A Multi-Size Compartment Vehicle Routing Problem for Multi-Product Distribution: Models and Solution Procedures

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ABSTRACT

This paper presents a Multi-Size Compartment Vehicle Routing Problem with Split Pattern (MC-VRP-SP). This MC-VRP-SP focuses on the delivery of multiple types of liquid products from depot to customers. The main objective is to minimize the total traveling cost. Multiple types of fluid product are loaded to the vehicles having several capacities with non-identical capacity of compartments. Then, two mathematical models are proposed to represent this MC-VRP-SP. Since the order quantity may be over the capacity of compartment, the splitting of order is needed. The original customers demand is split according to the pre-determined patterns so that it can be loaded to compartments appropriately. However, the split demands must be delivered by the same truck and one compartment can support only a single split demand. The optimization approach is utilized to solve these mathematical models within the limited computational time. A homogeneous and heterogeneous fleet of multi-compartment trucks are considered in the numerical experiment. The results from numerical examples show that the optimization approach can yield us the optimal solution only in small-scale problem. For large-scale problem, the optimization approach can determine only the feasible solutions. Thus, the local search so called 2-opt is employed to make an improvement on solution of the large-scale problem. Moreover, a saving algorithm is utilized to separate the customers into clusters. Then each cluster is solved separately by CPLEX. The results show that this clustering can significantly improve the solutions on the large-scale problem.

Keywords: Multi-size compartment vehicle routing problem, Multi-product, Homogeneous and heterogeneous fleet, Split pattern, Clustering, Local search.

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1 INTRODUCTION

The vehicle routing problem (VRP) is widely popular problem in logistics and supply chain. The original VRP first introduced by Dantzig and Ramser (1959) which classify as the classical capacitated vehicle routing problem (CVRP). The CVRP is concerned with delivery goods to a set of customers that have certainly demands. The objective of problem is to determine shorted distance routing for serve customers. The typical constraints of CVRP are the truck capacity, the time limit for traveling. The literatures on the CVRP have received much attention in Laporte et al (2000) and Toth and Vigo (2002). The vehicle with several compartments or multi-compartment vehicle routing problem (MC-VRP) is a variant of the capacitated VRP. The MC-VRP consists of designing set of routes to serve the demand of multi-product of a set of customers. In many real world problems, the MC-VRP can be applied in situation that the goods are different in types, for example milk (cows and goats) and fluid product (gasoline and diesel), different in quality, for instance expired dates, different in treatment conditions, for instance level of storage temperatures, and so on. To save transportation costs, vehicles with several compartments are employed in order to allow transporting the heterogeneous goods together on the same vehicle, but in different compartments.

The MC-VRP has widely practical applicability, however the MC-VRP has received not much attention in the literature. The MC-VRP is a variant of the capacitated vehicle routing problems (CVRP). A review on VRP can be found in Cordeau et al (2007). They discussed on the formulations of VRP, exact algorithm, heuristic algorithms, and metaheutistics. A vehicle with multiple compartments in the distribution of foods to convenient stores can be found in Chajakis and Guignard (2003). This paper tries to minimize both delivery and cooling costs. The cooling costs are the costs of keeping non-ambient temperature items at suitable temperatures during deliveries. The mathematical models which are 0-1 programming and mixed integer programming were formulated. The Lagrangean Relaxations was utilized to solve this problem. Derigs et al (2011) presented a good review and summary on the vehicle routing with compartments in terms of applications, modeling, and heuristics. This paper presented a model formulation, an integer programming model of the MC-VRP. The demand of customer order cannot be split. Many heuristics and metaheuristics are reviewed and the comparisons on solutions of the benchmark problems are presented as well. Other literatures on the modeling, optimizations, heuristics and evolutionary optimizations can also be found in Raza and Vidyarthi (2009), Farahani et al (2012), and Purcaru et al (2013).

The deliveries of liquid fuels to petrol station using vehicle with multiple compartments was presented by Abdelaziz et al (2002). The adjustment of order quantities was employed in order to fit vehicle compartments before starting dispatch. This paper proposed an integer linear programming and applied the variable neighborhood search heuristics to determine the solution. Another research on the petrol delivery was done by Surjandari et al (2011). This paper studied on the petrol delivery assignment with multi-product, multi-depot, split deliveries, and time windows. The purpose of this paper is to determine the fleet assignment and to design the delivery routes to petrol station using the available trucks with minimum total traveling cost. Each compartment has equal capacity and is not equipped with a flow meter. The demand quantities must be equal or an integer multiple of compartments capacity. The fluid product in each compartment has to be entirely unloaded into a storage tank. Each petrol station requires the delivery within the time window. Each truck can be assigned for several trips and each petrol station can have multiple visits. The Tabu search algorithm has been used to solve this problem. Avella et al (2004) also proposed solving a fuel delivery problem by heuristic and exact approaches. The customer order is an integer multiple of one thousand litres and may require one or more compartments. Each compartment must be completely emptied after replenishment. All orders of a customer have to be delivered by one truck. The branch-and-price is employed to improve solution obtained from heuristic which is the packing/routing heuristic.

The improvement on logistic of multi-compartment chemical tankers was also proposed by Jetlund and Karimi (2004). Cornillier et al (2008) also proposed the petrol station replenishment problem (PSRP). The objective of their work is to deliver petroleum products to petrol stations by an unlimited heterogeneous fleet of compartmented tank trucks. The solution consists of determining the quantities to deliver within a given interval, allocating products to compartments and designing the routes to stations. The possibly standard ILP algorithm was proposed to solve the loading problem. The routing problem is handled using a matching approach and a column generation scheme. They proposed a formulation of a mixed-integer linear programming (MILP) and a heuristic algorithm to maximize the total profit for a single ship. The petrol station replenishment problem (PSRP) with time window can be found in Corniller et al (2009). Their research consider the quantity of petroleum products to be delivered to petrol stations by a limited heterogeneous fleet of tank-trucks, the assignment of products to compartments, truck routing and scheduling. A mathematical formulation and two heuristics were presented to determine the appropriate solution. A paper of a generalized version of the trip packing problem was proposed by Boctor et al (2011). The solution procedure is to assign a number of trips to a fleet of a limited number of non-identical tank-trucks. Working time of vehicles is limited and the net revenue of each trip depends on the truck used. This paper provides a mathematical formulation and proposes some construction, improvement and neighborhood search.

