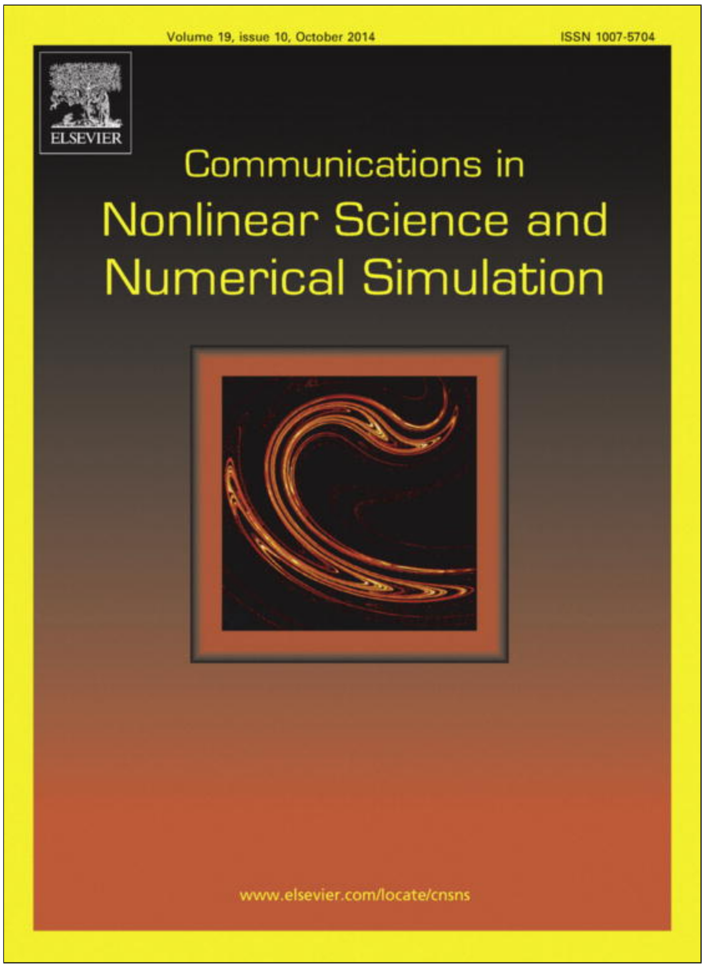
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A 2D chaotic path planning for mobile robots accomplishing boundary surveillance missions in adversarial conditions

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ABSTRACT

The path-planning algorithm represents a crucial issue for every autonomous mobile robot. In normal circumstances a patrol robot will compute an optimal path to ensure its task accomplishment, but in adversarial conditions the problem is getting more complicated. Here, the robot's trajectory needs to be altered into a misleading and unpredictable path to cope with potential opponents. Chaotic systems provide the needed framework for obtaining unpredictable motion in all of the three basic robot surveillance missions: area, points of interests and boundary monitoring. Proficient approaches have been provided for the first two surveillance tasks, but for boundary patrol missions no method has been reported yet. This paper addresses the mentioned research gap by proposing an efficient method, based on chaotic dynamic of the Hénon system, to ensure unpredictable boundary patrol on any shape of chosen closed contour.

1. Introduction

Mobile autonomous robots are intelligent real-time systems that operate in structured or unstructured environments without explicit human involvement. Their real-life applications are spread in a wide variety of domains where tedious or hazardous tasks must be precisely accomplished, from elderly and disabled care [1], precise agriculture [2–4], disaster intervention [5,6], to complex industrial activities [7].

The motion planning represents a fundamental issue for such robots, being tackled by numerous researchers over time. Basically a path-planning algorithm has to provide an optimal trajectory, when diverse constraints are applied, for the robot to accomplish its tasks. This is not a simple problem to solve due to the frequent and random changes in the environment. Moreover, when autonomous robots evolve in adversarial conditions, a new attribute must be considered by the path-planning mechanism for coping with possible opponents: unpredictability of the trajectory for any external observer.

