al.* (2007) proposed a new stability analysis method of fuzzy control systems for nonlinear processes. Heuristic fuzzy rules was used to design Takagi-Sugeno fuzzy logic controler (T-S FLC). They proved that for a specific condition the nonlinear system under consideration will be asymptotically stable. Precup *et al.* (2009) did a similar fuzzy logic stabilization of T-S FLC for a class of single input-single output nonlinear time varying process. Herrera-Viedma and López-Herrera (2010) present a survey of some fuzzy linguistic information access systems. They showed that tools of fuzzy linguistic modelling have found succesfull application in information retrieval systems, filtering systems, recommender systems, and web quality evaluation tools. The fuzzy linguistic modelling is proved to be superior in handling subjectivity, vagueness and imprecision that allow us to access to quality information in a flexible way. Some authors, such as Muallem *et al.* (2015), used fuzzy inference to develop a robust face and pose detection. Other authors showed the succesful applications of fuzzy logic and analysis in manufacturing and maintenance of industrial process equipment (Niraj and Kumar, 2011; Qi *et al.*, 2012; Feriyonika and Dewantoro, 2013).

In this paper, we apply fuzzy function to study the effect of vagueness, such as fuzzy usage rate, in lease contracts for used and new equipment in order to minimize the maintenance cost for both equipment with PM and a servicing strategy allowing more than one imperfect repair during the contract. For comparison we begin with the discussion of the lease contract for crisp usage rate. The paper is organized as follows. Section 2 deals with model formulation for the service contracts studied. Sections 3 and 4 give model analysis to obtain the optimal cost structure for the OEM. In Section 5 we present the numerical results and discussion of the results. Finally, we conclude with topics for further research in Section 6.

1. MODEL FORMULATION

We consider used repairable products *i.e.* a dump truck sold by the OEM. The OEM selling used products often performs some upgrading and inspection actions before sale in order to make the used product in a better condition. The inspection and upgrading actions aim at improving the reliability of the used product bought from the first user. One can model the reliability improvement through either a reduction in age or failure rate level. In this, we use an age reduction to represent an improvement of the used product after inspection and upgrading actions.

2.1. Symbols and Notations

To proceed further, the following notation of parameters will be used in subsequent sections:

Symbol	**Meaning**
$\Omega_l = [0,\Gamma_0)\times[0,U_0)$	: lease coverage
δ_y	: improvement level ($0<\delta<1$)
ψ_y	: parameter of lease contract type 1 ($0<\psi_y\le\Gamma_y$)
K_y	: threshold time of parameter lease contract type 1 ($0<K_y\le\psi_y/2$)
$Z_y, V_y, \psi_y, \zeta_y$	: the first failure after K_y and the first failure after $z_y + K_y$, ($0\le z_y\le\psi_y\le v_y\le\zeta_y\le\Gamma_y$)
$F_y(t)$, $f_y(t)$, $h_y(t)$	: failure distribution, density and hazard rate functions for a given *y*
$A_i(t)$	: virtual age after i^{th} imperfect with $i=1,2$
Z_1	: random variable of first failure after τ (S_1) for lease contract 1 (2) with distribution function $F_1(z_1)$
Z_2	: random variable of first failure after $Z_1+\tau$ (S_2) for lease contract 1 (2) with distribution function $F_2(z_2)$
c_m	: cost of minimal repair
c_p	: cost of perfect repair
$c_{im}(\delta)$	:cost of imperfect repair maintenance as a function of δ
$c_{ip}(\delta)$	: cost of imperfect PM as a function of δ
$J_y(\delta_y, K_y, \psi_y)$	: expected total lease contract cost

Let Y denote the age of a used product collected (or bought) from its first user. It is assumed that Y has a distribution function $G(y)$. We consider that the age of the used product after inspection and upgrading, is *A* which is smaller than *y*. In other words, the inspection and upgrading effort reduces the age by $x(=y-A)$. The inspection and upgrading cost adjusted from (Chattopadhyay and Murthy, 2006) is given by

$$C_u = \kappa E[Y-A]^{\xi} p^{\varphi}; \kappa, \varphi, \xi > 0, \tag{1}$$

where, $E[Y-A]=\int_{A}^{y_{\max}}(y-A)dG(y)$. Hence, the cost for the OEM to make a used product ready for sale is the sum of the acquisition cost (ϕ) plus inspection and upgrading cost given by

$$C_d = \phi + C_u. \tag{2}$$

.