The split demand of the order quantity is typical found in the split delivery vehicle routing problem (SDVRP). Most papers proposed the splitting of customers demand among vehicles on the different routes rather than the splitting of an order quantity and assigning the split order quantities to compartments in one vehicle. The split delivery in VRP (SDVRP) and the multi-depot VRP (MDVRP) can be found in Gulczynski et al (2011), which the demand of customer can be split into two or more trucks and multi-trip must be enable. A customer can be visited more than one truck. An integer programming-based heuristic was utilized to search for the high-quality solution. Muyldermans and Pang (2010) presented a study on a MC-VRP with demands of different products. The initial solution is obtained by utilizing the Clarke and Wright saving algorithm (Clarke and Wright, 1964). Then the local search including 2-opt, cross, exchange, relocate are used to improve the solution. The neighbor lists and marking and the guided local search are also combine with the local search to determine the better solution. Asawarungsaengkul et al (2012) presented a MC-VRP with splitting of the order quantity in delivery of the multiple types of fluid products to petrol stations. They propose two mathematical models with the fixed-split pattern and float-split pattern. LINGO12 is used to determine the solutions.

This paper focuses on the delivery of multiple types of fluid products by multi-size compartment vehicles to customers. The capacity of each compartment for storing the fluid product cannot be adjusted and is different in capacity. If the customers order of a fluid product is over than the compartments capacity, that demand order will be split so that the order of a customer can be loaded to exactly one vehicle. Thus this paper presents the procedure of splitting the order quantity and assigning the split order to multi-compartment trucks in the delivery of multiple types of fluid products from a supplier to petrol retailers. The problem can be formulated as the mixed integer programming (MIP). The optimization software namely CPLEX12.2 are utilized to solve this specific MC-VRP with two types of splitting patterns. The numerical experiment with various sizes of customers will be provided and both homogenous and heterogeneous fleets of multi-compartment trucks are in our interest. The 2-opt local improvement will be utilized to improve the solution obtained from optimization software. Moreover, the clustering technique that divides the large-scale problem into the small-scale problems will also be presented in the last session as well.

2 PROBLEM DESCRIPTIONS OF THE PROPOSED MC-VRP-SP

This paper proposes a MC-VPR-SP for the distribution of the multiple types of fluid products to customers. The order quantity for each type of fluid products is less than trucks capacity and therefore, the orders from several customers will be loaded to the same truck on delivery routes. The objective of this paper is to minimize the total traveling cost. The main activities of this problem is to split the order quantity and load each split demand to the compartments with various capacities and then determine the optimal routes.

The characteristics of the customer order and the truck routing are subject to the following assumptions and constraints:

1. each customer must be visited only once a day,
2. the orders are known and the planning period is made for one day delivery,
3. each truck must not be loaded over its capacity for a single route,
4. one compartment can contain only a single customers demand,
5. there are many types of fluid product and several compartment capacities in the homogeneous and heterogeneous fleet of trucks,
6. the unload time at customer is assumed to be constant,
7. each order for each fluid product type must be a multiple of 2 kilolitres, and
8. the vehicle is not equipped with a flow meter which means that all compartments will be emptied after replenishment.

In MC-VRP-SP for fluid product delivery, each type of fluid product is incompatible. Each order can easily be split and delivered in independent compartments. Let Y_{jpq} be a predetermined integer number. The split pattern Y_{jpq} is used to divide the original order quantity of customer j of product p (denoted by D_{jp}) by split pattern q where q = $1, ..., Q$. After splitting, quantity of each split order will be identical as following equation: $D^{'}_{jpq} = \dfrac{D_{jp}}{Y_{jpq}}$.

However, it is not allowed to mix between two customers orders; even if they are the same type of the fluid product. At the customer, driver just replenishes the fluid products to the big tanks.

3 MATHEMATICAL FORMULATION OF THE MC-VRP-SP

The mathematical models for this MC-VRP were proposed by Rattanamanee and Asawarungsaengkul (2011). This paper makes a revision on those mathematical models. The integer nonlinear program which is hard to solve is modified. The time constraint is added to the models. To deal with the MC-VRP-SP for fluid product delivery, two mathematical models are proposed and optimization software CPLEX is employed to provide us the practical solution for the planner of this problem. CPLEX determines the solution for MIP by using branch and cut technique.

We consider the splitting of the order quantity by integer number as described in pervious section and design of truck routes. The decision variables are the integer number used for splitting the order quantity, the selected compartments for each split-order quantity, and the truck routes. The constraints of this MC-VRP are the trucks capacity, the compartments capacity, and the working hours per day. The multi-product, multi-compartment, different capacity among compartments, homogeneous fleet of trucks, and heterogeneous fleet of trucks are the main characteristics taken into account in this research. There is only one depot in this MC-VRP. Next, the proposed mathematical models are grouped into two categories based on the splitting pattern available. Model 1 is the representation for the single-split pattern while model 2 is considered as the multi-split pattern. In the single-split pattern, there is an integer number available for splitting the order quantity. For the multi-split pattern, each order quantity can be divided by an integer number selected from the available set which depends upon the order size.

3.1 The MC-VRP-SP for Single-Split Pattern: Model 1

Model 1 is formulated as the mixed integer linear programming (MILP). The model formulation is based on the following notations.

TC	total cost of traveling
N	number of customers including one depot
d_{ij}	the shortest path distance from i to j; i, j = 1, 2, 3, ..., N; where 1 is depot
P	number of products p = 1, 2, 3, ..., P
K	number of available trucks
C_k	variable cost of truck k (bath/km), k = 1, 2, 3, ..., K
W_k	number of compartments for truck k, w = 1, 2, 3, ..., W_k
D_{jp}	demand of customer j on product p (kl)
Cap_k	capacity of truck k
$Capc_{kw}$	capacity of compartment w of truck k; w = 1, 2, 3,..., W_k
U_i	arbitrary real numbers which specify the position of node i
Y_{jp}	integer number of customer j on product p
D'_{jp}	split-order quantity which equals to D_{jp}/Y_{jp}
Un	unload time which is assumed a constant value
Ve	the average velocity of a truck

$Tmax$	Time limit for one trip

The decision variables are:

Z_{kwj}^{p} 1 if split order quantity of product p for customer j is loaded to compartment w of truck k
0 otherwise

X_{ijk} 1 if truck k travels from node i to node j
0 otherwise

E_{jk} 1 if truck k travels to node j
0 otherwise

The mathematical model can be written as following equalities and inequalities.