Previous research addressed two basic types of robot surveillance missions: monitoring an area and monitoring points of interest. When speaking about the third kind of surveillance missions – boundary surveillance, from our knowledge, no approach for unpredictable trajectories has been reported so far. This paper fills this research gap by proposing a robot path-planning methodology based on Hénon discrete chaotic system. We started with the analysis of a mobile coordinate frame in which a Hénon chaotic system evolves. If this frame is translated with a finite velocity along a closed contour marked in a stationary frame (the origin of the mobile frame performs a periodic motion along the closed contour), we observed that a brand-new chaotic system was constructed. As a remarkable fact we find that the Lyapunov exponents are

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conserved during this transformation even though the new-type trajectories have other shapes. Practically speaking, the arbitrarily-chosen boundary that must be monitored by the patrol robot is chaotified with the means of kinematic relative motion and the use of Hénon system.

The rest of the paper is organized as follows. Section 2 presents the state of the art in the field of generating chaotic trajectories for mobile robots. Section 3 provides the theoretical support for our methodology, demonstrating that the new robot trajectories are indeed chaotic. Section 4 thoroughly presents the novel step-by-step methodology, accompanied by three illustrative examples described in Section 5. The last section outlines the conclusions and final remarks.

2. Related work

In adversarial conditions, the unpredictability of a robot trajectory represents a crucial issue. It can be addressed using either random or chaotic sequences of waypoints. In both cases, the enemy entities cannot predict the future trajectory, but due to its deterministic nature, the chaotic path represents a better alternative. The reason lies in the fact that ally entities, knowing the initial conditions and formulas of chaotic system, are able to predict the robot path and, as a result, to make proper decisions.

Chaotic systems, due to their "sensitivity to initial conditions" feature [8], provide the much-needed framework in achieving the unpredictability in all the three basic types of surveillance missions: monitoring an area; monitoring a set of points of interest (set of precise objectives); or monitoring the boundary of a specified area.

The area surveillance missions presume an efficient coverage of all sections of a specified perimeter. This particular topic was addressed by some relevant research papers [9–15].

Nakamura and Sekiguchi [9] designed and implemented a chaotic motion controller for mobile robots able to sense the workspace boundary when arriving in its proximity. Their idea was to interconnect the control variables with state variables of the Arnold equations and by this to impart the chaotic behavior of incompressive fluid flow to the robot.

Martins-Filho and Macau proposed an ingenious path-planning mechanism where the sequence of intermediary goal positions is obtained using the well-known Chirikov–Taylor standard map [10,11]. This area-preserving chaotic map, besides its chaotic features originating from the dynamics of a kicked rotor, eliminates the need for boundary sensing.

Volos, Kyprianidis and Stouboulos [13,14] based their trajectory planning methodology on the use of the Logistic map. Here, a chaotic random bit generator provides a time-ordered succession of future robot positions, with the experimental results proving highly efficient and opportunistic area coverage. Another implementation based on the Logistic chaotic map, but this time improved by arcsine and arccosine transformations, is presented in [15].

The second type of surveillance missions was tackled in two of our previous papers. The first one presented an original method to monitor two points of interest based on a modified Lorenz system [16] and was followed by a generalized method that uses two types of 3D chaotic systems (Lorenz and Chen) to develop unpredictable trajectories for surveilling an indefinite number of specified points [17].

This paper addresses another kind of patrol robot mission - boundary surveillance, with the proposed methodology being based on the kinematic relative motion concept and the chaotic nature of the Hénon system.

3. Theoretical framework

In adversarial conditions an autonomous patrol robot must follow a path that cannot be easily predicted or understood by opponents. For boundary patrol missions, the problem can be formulated as follows:

> Problem formulation. Consider a given closed contour C in a two-dimensional Cartesian frame that has to be monitored by a patrol robot. The objective is to design a trajectory, unpredictable for possible opponents, developed in the proximity of C that assures efficient boundary surveillance.

In our vision, two ways can be pursued in solving this problem: (a) to find an already known chaotic system with the same shape as the given bounding line and adapt it via diverse transformations (e.g. affine transformations); or (b) to construct a new chaotic system tailored for this specific application. The first possibility is applicable for a limited number of boundary shapes so, a general method to create customized chaos may be the proper solution.

In this endeavor we started with the analysis of the well-known Hénon chaotic system [18]. We will demonstrate that if we slide an evolving Hénon system on a closed contour, a chaotic trajectory will be obtained. Practically speaking, with the means of kinematic relative motion concept we can design unpredictable paths in the vicinity of any given closed contour.