2.2. Maintenance Lease Contract

A lessor starts to operate the used product at time *t* = 0, time instant of sale. We consider that the used products sold are all identical items in the sense that the age of the used items at time instant of sale is *A*. Let X_1 denote time to first failure for the used product of age *A*. The distribution of X_1 is given by $H(x)=F(A+x)$, where is the distribution of time to first failure of the new product. Failure rate function associated with $H(x)$ is given by $r(x)=\lambda(A+x)$, where $\lambda(t)=f(t)/1-F(t)$. The cumulative failure rate function of the used product is $R(t)=\int_0^t r(x)dx$.

We consider that a mining company operates a number of leased dump trucks and each covered by a rectangle region $\Omega=[0,\Gamma_0)\times[0,U_0)$ (see Figure 1), where Γ_0 and U_0 are the time, and the usage limits of the warranty. All failures under lease contract are rectified at no cost to the lessee. The contract ceases at Γ_y (the first instance) when the age of the product reaches Γ_0 or its usage reaches U_0, whichever occurs first Γ_y is given by $\Gamma_y=\Gamma_0$ for $y\le U_0/\Gamma_0$, $\Gamma_y=\Gamma_1(=U/y)$, for $y>U_0/\Gamma_0$ (see Figure 1).

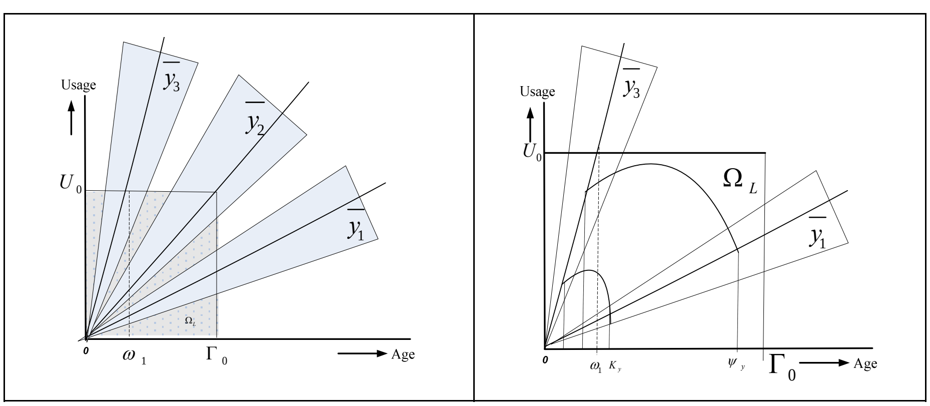

Figure 1. Lease region and average usage rate (left). Parameter of strategy 1-2 D with two imperfect repairs (right)

Using one dimensional approach, we consider that the usage rate Y varies from lessee to lessee but is constant for a given lessee (or a given product). Y is a random variable with density function $g(y), 0 \leq u < \infty$. Conditional on $Y = y$, the total usage u at age x is given by $u = yx$ (Husniah *et al.*, 2013).

The lessee are divided based on the usage rate of the leesed product into three types - light, medium and heavy usage. The usage rate sub-intervals are given by $[y_{i-1}, y_i]$, i = 1, 2, 3 (coresponding to light, medium and heavy usage, respectively) where $y_0 = Y_L$ and $y_i = Y_L + (Y_U - Y_L)/3$ for i = 1, 2, 3 (see Figure1-left). Then, each usage type is represented by its average usage rates given by $\bar{y}_i = \int_{y_{i-1}}^{y_i} \frac{sg(s)}{G(y_i) - G(y_{i-1})} ds$ for $i = 1, 2, 3$. For a given usage rate y, $h(x|y) \geq 0$ represents the conditional hazard (failure rate) function for the time to first failure and it is is a non-decreasing function of the item age x and y. For the case where all repairs are 'minimal' and repair times are negligible, then $\mathrm{r}(x|y) = h(x|y)$. All failures under lease contract are rectified at no cost to the lessee (consumer).