$$\text{Minimize } TC = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{K} d_{ij}X_{ijk}C_k \tag{1}$$

subject to

$$(N-3)X_{jik} + (N-1)X_{ijk} + U_i \le U_j + (N-2) \qquad \text{for } i,j = 2,3,...,N; i \ne j;\ \forall k \tag{2}$$

$$\sum_{i=1}^{N} X_{ihk} - \sum_{j=1}^{N} X_{hjk} = 0 \qquad \text{for } h = 2,3,...,N;\ \forall k \tag{3}$$

$$\sum_{i=1}^{N} X_{ijk} = E_{jk} \qquad \text{for } j = 2,3,...,N; i \ne j;\ \forall k \tag{4}$$

$$\sum_{k=1}^{K} E_{jk} = 1 \qquad \text{for } j = 2,3,...,N \tag{5}$$

$$\sum_{j=2}^{N} X_{1jk} \le 1 \qquad \forall k \tag{6}$$

$$\sum_{k=1}^{K}\sum_{w=1}^{W_k} Z_{kwj}^{p} = Y_{jp} \qquad \forall j,p \tag{7}$$

$$\sum_{j=1}^{N}\sum_{p=1}^{P} Z_{kwj}^{p} \le 1 \qquad \forall k,w \tag{8}$$

$$E_{jk} \ge Z_{kwj}^{p} \qquad \forall k,w,j,p \tag{9}$$

$$\sum_{j=1}^{N}\sum_{w=1}^{W_k}\sum_{p=1}^{P} Z_{kwj}^{p} D_{jp}^{'} \le Cap_k \qquad \forall k \tag{10}$$

$$Z_{kwj}^{p} D_{jp}^{'} \le Capc_{kw} \qquad \forall k,w,j,p \tag{11}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \frac{X_{ijk}d_{ij}}{Ve} + \sum_{i=1}^{N}\sum_{j=2}^{N} X_{ijk}Un \le Tmax \qquad \forall k \tag{12}$$

$$X_{ijk} \in \{0,1\}, E_{jk} \in \{0,1\}, Z_{kwj}^{p} \in \{0,1\} \qquad \forall k,w,j,p \tag{13}$$

In this mathematical model, the objective function is to minimize the total cost of fuel distribution (Equation (1)). Constraint (2) represents a sub-tour elimination adopting from Desrochers and Laporte (1991). Constraint (3) states that a truck that visits node j must leave node j. Constraint (4) and (5) forces each customer to have only one vehicle visited. Constraint (6) states that each vehicle has only one route. Constraint (7) ensures that the amount of product p (after splitting) of customer j must be delivered. One compartment can contain only a single split order quantity is stated in Constraint (8). Constraint (9) means that each compartment can contain the split demand if and only if the customer j is visited by truck k. Constraint (10) ensures all trucks must not be loaded over theirs capacity for a single route. Constraint

(11) compels each compartment not to be loaded over its capacity. Constraint (12) is used to make sure that the traveling time for each route does not exceed the time limit. Constraint (13) defines decision variable to be binary.

3.2 MC-VRP-SP for Multi-Split Patterns (MILP): Model 2

Some variables are needed to be defined for model 2. The additional notations are used in the model 2 are listed below.

Y_{jpq}	integer number q of customer j on product p
Q	number of patterns; $Q \in \{1, ..., Q_{max}\}$
Q_{max}	maximum number of split pattern available
D'_{jpq}	split demand order equals D_{jp}/Y_{jpq}

The customers order can be split by dividing the order quantity with an available integer number selected from a set of predetermined integer number Y_{jpq}.

The original of the mathematical model 2 proposed by Rattanamanee and Asawarungsaengkul (2011) is classified as mixed integer nonlinear programming (MINLP). In this paper, mathematical model 2 is modified to be a mixed integer linear programming (MIP). A decision variable $S^{pq}_{kwj} \in \{0, 1\}$ is added in to model 2. The variable S^{pq}_{kwj} is used to make linear relation between variable Z^{pq}_{kwj} and V_{jpq}. The linear relation is shown in equation (27) and (28). The model 2 in a mixed integer programming (MIP) form can be solved by CPLEX more effectively.

The mathematical model for the multi-split patterns MC-VRP can also be written as following equalities and inequalities.

$$\text{Minimize}\, TC = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{K} d_{ij} X_{ijk} C_k \tag{14}$$

subject to

$$(N-3)X_{jik} + (N-1)X_{ijk} + U_i \le U_j + (N-2) \qquad \text{for } i, j = 2, 3, ..., N; i \ne j;\ \forall k \tag{15}$$

$$\sum_{i=1}^{N} X_{ihk} - \sum_{j=1}^{N} X_{hjk} = 0 \qquad \text{for } h = 2, 3, ..., N;\ \forall k \tag{16}$$

$$\sum_{i=1}^{N} X_{ijk} = E_{jk} \qquad \text{for } j = 2, 3, ..., N; i \ne j;\ \forall k \tag{17}$$

$$\sum_{k=1}^{K} E_{jk} = 1 \qquad \text{for } j = 2, 3, ..., N \tag{18}$$

$$\sum_{j=2}^{N} X_{1jk} \le 1 \qquad \forall k \tag{19}$$

$$\sum_{k=1}^{K} \sum_{w=1}^{W_k} Z^{pq}_{kwj} = Y_{jpq} V_{jpq} \qquad \forall j, p, q \tag{20}$$

$$\sum_{j=1}^{N} \sum_{p=1}^{P} \sum_{q=1}^{Q} Z^{pq}_{kwj} \le 1 \qquad \forall k, w \tag{21}$$

$$E_{jk} \ge Z^{pq}_{kwj} \qquad \forall k, w, j, p, q \tag{22}$$

$$\sum_{j=1}^{N} \sum_{w=1}^{W_k} \sum_{p=1}^{P} \sum_{q=1}^{Q} S^{pq}_{kwj} D'_{jpq} \le Cap_k \qquad \forall k \tag{23}$$

$$S^{pq}_{kwj} D'_{jpq} \le Capc_{kw} \qquad \forall k, w, j, p, q \tag{24}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{X_{ijk}d_{ij}}{Ve}+\sum_{i=1}^{N}\sum_{j=2}^{N}X_{ijk}Un \leq Tmax \qquad \forall k \tag{25}$$

$$\sum_{q=1}^{Q}V_{jpq}=1 \qquad \forall j,p \tag{26}$$

$$Z_{kwj}^{pq}+V_{jpq}-S_{kwj}^{pq}-1\leq 0 \qquad \forall k,w,j,p,q \tag{27}$$

$$-Z_{kwj}^{pq}-V_{jpq}+2S_{kwj}^{pq}\leq 0 \qquad \forall k,w,j,p,q \tag{28}$$

$$X_{ijk}\in\{0,1\},E_{jk}\in\{0,1\},Z_{kwj}^{pq}\in\{0,1\},V_{jpq}\in\{0,1\},S_{kwj}^{pq}\in\{0,1\} \qquad \forall k,w,j,p,q \tag{29}$$

In model 2, the objective function is minimizing the total traveling cost of fuel distribution. Most constraints of model 2 are not different from those of model 1 except constraint (20), (23), (24), and (26). These constraints are utilized to ensure that one of an integer number Y_{jpq} will be selected to split the order quantity and all split demands will be loaded appropriately.