3.1. Hénon chaotic system

Analyzing the Lorenz chaotic system [19], the French astronomer and mathematician Hénon [18], discovered a two-dimensional discrete map exhibiting similar properties. His system is described by the following set of equations:

$$\begin{cases} x_{n+1} = y_n + 1 - a \cdot x_n^2 \\ y_{n+1} = b \cdot x_n \end{cases} \quad (1)$$

where the standard values of parameters, $a = 1.4$ and $b = 0.3$, provide a chaotic behavior (Fig. 1).Four characteristics of the Hénon chaotic map are worth mentioning in the context of our methodology:

1. The Lyapunov exponents [20], which characterize the average rate of separation of two nearby Hénon trajectories for x and y axes, are $\lambda_1 = 0.408$ and $\lambda_2 = -1.62$. Since the maximum Lyapunov exponent (λ_1) has a positive value, the chaotic behavior is confirmed [21];
2. The bifurcation diagram, presented in Fig. 2, underlines the chaotic behavior of the system for $a = 1.4$ and $b = 0.3$.
3. The trapping region (i.e. a region for which all entering trajectories will thereafter never leave) of the Hénon attractor [18] is represented by the quadrangle $\Omega = \overline{ABCD}$, with the vertices having the following positions $A = (-1.33; 0.42)$, $B = (1.32; 0.133)$, $C = (1.245; -0.14)$ and $D = (-1.06; -0.5)$;
4. The Hénon system exhibits computational simplicity when implemented on resource-constrained digital devices (only four multiplications and two additions are needed to obtain the coordinates of the next point).

The Lyapunov exponents and the bifurcation diagram, corresponding to the Hénon system, will be used to prove that the new robot's paths provided by our methodology are chaotic, while the trapping region is used for tuning the vicinity of the boundary where the robot will evolve. The illustration of the Hénon map and its trapping region is depicted in Fig. 1, while the bifurcation diagram when $b = 0.3$ is presented in Fig. 2.

This simple two-dimensional chaotic system is the source of unpredictability in our methodology, the new trajectories being obtained by means of the kinematic relative motion concept.

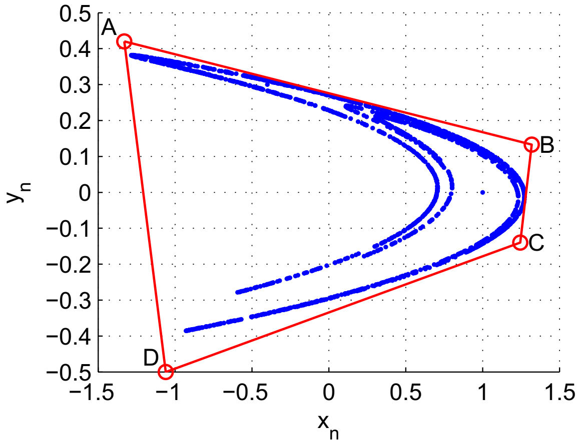

Fig. 1. The Hénon map and its trapping region.

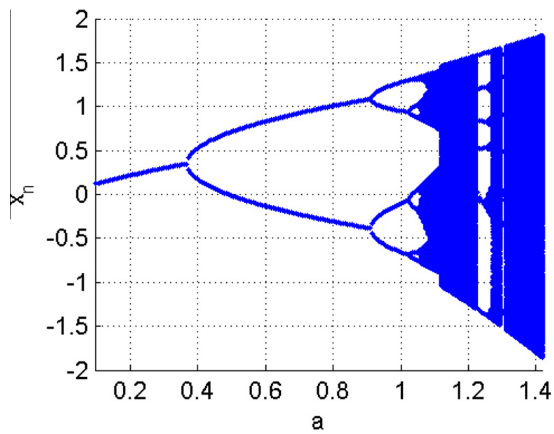

Fig. 2. Hénon bifurcation diagram for $b = 0.3$.

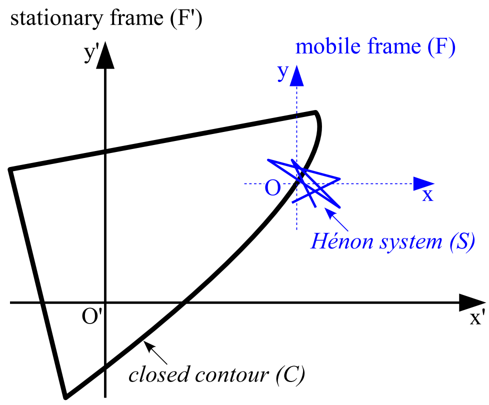

Fig. 3. Obtaining the chaotic path via relative motion.