A penalty cost incurs the OEM if the actual down time falls above the target $(\mathfrak{T})$. If $\mathscr{D}$ is down time (consisting repair time and waiting time) for each failure occuring during the contract, then the OEM should pay a penalty cost ($\mathscr{C}_{\mathscr{P}}$) when $\mathscr{D} > \mathfrak{T}$. The amount of the penalty cost is assumed to be proportional to $\Delta = \mathscr{D} - \mathfrak{T}$. The decision problem for the OEM is to determine the optimal maintenance level such that to minimize the expected maintenance cost. If the distribution function for *T* is given by *F* (*T*, *α*), where *α* is the scale parameter, then the distribution function for *T* is $F(t; \alpha_y) = 1 - \exp(-(t + A) / \alpha_y)^{\beta}$, and its hazard function is $r_y(t) = \beta\left(t^{\beta-1}/(\alpha_y)^{\beta}\right)$.

2.3. Servicing Strategy

We define the servicing strategy as follows. For a given product (or a usage rate *y*), imperfect repair is done at failure (at time *t*, $K_y < t \leq \psi_y$) if time elapsed since the last imperfect repair (or the beginning of the operation, *t*=0) is greater than K_y (a threshold value) (See Figure 1-right). All other failures are fixed by the minimal repair. For a given y, let z_y and v_y denote the first failure after K_y and the first failure after $z_y + K_y$, respectively. Then, we have $K_y < z_y < v_y \leq \psi_y$ and $(v_y - z_y) > K_y$ where $(v_y - z_y)$ is the time elapsed since the first imperfect repair. As a result, this servicing strategy allows more than one imperfect repair. It is assumed that imperfect repair improves the reliability of the item –by reducing the age of the item. Each imperfect repair will result in reducing the age with improvement factor δ_y. It is assumed that Imperfect repairs improve the reliability of the repaired item and it is represented by the hazard rate of the item after repair being smaller than that before failure. We can model the effect of imperfect repair through either hazard rate or age reduction models (Doyen and Gaudoin, 2011). In this paper we use the age reduction model and it is described as follows.

We model the effect of the imperfect repairs by reducing the virtual age of the repaired item. Let $A(t)$ be the virtual age of the item at time *t* for a given usage rate y. The hazard rate of the item is as function of $A(t)$, denoted by $h_y(A(t))$. If the imperfect repair is done at age z_y with improvement level δ_y then the virtual age and the hazard rate after repair are given by $A_z(t) = t - \delta_y z_y$ and $h_{z|y}(t) = h(t - \delta_y z_y)$ for $t > z_y$. For the second imperfect repair occuring at age v_y the virtual age and hazard rate are given by $A_v(t) = t - \delta z_y + \delta^2 z_y - \delta v_y$ and $h_{v|y}(t) = h_v(t - \delta z_y + \delta^2 z_y - \delta v_y)$ for $t > v_y$, respectively. For the case where all failures are rectified by minimal repairs then $A(t)$ is equal to *t* that is called actual age. With a wider coverage of a lease contract, it needs several imperfect repairs required to reduce the the number of failures and maintenance costs. For both case, $y \le \gamma$ and $y > \gamma$ we confine to at most two imperfect repairs over the lease contract coverage. Now, we obtain distribution functions for z_y and v_y defined as $F_{z|y}(z_y)$ and $F_{v|y}(v_y)$] which are important in modelling the expected maintenance cost for the servicing strategy studied. As failures occuring in $(0, K_y]$ and $(z_y, L_y]$ are fixed by minimal repair, then

$$F_{z|y}(z_y) = 1 - \exp[H_y(K_y) - H_y(z_y)]\,, \tag{3}$$

where $H(x) = F(A + x)$ since it is a failure distribution for a used product and then $H_y(t) = \int_0^t h_y(u)du$.