4 NUMERICAL EXAMPLES

This section discusses the numerical example and the solution procedure by utilizing these two mathematical models in section 3. The delivery of fuel oils to petrol stations is used to demonstrate the solution procedures for MC-VRP-SP. The hypothetical problems which vary the number of customer range from 5 to 50 customers are generated with 2 replicates of each problem size. The optimization software namely IBM ILOG CPLEX v.12.2.2 is used to find solution. The constant variables for these problems are defined and will be used for all problems. The maximum order quantity is 16 kiloliters, the unload time is assumed a constant value of 20 minutes per customer, the average velocity of a truck is 40 km/hour, and time limit for one trip is 8 hours. For single-split pattern, the denominators Y_{jp} are as following:

$$Y_{jp}=\begin{cases}1;\ \text{if } D_{jp}=8,12,16\\ 2;\ \text{if } D_{jp}=10,12,14,16\\ 0;\ \text{if } D_{jp}=0\end{cases}$$

For multi-split pattern model 2, we set Q_{max} = 3 and the denominators Y_{jpq} are depended upon the order quantity D_{jp}. Parameter Q is defined as

$$Q=\begin{cases}3;\ \text{if } D_{jp}=8,12,16\\ 2;\ \text{if } D_{jp}=4,6\\ 1;\ \text{if } D_{jp}=2,10,14\end{cases}$$

when Q = 3 denominators can be $Y_{jpq}=\begin{cases}2,4,8;\ \text{if } D_{jp}=16\\ 2,3,6;\ \text{if } D_{jp}=12\\ 1,2,4;\ \text{if } D_{jp}=8\end{cases}$,

when Q = 2 denominators can be $Y_{jpq}=\begin{cases}1,2;\ \text{if } D_{jp}=4\\ 1,3;\ \text{if } D_{jp}=6\end{cases}$,

and when Q = 1, denominator can be$Y_{jpq}=\begin{cases}2;\ \text{if } D_{jp}=4\\ 1;\ \text{if } D_{jp}=6\\ 0;\ \text{if } D_{jp}=0\end{cases}$.

Several numerical examples are tested by utilizing mathematical models in section 3. The customer positions locate in x-y coordinates which have the boundary between -50 to 50 kilometers away from the depot where the depot is located at [0, 0] and the order quantity is an integer of multiple of 2,000 liters. The numerical examples are separated into two groups which are the MC-VRP-SP with the homogeneous fleet of trucks and the MC-VRP-SP with the heterogeneous fleet of trucks. The maximum computational time is 7200 seconds.

4.1 MC-VRP-SP with a homogeneous fleet of trucks

In this section, the single-split pattern and multi-split patterns MC-VRP will be examined with the homogeneous fleet of trucks. The information of customer including locations, demand order, available vehicles, and results are displayed in Table 1, 2, 3, and 4, respectively.

Table 1: X-Y coordinates for 10 customers with a homogeneous fleet of trucks

Coordinate	Customer									
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
X	-22.25	-4.75	-50.00	-15.00	-31.75	45.50	-17.00	2.50	36.50	34.75
Y	-45.00	-23.00	-15.00	25.50	4.25	5.50	-37.50	30.25	-39.25	-20.00

Table 2: Demands (kl) of each customer for 10 customers with a homogeneous fleet of trucks

Product	Customer									
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	6	0	4	0	0	0	0	0	6	0
2	0	0	0	0	2	4	0	0	4	0
3	0	8	0	16	6	0	0	0	0	12
4	0	0	0	4	0	0	2	0	0	4
5	2	0	0	0	0	4	0	8	0	0

Table 3: Data of available homogeneous fleet of trucks

Type	Amount	Capacity (kl)	Compartment Capacity (kl)	Cost (Bath/km)
1	6	40	8 8 6 6 4 4 2 2	19

Table 4: Result for a problem of 10 customers with a homogeneous fleet of trucks

Truck	Routes - Model 1	Truck	Routes - Model 2
1	D→C9→C10→D	1	D→C8→C4→D
2	D→C2→C7→C1→C3→C5→C8→D	2	D→C9→C10→C6→D
3	D→C4→C6→D	3	D→C2→C7→C1→C3→C5→D
Total Cost	8,846 Baht	Total Cost	7,135 Baht
State	optimal	State	optimal
Time (sec.)	28.39	Time (sec.)	116.80

Note: D = depot

Other problem sets ranging from 5 to 50 customers are also examined where each problem set consists of 2 subproblems. The upper bound (UB), lower bound (LB), and calculation time of all problems with a heterogeneous fleet of trucks can be found in Table 5 - 8.

Table 5: Problem 1 of 5, 10, 15 customers with a homogeneous fleet of trucks

Model	Total cost (Baht) for each problem					
	5	Time	10	Tme	15	Time
1	5,517 UB	1.17	7,572 UB	17.70	8,349 UB	7200
	5,517 LB		7,572 LB		7,898 LB	
2	4,960 UB	1.20	7,572 UB	138.8	8,349 UB	7200
	4,960 LB		7,572 LB		6,609 LB	

Table 6: Problem 2 of 5, 10, 15 customers with a homogeneous fleet of trucks

Model	Total cost (Baht) for each problem					
	5	Time	10	Tme	15	Time
1	4,226 UB	1.20	8,446 UB	28.39	10,161 UB	7200
	4,226 LB		8,446 LB		9,112 LB	
2	4,075 UB	0.56	7,135 UB	116.8	9,613 UB	7200
	4,075 LB		7,135 LB		7,468 LB	

Table 7: Problem 1 of 20, 30, 40 and 50 customers with a homogeneous fleet of trucks

Model	Total cost (Baht) for each problem							
	20		30		40		50	
1	12,285	UB	18,217	UB	*22,816	UB	*40,439	UB
	8,179	LB	9,317	LB	7,585	LB	8,427	LB
2	11,853	UB	17,767	UB	-	UB	-	UB
	5,048	LB	6,820	LB	7,820	LB	8,239	LB

Table 8: Problem 2 of 20, 30, 40 and 50 customers with a homogeneous fleet of trucks

Model	Total cost (Baht) for each problem							
	20		30		40		50	
1	12,512	UB	18,104	UB	*23,701	UB	*39,739	UB
	8,260	LB	7,749	LB	7,949	LB	8,308	LB
2	12,007	UB	18,686	UB	-	UB	-	UB
	4,952	LB	7,628	LB	7,533	LB	8,239	LB

Note: *CPLEX was terminated because of out of memory. The computational time limit was reduced to handle this problem.