3.2. Obtaining the chaotic trajectories by means of kinematic relative motion

First, let us describe the context. We consider a two-dimensional domain with its stationary coordinate system F' (Fig. 3). In this reference frame we establish an arbitrary closed contour C on which the origin O of a moving translation frame F (each of the F axes remains parallel with corresponding axes of F', any kind of rotation being forbidden) will slide with a constant velocity performing a periodic motion.

The relative motion is described by the following vector equation:

$$\vec{r'}(t) = \vec{r}(t) + \vec{r'_O}(t), \tag{2}$$

where $\vec{r'}(t)$ and $\vec{r}(t)$ are the position vectors of a trajectory point in the stationary and mobile coordinate frames at time t and $\vec{r'}_O(t)$ is the position vector of the origin O of the mobile frame with respect to the fixed frame (in our case indicates a position along the closed contour).

As a matter of notation, in what follows we will denote with prime the variables in stationary coordinate frame and without prime the variables in the mobile frame.

The combined motion can be described by the following set of equations:

$$\begin{cases} \left.\begin{aligned} x'_{n+1} &= x_{n+1} + \hat{x'}_{n+1} \\ y'_{n+1} &= y_{n+1} + \hat{y'}_{n+1} \end{aligned}\right\} \textit{composite dynamics based on relative motion concept} \\ \left.\begin{aligned} x_{n+1} &= y_n + 1 - a \cdot x_n^2 \\ y_{n+1} &= b \cdot x_n \end{aligned}\right\} \textit{Henon chaotic system} \\ (\hat{x'}_{n+1}, \hat{y'}_{n+1}) = f(\hat{x'}_n, \hat{y'}_n) \textit{periodic motion of the origin } O \\ \qquad\qquad \textit{of the mobile frame on the closed contour } C \end{cases} \tag{3}$$

The remainder of this section is dedicated to the proof of chaotic properties of the newly generated trajectory obtained by sliding an evolving Hénon system on a closed contour. In other words, we will demonstrate that the system described by (3) is chaotic. For this, we will appeal to a working chaotic system definition. During the time, a lot of definitions for chaos [8,22,23] have been proposed but none of them has gained unanimous acceptance [24–26]. A working definition was formulated by Strogatz [27] stating that chaos represents an aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions. A pragmatic approach to prove that a system is chaotic is to demonstrate that it displays the following features [20]: the system is deterministic; its trajectories are bounded when starting from a point within its attracting basin; and it exhibits at least one positive Lyapunov exponent. With these three features in mind, we will start a brief proof of a lemma which represents the cornerstone of our methodology.

Lemma: When translating the coordinate frame F in which a Hénon system evolves, the Lyapunov exponents related to the fixed coordinate frame F' are conserved.

Proof: The demonstration is presented here only for the x′ coordinate, for y′ being similar. Let us consider an infinitesimal difference $\delta_{x'}(0) = \Delta_{x'}(0) = x'_1(0) - x'_2(0)$ between the starting points $x'_1(0)$ and $x'_2(0)$ of two adjacent trajectories, coordinates being given in the stationary frame. The Lyapunov exponent [27] corresponding to the x′ coordinate for the system evolving in the fixed frame is defined as:

$$\lambda_{xfixedframe} = \lim_{t\to\infty} \lim_{\delta_{x'}\to 0} \frac{1}{t} \cdot ln \frac{x'_1(t) - x'_2(t)}{x'_1(0) - x'_2(0)} \tag{4}$$

By decomposing the general vector equation of relative motion (2), let us evaluate the distance between the two trajectories at the t moment in time:

$$\Delta_{x'}(t) = x'_1(t) - x'_2(t) = (x_1(t) + r'_{0x}(t)) - (x_2(t) + r'_{0x}(t)) = x_1(t) - x_2(t) = \Delta_x(t) \tag{5}$$

where $r'_{0x}(t)$ represents the displacement for x'-coordinate at t moment in time of the moving coordinate system versus the fixed one. Eq. (5) confirms that the distance between the two trajectories is the same with report to stationary or mobile coordinate frame even if an additional movement on the closed contour was introduced.