Differentiating (2) with respect to z_y yields

$$f_{z|y}(z_y) = h_y(z_y)\exp[H_y(K_y) - H_y(z_y)]\,. \tag{4}$$

On the condition that $Z_y = z_y$ and then unconditioning it, we have $F_{v|y}(v_y)$ given by

$$F_{v|y}(v_y) = \int_{K_y}^{\psi_y} \left\{1 - \exp\left[H_{z|y}\left(K_y + z_y\right) - H_{v|y}\left(v_y\right)\right]\right\} h_y(z_y)\exp[H_y(K_y) - H_y(z_y)]dz_y \tag{5}$$

where $H_{z|y}(t) = \int_0^t h_{z|y}(u)du$ and $H'_{z|y}(t) = \int_0^t h'_{z|y}(u)du$. The density function is given by

$$f_{v|y}(v_y) = \int_{K_y}^{\psi_y} h_{z|y}\left(v_y\right)\exp\left[H_{z|y}\left(K_y + z_y\right) - H_{z|y}\left(v_y\right)\right] h_y(z_y)\exp[H_y(K_y) - H_y(z_y)]dz_y \tag{6}$$

and it can be found in (Husniah *et al.*, 2013).

3. MODEL ANALYSIS

We consider a situation where the lessor needs to perform PM more than one imperfect repair for reducing the maintenance cost. And also we assume a situation where the lessor incurs repair cost for each failure and PM cost. Here, we confine to at most two imperfect repairs over the contract. We first obtain the expected total maintenance cost lease contract with proposed servicing strategy and PM for the used equipment and then compare it with the lease contract for new equipment.

3.1. Lease Contract for Used Equipment

The total maintenance cost consists of penalty cost (when down time above the target), repair cost and PM cost. Hence, the expected cost is given by $E[\pi_y]=E[Penalty\ Cost]+E[Repair\ and\ PM\ Costs]$. We then obtain the expected repair cost and expected penalty cost in (0, Γ0]. The expected maintenace cost with PM and imperfect repair is obtained by a conditional approach. If z_y and v_y fall in $(0,\psi_y]$ then two imperfect repairs occur in the leasing period. Conditional on $Z_y = z_y$ and $V_y = v_y$, the expected warranty servicing cost for Strategy 1, $J_{z|y}(\delta_y, K_y, \psi_y \,|\, Z_y = z_y, V_y = v_y)$ is given by

We obtain the expected repair cost and expected penalty cost in (0, Γ_0] as follows.

$$J_{1|y}(\delta_y, K_y, \psi_y \,|\, Z_y = z_y, V_y = v_y) = \begin{cases} c_m\left[H_y(K_y) + H_y(W_y) - H_y(\psi_y)\right] & z_y > \psi_y \\ c_i(\delta_y) + c_m\left[H_y(K_y) + H_{z|y}(\psi_y) - H_{z|y}(z_y)\right] & \psi_y - K_y < z_y \le \psi_y,\ v_y > \psi_y \\ c_i(\delta_y) + c_m\begin{bmatrix} H_y(K_y) + H_{z|y}(z_y + K_y) - H_{z|y}(z_y) \\ + H_{z|y}(W_y) - H_{z|y}(\psi_y) \end{bmatrix} & K_y < z_y \le \psi_y - K_y,\ v_y > \psi_y \\ 2c_i(\delta_y) + \left[H_y(K_y) + H_{z|y}(z_y + K_y) - H_{z|y}(z_y) + H_{v|y}(W_y) - H_{v|y}(v_y)\right] & K_y < z_y \le \psi_y - K_y,\ v_y \le \psi_y \end{cases} \tag{7}$$

where $H_{v|y}(t) = \int_0^t h_{v|y}(u)du$ and $H'_{v|y}(t) = \int_0^t h'_{v|y}(u)du$.