CPLEX can provide us the global optimal solutions up to the problem of 10 customers. Mostly, CPLEX is able to yield us the best solution when utilizing the multi-split patterns model (model 2). For the large-scale problem (40 and 50 customers), solving the single-split pattern model enable us to have the best solution. We found that CPLEX is unable to find the feasible solution within 2 hours when solving the problem of 50 customers and utilizing the multi-split pattern (model 2). It is noticed that the multi-split pattern models are more difficult to find the feasible solution for the large-scale problem.

4.2 MC-VRP-SP with a heterogeneous fleet of trucks

In this section, the single-split and multi-split pattern MCVRP will be examined with a fixed heterogeneous fleet of trucks. A problem of 10 customers is solved and the information of customer locations, demands of each customer, available vehicles, and results are displayed in Table 9, 10, 11, and 12, respectively.

Table 9: X-Y coordinates for 10 customers with a heterogeneous fleet of trucks

Coordinate	Customer									
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
X	39.25	7	-20.75	-22	-5.5	22.25	-3	-22.75	-18	7.5
Y	-25.25	-33.75	17.5	-1.5	45.5	21.5	11.5	13.5	-28.25	6.25

Table 10: Demands (kl) of each customer for 10 customers with a homogeneous fleet of trucks

Product	Customer									
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	8	0	14	4	0	12	4	0	16	0
2	0	0	0	0	0	0	0	4	0	8
3	0	0	0	0	6	0	6	2	0	0
4	0	0	0	8	0	2	0	0	0	0
5	0	4	0	0	0	0	0	0	0	0

Table 11: Data of available heterogeneous trucks

Type	Amount	Capacity (kl)	Compartment Capacity (kl)	Cost (Bath/km)
1	1	16	4 4 2 2 2 2	15
2	1	16	4 4 4 4	15
3	1	40	8 8 8 8 8	19
4	1	40	8 8 6 6 4 4 2 2	19
5	1	32	6 6 6 6 4 4	17
6	1	32	8 8 4 4 4 4	17

Table 12: Result for a problem of 10 customers with a heterogeneous fleet of trucks

Truck-Type	Routing - Model 1	Truck-Type	Routing - Model 2
1-3	D→C10→C1→C2→C9→D	1-1	D→C10→D
2-4	D→C4→D	2-4	D→C3→C8→C4→D
3-5	D→C6→C5→C7→D	3-5	D→C6→ C5→ C7→ D
4-6	D→ C8→C3→ D	4-6	D→ C9→ C2→ C1→D
Total Cost	6,548 Baht	Total Cost	5,894 Baht
State	optimal	State	optimal
Time (sec.)	4.63	Time (sec.)	171.6

Other problem sets ranging from 5 to 50 customers are also examined where each problem set consists of 2 problems. The computational time limit is 7200 seconds. The UB and LB of all problems with a heterogeneous fleet of trucks are in Table 13 - 16.

Table 13: Problem 1 of 5, 10, 15 customers with a heterogeneous fleet of trucks

Model	Total cost (Baht) for each problem					
	5	Time	10	Tme	15	Time
1	4,300 UB	1.08	6,548 UB	4.63	8,642 UB	525
	4,300 LB		6,548 LB		8,642 LB	
2	4,211 UB	1.03	5,894 UB	171.6	8,642 UB	7200
	4,211 LB		5,894 LB		4,825 LB	

Table 14: Problem 2 of 5, 10, 15 customers with a heterogeneous fleet of trucks

Model	Total cost (Baht) for each problem					
	5	Time	10	Tme	15	Time
1	2,004 UB	1.37	5,891 UB	10.94	9,090 UB	1684
	2,004 LB		5,891 LB		9,090 LB	
2	2,004 UB	5.5	5,666 UB	268.4	8,853 UB	7200
	2,004 LB		5,666 LB		4,992 LB	

Table 15: Problem 1 of 20, 30, 40 and 50 customers with a heterogeneous fleet of trucks

Model	Total cost (Baht) for each problem							
	20		30		40		50	
1	12,798	UB	16,198	UB	*23,044	UB	*29,695	UB
	6,195	LB	6,808	LB	7,585	LB	7,550	LB
2	12,985	UB	16,572	UB	*28,092	UB	-	UB
	5,029	LB	5,900	LB	6,995	LB	7,394	LB

Table 16: Problem 2 of 20, 30, 40 and 50 customers with a heterogeneous fleet of trucks

Model	Total cost (Baht) for each problem							
	20		30		40		50	
1	10,666	UB	14,354	UB	*26,213	UB	*30,351	UB
	5,417	LB	6,802	LB	7,281	LB	7,797	LB
2	10,895	UB	14,704	UB	*24,004	UB	-	UB
	4,793	LB	5,932	LB	6,575	LB	6,914	LB

4.3 Clustering Method and 2-opt Algorithm

In large-scale problem, we can observe the big gap between UB and LB. Thus, the Clarke & Wright's saving algorithm in Clarke and Wright (1964) is employed to perform the clustering of customers and then each cluster is separately solved model 2. Equation (30) represents the saving algorithm. Fig. 1 (a) and 1 (b) show how to save the distance when visit 2 two node before traveling back to the depot.

$$S_{ij} = D_{0i} + D_{j0} - D_{ij} \tag{30}$$

where

D_{0i} is the distance from depot 0 to node j
D_{ij} is the distance from node i to node j
S_{ij} is a saving value traveling directly between node i to node j

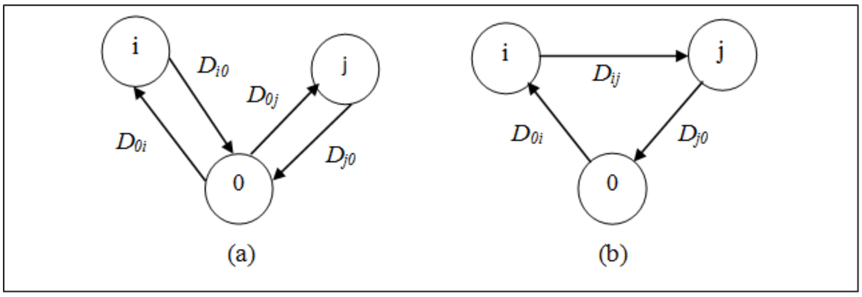

Figure 1: Saving Algorithm

The clustering technique is used to arbitrarily separate the customers into the smaller groups. In this paper, each cluster has 10 customers. Then, each cluster is separately solved by utilizing model 2 for the Problem of 40 and 50 customers. Each cluster will be obtained the optimal solution.