By particularizing (5) for $t = 0$, we obtain:

$$\delta_{x'}(0) = \Delta_{x'}(0) = \Delta_x(0) = \delta_x(0) \tag{6}$$

Using (5) and its particularized value for initial conditions (6), the Eq. (4) becomes:

$$\lambda_{x\ fixed\ frame} = \lim_{t\to\infty} \lim_{\delta_{x'}\to 0} \frac{1}{t} \cdot ln \frac{x'_1(t) - x'_2(t)}{x'_1(0) - x'_2(0)} = \lim_{t\to\infty} \lim_{\delta_x\to 0} \frac{1}{t} \cdot ln \frac{x_1(t) - x_2(t)}{x_1(0) - x_2(0)} = \lambda_{x\ mobile\ frame} \tag{7}$$

This demonstrates that the Lyapunov exponents are conserved by the kinematic relative motion.

Remark: The above lemma is a necessary but not sufficient condition for trajectories obtained via relative motion to be chaotic. For sufficiency, we also need to assure their boundedness. This key issue is solved when the mobile frame is translated on a closed contour.

Theorem. Let us consider a Hénon system S that evolves in a mobile coordinate frame F. If this reference frame is translated with a finite velocity along a given two-dimensional closed contour C, a new chaotic system is obtained. Moreover, the Lyapunov exponents are conserved via this transformation.

Proof: We will demonstrate the three mentioned conditions for a dynamical system to be chaotic.

- *Determinism* – Let us consider an initial state of the system that includes the absolute position of the origin O of the mobile frame $\overrightarrow{r_O}(0)$ and the initial state of the Hénon system inside the mobile frame. Because S is deterministic and the equation of periodic motion of origin O on the closed contour C is known and deterministic, we can precisely determine every future states of the system. As can be seen, no randomness is implied, thus, the new system is deterministic.
- *Boundedness* – Let us consider that the initial point (x_0,y_0) of the Hénon system S belongs to its attracting basin [28]. Knowing that such trajectories are bounded [22], their coordinates x and y (correspond to x' and y') in the mobile frame are also bounded: $x_S \in (x_{Smin}, x_{Smax})$, $y_S \in (y_{Smin}, y_{Smax})$. Let us consider that the lower and upper limits of the coordinates x' and y' for the established closed contour C are $x'_{C,min}$ and $x'_{C,max}$, respectively $y'_{C,min}$ and $y'_{C,max}$. We can conclude that the x' coordinate for the new system, denoted by $x'_{S'}$ will evolve inside the interval $x'_{S'} \in (x'_{C,min} - x_{Smin}, x'_{C,max} + x_{Smax})$ while for the y coordinate we will have $y'_{S'} \in (y'_{C,min} - y_{Smin}, y'_{C,max} + y_{Smax})$. As a result, the obtained system is bounded.
- *Lyapunov exponents* – Because S is a chaotic system (Hénon), at least one of its Lyapunov exponents is positive (i.e. $\lambda_1 = 0.408$). As shown by the above lemma, all of the Lyapunov exponents are conserved in the case of our combined motion. This leads us to the conclusion that the new system (3) exhibits at least one positive Lyapunov exponent, equal to the one exhibited by the Hénon system: $\lambda_1 = 0.408$.

Because all the three conditions are met, the new system, obtained by translating the Hénon system along a closed contour, is indeed chaotic.

Regarding this theorem, two important observations are worth mentioning:

(i) as can be seen, each newly introduced element around the original chaotic system (Hénon system) contributes to the purpose of preserving the chaotic properties: the contour is closed to assure confined trajectories, the velocity of the mobile frame is bounded for the same reason and the relative motion is used because it conserves the Lyapunov exponents; and

(ii) there is no restriction related to the guiding closed contour, so we can use any type of closed line, including self-intersecting, contours with overlapping sections, or even a simple line segment (e.g. can be considered a closed contour in the shape of a triangle with two coincident vertices).

4. Methodology

Based on the above demonstrated theorem, we designed a simple and efficient methodology for obtaining new chaotic trajectories in the proximity of any arbitrarily chosen closed contour for boundary surveillance purposes. This methodology comprises the following four steps:

4.1. Select a closed contour (C) in the two-dimensional space

The closed contour established in this step represents the guiding line for the new chaotic trajectory (the origin of the mobile frame is moving in a periodic motion along it) and has to be chosen in the proximity of the boundary that has to be monitored. It can be the boundary itself in the case that the robot can go outside the perimeter fence or a parallel line of the boundary, placed inside the perimeter, if the robot cannot exceed the boundary contour.