Removing the conditional form in (7) yields

$$\begin{aligned} & J_{1|y}(\delta_y, K_y, \psi_y) = c_m\left[H_y(K_y) + H_y(W_y) - H_y(\psi_y)\right]\exp[H_y(K_y) - H_y(\psi_y)] + \\ & \int_{\psi_y - K_y}^{\psi_y} \left\{c_i(\delta) + c_m\left[H_y(K_y) + H_{z|y}(W_y) - H_{z|y}(z_y)\right]\right\} h_y(z_y)\exp[H_y(K_y) - H_y(z_y)]dz_y + \\ & \int_{K_y}^{\psi_y - K_y} \left\{c_i(\delta) + c_m\left[H_y(K_y) + H_{z|y}(z_y + K_y) - H_{z|y}(z_y) + H_{z|y}(W_y) - H_{z|y}(\psi_y)\right]\right\} \\ & \exp[H_{z|y}(z_y + K_y) - H_y(\psi_y)]h_y(z_y)\exp[H_y(K_y) - H_y(z_y)]dz_y + \\ & \int_{K_y}^{\psi_y - K_y}\int_{z_y + K_y}^{\psi_y} \left\{2c_i(\delta_y) + c_m\left[H_y(K_y) + H_{z|y}(z_y + K_y) - H_{z|y}(z_y) + H_{v|y}(W_y) - H_{v|y}(v_y)\right]\right\} \\ & h_{z|y}(v_y)\exp\left[H_{z|y}\left(z_y + K_y\right) - H_{z|y}\left(v_y\right)\right]h(z_y)\exp[H_y(K_y) - H_y(z_y)]dv_y dz_y. \end{aligned} \tag{8}$$

<u>Expected of Penalty Cost:</u>

Let $\mathscr{D}$ and $\mathfrak{T}$ denote down time (consisting repair time and waiting time) for each failure occuring during the contract and down time allowed. The expected penalty cost is given by

$\mathcal{C}_{\mathcal{P}}\bar{G}(\mathfrak{I})H_y(\delta_y, K_y, \psi_y)$ where $\mathcal{C}_{\mathcal{P}}$ is the penalty cost and $H_y(\delta_y, K_y, \psi_y)$ denotes the expected total number of failure during the lease contract. As a result, the total expected cost of the OEM is

$$E\left[\pi_y\right] = J_y(\delta_y, K_y, \psi_y) + \mathcal{C}_{\mathcal{P}}\bar{G}(\mathfrak{I})H_y(\delta_y, K_y, \psi_y)\,. \tag{9}$$

4. NUMERICAL EXAMPLES

Since the complexity of integral equations involved in (8), it is not possible to obtain the optimal values δ_y, K_y, ψ_y analytically. Hence, a computational approach will be used to obtain the sub optimal solution. We consider that $F_y(t;\alpha_y)$ the time to the first failure for a given usage rate *y* is given by the Weibull distribution with $F_y(t;\alpha_y) = 1 - \exp(-t/\alpha_y)^{\beta}$, and its hazard function is $r_y(t) = \beta\left(t^{\beta-1}/(\alpha_y)^{\beta}\right)$ where α_y as in (1). The other parameter values be as follows. β = 2, *A*=4 (years), Γ_0 = 5(years), *U*=5 (x10^4 Km) (γ = *U/W* = 1), y_0 = 1, and $c_m = 0.5.c_p$, $\mathfrak{I}$ = 1 (year), $\mathcal{C}_{\mathcal{P}} = 25c_p r$. The cost of imperfect repair is a function of δ_y given by $c_i(\delta_y) = c_m + (c_p - c_m)\delta_y^{\,4}$ as in (Yeh and Chang, 2007). The down time distribution is given by the Weibull distribution with $\alpha = \beta$ = 0.5. The values of ρ for three different land contours are 1.7, 2.0, and 2.3 coresponding to light incline, high incline and very hilly, respectively. In the following computation we consider two types of usage rates, *i.e.* the crisp usage rate and the fuzzy usage rate for three different level representing light, medium, and heavy usage rates. The results are presented in Table 1 for the crisp usage rate leveling and in Figures 2 and 3 for the fuzzy usage rate leveling.