In the problem with a homogeneous fleet of trucks, the clustering technique reduce the total cost of traveling from 22,816 baht to 20,525 baht in problem 1 of 40 customers as show in Table 17 and from 23,701 baht to 22,493 baht in problem 2 of 40 customers (see Table 21). For the problems of 50 customers, the total cost can be reduced from 40,439 baht to 28,815 baht in problem 1 as shown in Table 18 and from 39,739 baht to 25,644 baht in problem 2 (see Table 21).

Table 17: Results of clustering method for problem 1 of 40 customers with a homogeneous fleet of trucks

Group	Customer	Cost (Baht)	Time (sec.)
1	8, 37, 10, 5, 23, 29, 25, 38, 15, 13	5,232	1753.0
2	20, 16, 31, 4, 34, 2, 11, 28, 18, 26	6,915	155.8
3	9, 32, 40, 1, 12, 24, 27, 30, 3, 22	5,530	1406.6
4	14, 6, 36, 39, 21,33, 7, 17, 35, 19	2,848	241.3
	Total	20,525	3556.7

Table 18: Results of clustering method for problem 1 of 50 customers with a homogeneous fleet of trucks

Group	Customer	Cost (Baht)	Time (sec.)
1	10, 39, 25, 32, 8, 28, 17, 44, 36, 33	7,430	1130.3
2	38, 49, 7, 21, 14, 42, 29, 26, 48, 9	6,369	2608.6
3	4, 31, 37, 2, 47, 23, 19, 46, 34, 30	5,272	1094.6
4	50, 11, 5, 24, 3, 22, 12, 41, 15, 1	5,146	1750.0
5	6, 13, 16, 18, 20, 27, 35, 40, 43, 45	4,598	75.7
	Total	28,815	6659.2

In the heterogeneous fleet of trucks, the total cost of traveling reduces from 23,044 baht to 21,802 baht in problem 1 of 40 customers as shown in Table 19 and from 24,004 baht to 19,783 baht in problem 2 of 40 customers (see Table 22). For problem of 50 customers, the total cost of traveling is reduced from 29,695 baht to 25,000 baht in problem 1 as shown in Table 20 and from 30,351 baht to 28,312 baht in problem 2 (see Table 22).

Table 19: Results of clustering method for problem 1 of 40 Customers with a heterogeneous fleet of trucks

Group	Customer	Cost (Baht)	Time (sec.)
1	17, 37, 4, 7, 20, 6, 18, 28, 30, 2	7,019	923
2	10, 23, 33, 25, 9, 31, 16, 14, 29, 34	5,990	1267.2
3	8, 19, 12, 27, 13, 5, 36, 35, 21, 40	5,516	376.5
4	15, 22, 24, 39, 38, 3, 32, 11, 1, 26	3,277	34.9
Total		21,802	2601.6

Table 20: Results of clustering method for problem 1 of 50 customers with a heterogeneous fleet of trucks

Group	Customer	Cost (Baht)	Time (sec.)
1	47, 36, 12, 30, 22, 23, 9, 5, 18, 31	7,316	341.3
2	21, 26, 25, 17, 11, 41, 46, 27, 14, 39	7,077	1178.2
3	34, 2, 50, 8, 6, 40, 4, 33, 28, 20	5,031	1290.1
4	13, 29, 7, 10, 48, 16, 19, 43, 44, 24	2,518	3229.3
5	1, 3, 15, 49, 45, 32, 35, 37, 38, 42	3,059	56.5
Total		25,000	6095.4

The 2-opt procedure works on improving single route by systematically exchange the route direction between two pairs of consecutive customers in the route and evaluate whether the routing cost of the route is improved or not. The exchange mechanism is illustrated in Fig. 2, in which the route direction between customers i –$(i + 1)$ and j –$(j + 1)$ are interchanged. If the routing cost of the modified route is better than the routing cost of original one, the route is updated with the modified one. This paper employs the 2-opt to improve the best solution of the problem of 40 and 50 customers which are obtained from CPLEX. The improvement in percent by implementing 2-opt are in the Table 21 and 22. The results show that 2-opt can reduce the cost of traveling for 7 out of 8 problems.

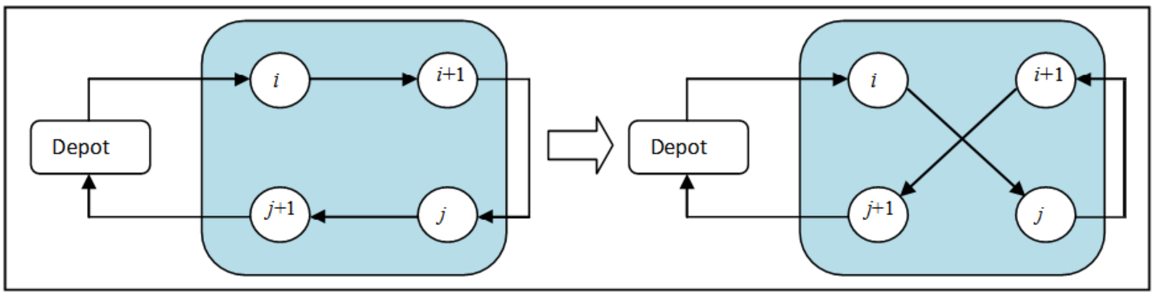

Figure 2: The 2-opt Algorithm

The comparisons on the several solution approaches are shown the Table 21 and 22. Fig. 3 to Fig. 6 are also used to illustrated the performance of each solution procedure. For small-scale problem (number of customers are less than or equal to 15), the solutions received from the CPLEX in solving 2 mathematical models is not significant difference. However, Model 2 can yield us the lowest cost. In the most cases of large-scale problem, using Model 1 can find the lowest traveling cost. The solutions in the problems of a homogeneous fleet of and heterogeneous trucks are in the same trend.