After selecting the C contour, a point-by-point motion along it can be formalized by:

$$\left(\widehat{x'}_{n+1}, \widehat{y'}_{n+1}\right) = f\left(\widehat{x'}_n, \widehat{y'}_n\right) \tag{8}$$

where f is a discrete periodic function and each pair of coordinates $(\widehat{x'}_i, \widehat{y'}_i)$ specifies a point on the guiding line:

$$\widehat{P}_i = \widehat{P}\left(\widehat{x'}_i, \widehat{y'}_i\right) \in C. \tag{9}$$

4.2. Choose the vicinity in which the robot must evolve;

Based on the guiding line established in the first step, we have to determine the area that surrounds it in which the robot must evolve. A practical way to select the vicinity in which the robot will develop its path is to establish a circular vicinity for each arbitrary point $\widehat{P}_i$ of the guiding line. This circle will have its center in the $\widehat{P}_i$ location and a diameter denoted by d (Fig. 4).

The diameter of this vicinity is chosen by considering an assortment of factors which includes: terrain configuration, obstacles located in the proximity of the boundary, other activities taking place in the area that can interfere with the robot's task, range of the robot's sensors that have to monitor the given boundary, average speed of the mobile robot, adopted control law for the robot when pursuing the point-by-point chaotic path, etc.

4.3. Adapt the Hénon chaotic system using affine transformations;

To ensure that Hénon system will evolve in the established surrounding area (chosen in step 2) of the guiding line, a tuning procedure must be performed. This can be accomplished using simple affine transformations like scalings, translations or rotations that are applied to the original Hénon system and are based on the Hénon system's trapping region presented in Fig. 1. As a consequence, the Hénon system (1) will be adapted using an appropriate change of variables.

For example, the following formulas that map the trapping region inside a circle of diameter d can be used:

$$\begin{cases} \tilde{x}_n = 0.3225 \cdot d \cdot x_n + 0.0875d \\ \tilde{y}_n = y_n \end{cases} \tag{10}$$

where x_i and y_i are the variables of the Hénon system.

4.4. Construct the new chaotic trajectory via relative motion

Having the guiding line and the adapted Hénon system, we can obtain a point-by-point chaotic trajectory by applying the relative motion concept depicted in paragraph 2.2. The system of equations describing the new chaotic path is similar to (3), with only one difference - the classic Hénon system is replaced, if needed, by an adapted version which encloses the variable change (10).

5. Case studies

In this section we illustrate our proposed methodology in a simple case of a circular guiding line and in two more complex cases where the closed contour is either a concave polygonal shape or a smooth closed curve. Moreover, for the circular guiding line, the chaotic properties of the newly designed trajectory will be verified by constructing the bifurcation diagram.

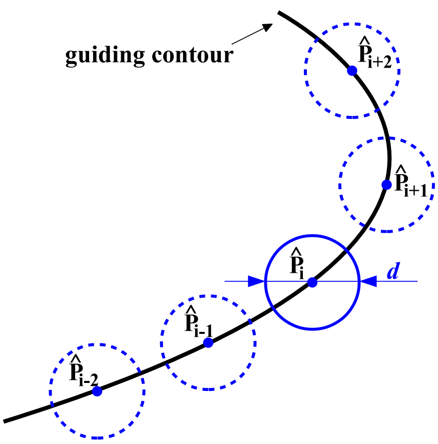

Fig. 4. Selecting the vicinity's dimension by choosing the diameter d.

In order to simulate the motion of a patrol robot along the chaotic path provided by our motion-planning approach, we made use of the kinematic closed-loop control law of a wheeled robot presented in [29–31].

5.1. Patrol robot kinematics model

Let us consider a differential mobile robot with three wheels: two parallel and independently driven active wheels and one passive caster wheel to ensure the robot's stability. The motion for this type of robots is controlled by modulating the active wheels' velocities $\omega_{left}(t)$ and $\omega_{right}(t)$, which is equivalent with considering the robot's linear and angular velocities denoted by $v(t)$ and $\omega(t)$ as control variables [30].

Our path-planning methodology provides a sequence of time-ordered waypoints that has to be followed by the mobile robot. The motion between two successive waypoints depends on the type of robot, its capabilities and the adopted motion control strategy. In the case of the differential mobile robot with three wheels considered here, the simplest mode to follow a point-by-point path is to consider that on each trajectory segment the motion is split in two: a rotation until the robot is positioned on the direction to the next waypoint and a linear motion directly to the waypoint. This approach has a major disadvantage in the case of adversarial conditions: when rotating, the robot is not changing its Cartesian coordinates and is more susceptible to attack. To overcome this drawback, we appeal to another motion control approach where the position is continuously changing, even if the real trajectory is not superposed on the straight segment between two consecutive waypoints.