Table 1. Results for Lease Contract for used and new product with 2 imperfect repair for C_p =2

	Strategy with 2 imperfect repair Used Product				Strategy with 2 imperfect repair New Product			
Usage rate	δ^*	K_y	ψ_y^*	J_1^*	δ^*	K_y	ψ_y^*	J_1^*
Light $\bar{y} = 0.80$	0.28	0	4.76	6.34	0.12	1.26	1.26	1.498
Medium $\bar{y} = 1.00$	0.38	0.66	4.73	9.007	0.21	1.23	1.23	1.791
Heavy								
$\bar{y} = 1.20$	0.43	0.76	3.93	10.784	0.23	1.01	1.01	1.931
$\bar{y} = 1.40$	0.48	0.76	3.37	12.812	0.26	0.85	0.85	2.088
$\bar{y} = 1.60$	0.15	0.0	1.53	18.493	0.28	0.73	0.73	2.263
$\bar{y} = 1.80$	0.56	0.7	2.6	17.492	0.30	0.64	0.64	2.454
$\bar{y} = 2.00$	0	0	0	29.011	0.32	0.57	0.57	2.660

a. Crisp usage rate

Firstly, we consider three crisp usage types, *i.e.* light $(0 < y < 1.0)$, medium $(1.0 \le y < 1.4)$ and heavy $(y \ge 1.2)$ usages with ρ = 1.7, 2.0 and 2.3 coresponding to light incline, high incline and very hilly, respectively. Tables 1 show optimal improvement level for used and new product by assuming these crisp three usage types with ρ = 1.7, 2.0 and 2.3 coresponding to light incline, high incline and very hilly, respectively. Here we assume c_p = 7 and $\alpha_0 = 2$. For a given *y*, and α_0 (representing reliability level), the optimal expected cost increases as the usage rate y increases. This is as expected since the increasing in *y* causes the failure rate to increase and this in turn increases the number of failures under Lease Contract, and also it requires a higher δ^*. Lease Contract with new product has low expected cost compared with used product for a high usage rate condition.

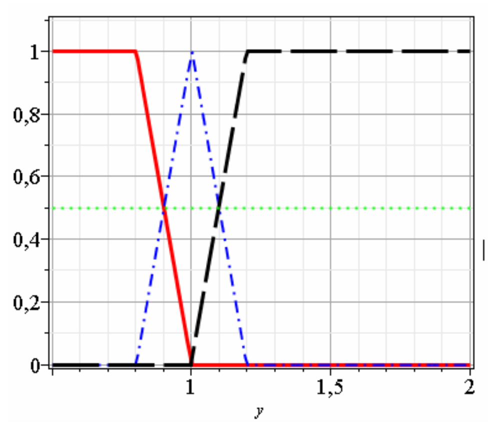

Figure 2. Fuzzy membership function of usage rate level: light (red solid), medium (blue dot-dash), and heavy (black dash) associated with the first column of Table 1.

b. Fuzzy usage rate

Secondly, we consider three fuzzy values of usage rates as the input to find the expected cost of the lease contract. In reality the membership function (MF) of the fuzzy numbers should be chosen appropriately according to the available data which are being used. There are some methods, like Genetic Algorithm and Neural Network, to assign an appropriate MF to a fuzzy quantity (Ross, 2010). But, since in this example the fuzzy quantity is hypothetical, we choose the simplest form of fuzzzy number, *i.e.* the triangular and trapezoidal types. It is assumed that the fuzzy membership function of the usage rate category is given as in Figure 2, *i.e.*:

Light,

$$\tilde{y} = \begin{cases} 1 & , y \le 0.8 \\ (y - 1)/(0.8 - 1) & , 0.8 < y \le 1 \\ 0 & , otherwise \end{cases}$$

Medium,

$$\tilde{y} = (0.8, 1, 1.2) = \begin{cases} 0 & , y \le 0.8 \\ (y - 0.8)/(1 - 0.8) & , 0.8 < y \le 1 \\ (y - 1.2)/(1 - 1.2) & , 1 < y \le 1.2 \\ 0 & , otherwise \end{cases}$$

Heavy,

$$\tilde{y} = \begin{cases} 0 & , y \le 1 \\ (y-1)/(1.2-1) & , 1 < y \le 1.2 \\ 1 & , otherwise \end{cases}$$

By assuming that the fuzziness of the independence variable “usage rate” propagates to the dependence variables (see Lee, 2005, pp. 152), we get the expected maintenance cost per lease period given in Figure 3.