When using 2-opt and clustering technique, it is noticed that the clustering technique using the saving algorithm can make a significant improvement on the traveling cost of both problems in the large-scale problem.

Table 21: Comparisons of the clustering method, the best UB, and 2-opt on a homogeneous fleet of trucks

Size	Problem	Best UB (Baht)	Best UB obtained from	2-opt (Baht)	Clustering solution (Baht)	* % Improvement	
						2-opt	Clustering
40	1	22,816	*Model 1*	21,914	20,525	3.95	10.04
	2	23,701	*Model 1*	22,924	22,493	3.28	5.10
50	1	40,439	*Model 1*	38,636	28,815	4.46	28.74
	2	39,739	*Model 1*	39,624	25,644	0.29	35.47

Note: * % Improvement compared to the Best UB

Table 22: Comparisons of the clustering method, the best UB, and 2-opt on a heterogeneous fleet of trucks

Size	Problem	Best UB (Baht)	Best UB obtained from	2-opt (Baht)	Clustering solution (Baht)	* % Improvement	
						2-opt	Clustering
40	1	23,044	*Model 1*	23,044	21,802	0.00	5.39
	2	24,004	*Model 2*	23,002	19,783	4.17	17.58
50	1	29,695	*Model 1*	28,517	25,000	3.97	15.81
	2	30,351	*Model 1*	30,060	28,312	0.96	6.72