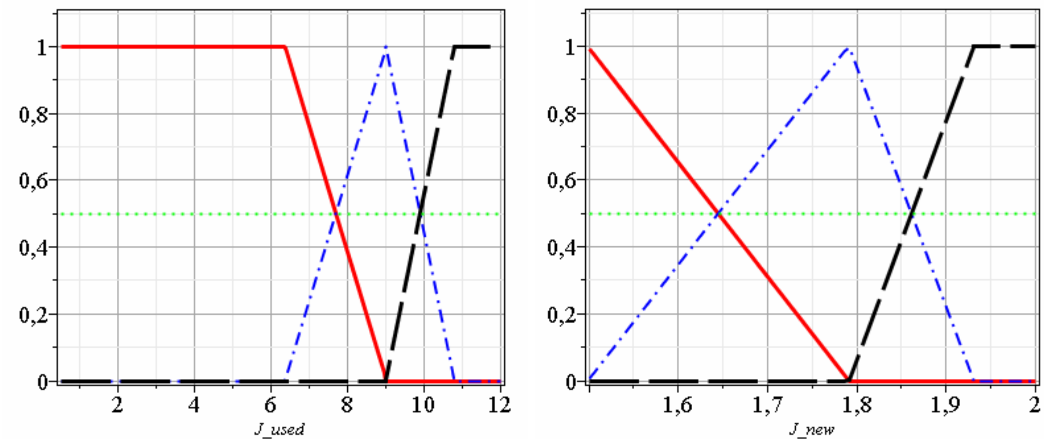

Figure 3. Fuzzy membership function of the resulting expected maintenance cost for three different usage rates level: light (red solid), medium (blue dot-dash), and heavy (black dash).

Figure 4 only gives the expected maintenance cost spent by the lessor during the lease period. If now we consider the value of the equipment that is leased, then the total cost are TC_{new} and TC_{used} for new and used product, respectively, with $TC_{new} = J_{new} + X_{new}$ and $TC_{used} = J_{used} + X_{used} = J_{used} + \xi X_{new}$. The symbol ξ represents the ratio of used selling price to new selling price, which is normally 40% to 80% depending on many factors as described earlier. Supposed that the ratio is uncertain and given by the triangular fuzzy number (40%;60%;80%), corresponding to a fuzzy number with the modal point 60%. with the new selling price is Rp. 1,200,000,000. It seems reasonable to assume the unit cost of the expected maintenance in Figure 3 to be Rp. 10,000,000. These assumptions give rise to the resulting total costs as in Figure 4. In this Figure we concentrate on the medium usage rate as the example. The computation is done using the α-level computation as described in (Barros *et al.*, 2017, pp. 34), or also known as α-cut arithmetic (Buckey *et al.*, 2002). Other method by considering the LR-type of fuzzy number such as described in (Panchal *et al.*, 2014) can also be applied.