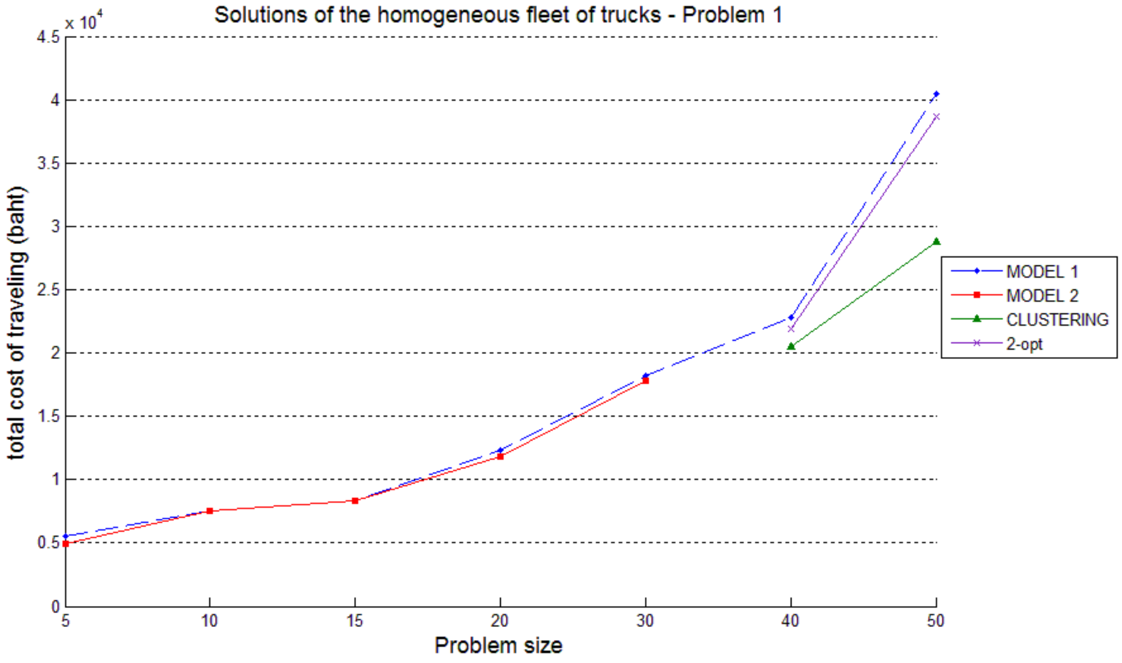

Figure 3: Solutions of the homogeneous fleet of trucks - Problem 1

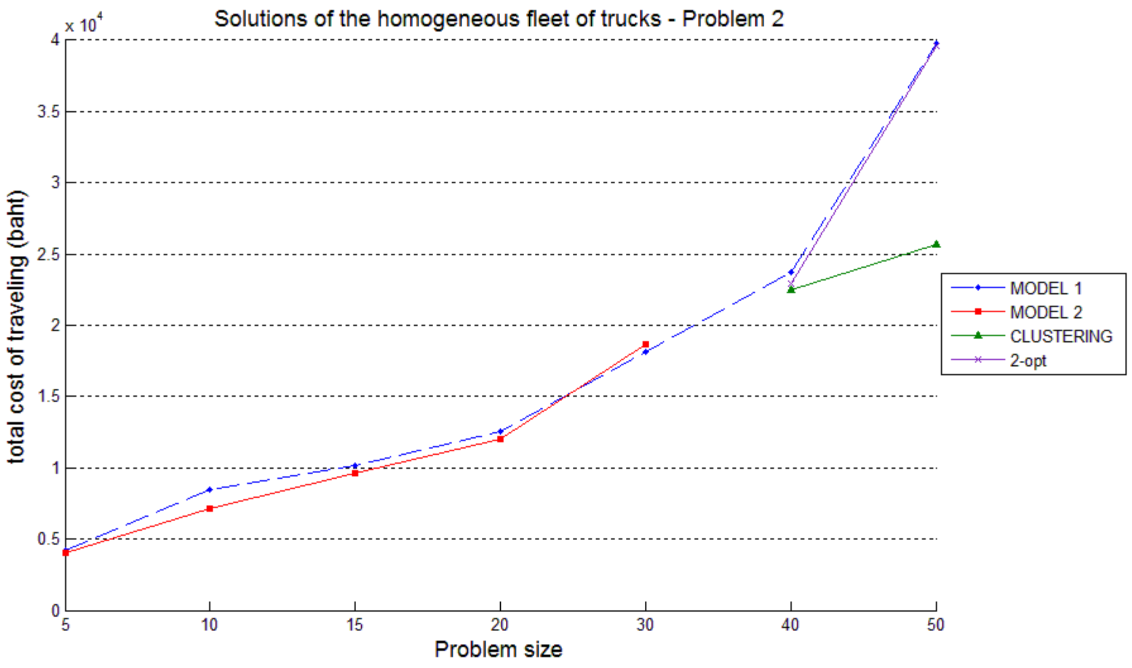

Figure 4: Solutions of the homogeneous fleet of trucks - Problem 2

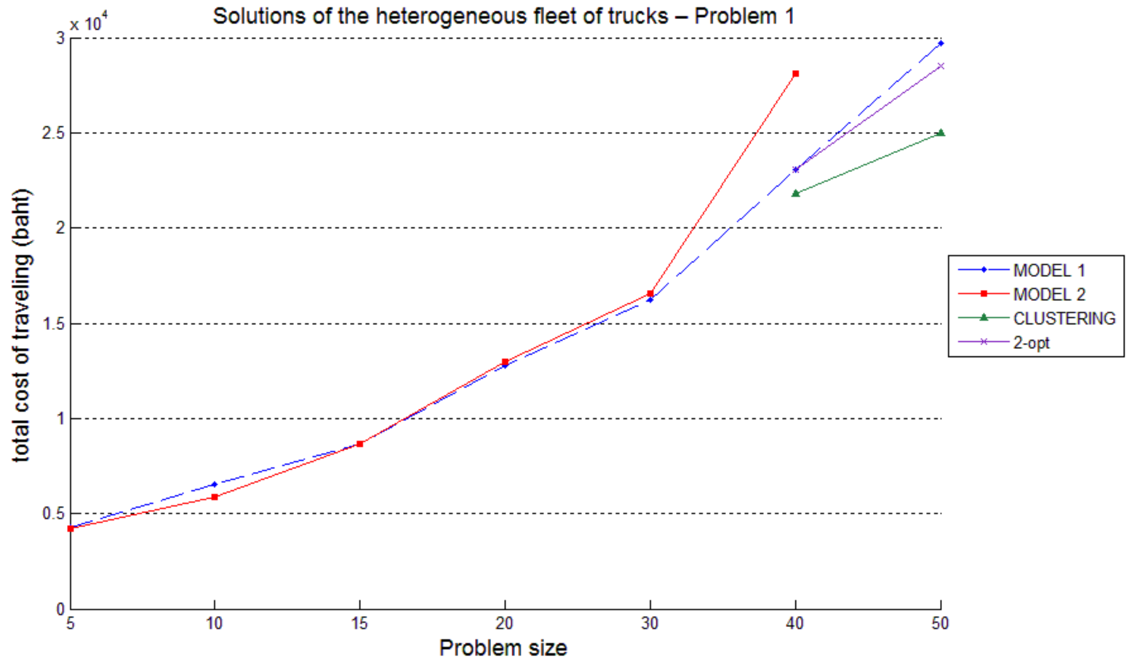

Figure 5: Solutions of the heterogeneous fleet of trucks - Problem 1

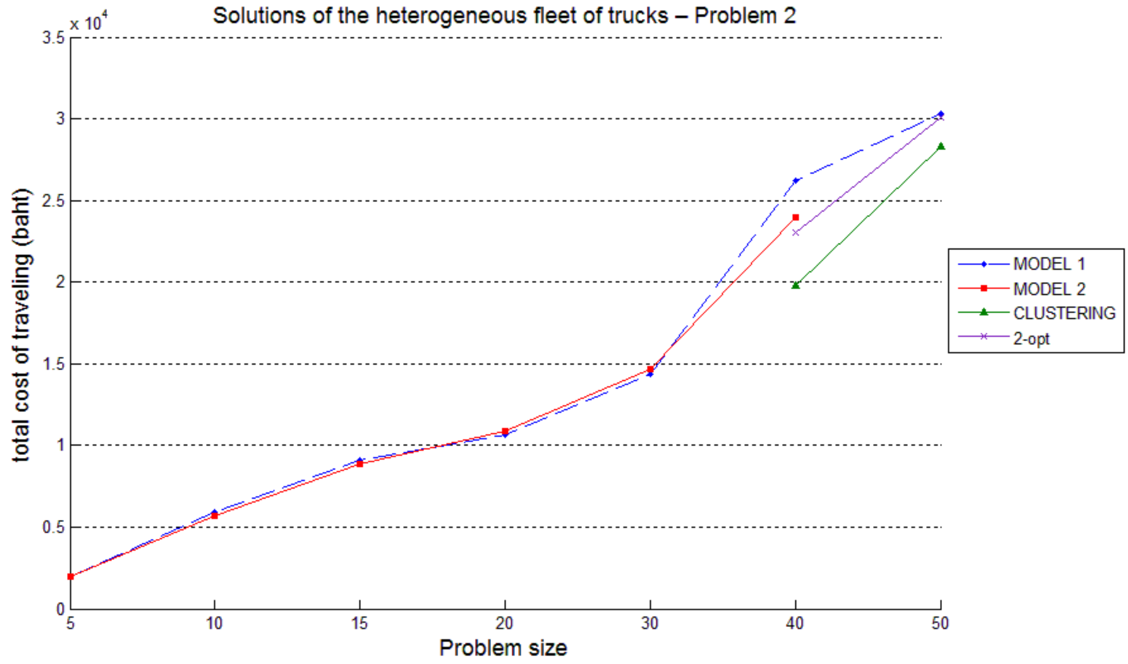

Figure 6: Solutions of the heterogeneous fleet of trucks - Problem 2

5 CONCLUSIONS AND DISCUSSIONS

This paper proposes three mathematical models and the solution procedures including the optimization approaches (CPLEX), 2-opt algorithm, and clustering technique. The objective of the mathematical models is to minimize the total traveling cost. The major constraint is that each compartment on a truck can contain a single customer's demand. Since the compartments are not identical, the predetermined split pattern including single- and multi-split patterns are proposed to separate the demand so that the split demand will not be over the compartment capacity. CPLEX can guarantee the optimality in the problem size up to 15 customers. In additions, using model 2 can potentially obtain the lower cost than the other alternatives. Two groups problem with a homogeneous fleet and a heterogeneous fleet yield us the solutions in the same fashion. Next, the 2-opt heuristic and the clustering using the saving algorithm are employed to improve the solution in the large-scale problem. It is noticed that 2-opt can improve the solutions received from CPLEX up to 4.6% in the problem of 50 customers with a homogeneous fleet of trucks. The clustering technique can yield us best solution compared to other approaches. Using the clustering can make an improvement on a solution of 50 customers by 35.47%. In addition, the cumulative computational time for all clusters is still lower than 7,200 seconds. These results reveal that separating the problem into the small-scale enables us to have the lower traveling cost. It is recommended that, for large-scale problem, the practical solution procedure is to make the clusters before searching for the optimal routes.

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