By concentrating on the medium usage rate, and following the α-level method, the fuzzy number of the expected maintenance cost is (63,400,000 + α (26,670,000) , 107,840,000 - α (17,770,000)) which is equivalent to the triangular fuzzy number (63,400,000; 90,070,000; 107,840,000) for used equipment. While for the new equipment the expected maintenance cost is (14,980,000 + α (2,930,000) , 19,310,000 - α (1,400,000)) which is equivalent to the triangular fuzzy number (14,980,000; 17,910,000; 19,310,000). Note that we use the comma symbol for two different purposes: decimal separation and interval. The α-level for the used equipment selling price is equivalent to the triangular fuzzy number (40%*1,200,000,000; 60%*1,200,000,000; 80%*1,200,000,000) which is given by (480,000,000 + α(240,000,000) , 960,000,000 – α(240,000,000)). Hence the resulting total cost of leasing for the used equipment is TC_{used} = (543,400,000 + α (266,670,000) , 1,067,840,000 – α(257,770,000)) which is equivalent to the triangular number (543,400,000; 810,070,000; 1,067,840,000). While the resulting total cost of leasing for the new equipment is TC_{new} = ((1,200,000,000+14,980,000) + α (2,930,000) , 1,200,000,000+19,310,000) - α (1,400,000)) = (1,214,980,000 + α (2,930,000) , 1,219,310,000 - α (1,400,000)) which is equivalent to the triangular number (1,214,980,000; 1,217,910,000; 1,219,310,000).

There several methods documented in the literature of how ordering fuzzy numbers (see Bortolan and Degani, 1985; Chang and Lee, 1994). However, Buckley (2006) argues that there is an easy way to determine if a fuzzy number *M* is less than another fuzzy number *N* or they are the "same", for many fuzzy numbers. First, it is easy to see that if the core of *N* lies completely to the right of the core of *M*, then the measure of how much *M* is less than or equal to *N* is one, *.i.e.* v (M $\leqslant$ N) = 1. According to this concept, it is easy to conclude that $v(TC_{used} \leqslant TC_{new})$ = 1 (see Figure 4). Literally, this means we know for sure that the total cost of the used equipment is lower than the total cost of the new equipment as expected. The light and the heavy usage rates case can be treated analogously.

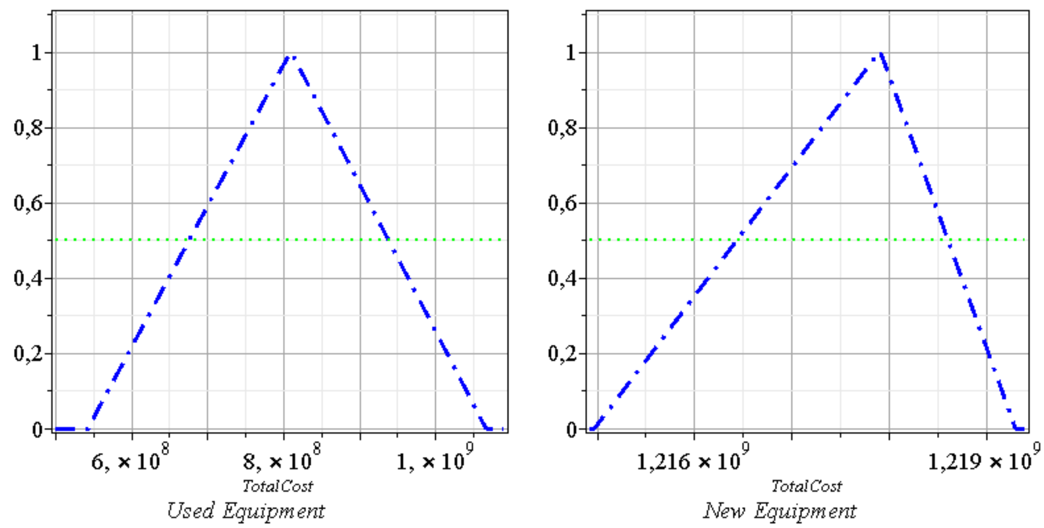

Figure 4. Graphical representation of the expected costs for the medium usage rate.

5. CONCLUSION

In this paper we have developed a mathematical model of lease contract involving imperfect repair for a repairable product (such as dump truck). Two cases of lease contracts have been studied - *i.e.* new and used equipment.The analysis was done from the lessor point of view by considering both crisp and fuzzy parameters. The results show that the total cost of the used equipment is far lower than the new equipment as expected. More general case which allows more than two imperfect repairs under the contract is interesting to investigate and this is a challenging topic for further research. Other approach of the fuzzy arithmetics with different types of fuzzy numbers is also worth to explore.

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