The motion control problem configuration, when the mentioned robot is on the route to the following waypoint, can be visualized in Fig. 5, where ρ is the position error (distance between present and target positions) and φ is the orientation error.

The motion between the current and the goal positions can be formalized by:

$$\begin{bmatrix} \dot{\rho} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} -\cos\varphi & 0 \\ \frac{1}{\rho}\sin\varphi & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}. \tag{11}$$

In this context, an efficient control law is the one provided by Lee et.al [29]:

$$\begin{cases} v = k_1 \rho \cos\varphi \\ \omega = -k_1 \sin\varphi \cos\varphi - k_2 \varphi \end{cases}, \tag{12}$$

which is proved to take the robot to the target point on a smooth path. Moreover, the controller's tuning process is simple, k_1 being used to minimize the position error and k_2 the orientation error.

5.2. Chaotic trajectory with a circular guiding line

We considered a circular guiding line without establishing constraints about the vicinity in which the robot will evolve (there is no need for adapting the Hénon system via change of variables). Under these circumstances, the new chaotic system is represented by the following set of equations:

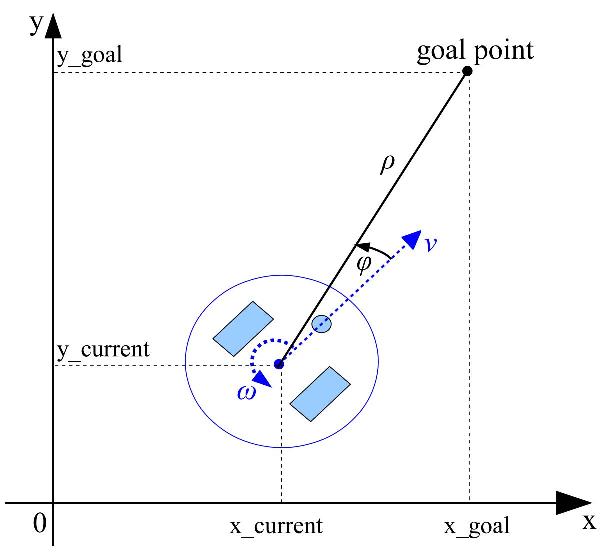

Fig. 5. Robot control configuration.

$$\begin{cases} \left.\begin{aligned} x'_{n+1} &= x_{n+1} + \widehat{x'}_{n+1} \\ y'_{n+1} &= y_{n+1} + \widehat{y'}_{n+1} \end{aligned}\right\} \textit{composite motion based on relative motion concept} \\ \left.\begin{aligned} x_{n+1} &= y_n + 1 - a \cdot x_n^2 \\ y_{n+1} &= b \cdot x_n \end{aligned}\right\} \textit{Henon chaotic system} \\ \left.\begin{aligned} \widehat{x'}_{n+1} &= R' \cdot cos(\theta'_0 + 2\pi \cdot \tfrac{n}{p}) + x'_{center} \\ \widehat{y'}_{n+1} &= R' \cdot sin(\theta'_0 + 2\pi \cdot \tfrac{n}{p}) + y'_{center} \end{aligned}\right\} \textit{periodic motion on a circle of } R' \textit{ radius} \end{cases} \tag{13}$$

where x'_{center} and y'_{center} are the circle's center coordinates, R' is the radius of the circle, θ'_0 is the initial central angle measured counterclockwise from the positive horizontal-axis, p is the number of points considered on the circle circumference (at equal central angles), x'_i and y'_i are the coordinates in the stationary frame, $\widehat{x'}_i$ and $\widehat{y'}_i$ are the positions on the closed contour. The chaotic path generated using (13), for $x'_{origin} = y'_{origin} = 0$, R' = 5 and p = 40, is depicted in Fig. 6.

For a supplementary verification of the chaotic feature for the designed path (the demonstration was already given by the theorem presented in paragraph 2.2), we drew the bifurcation diagram corresponding to the new chaotic system (13) for b = 0.3 (Fig. 7). It starts with 21 branches (there are 21 possible values for $\widehat{x'}_n$ on the circular guiding line if p = 40 and $\theta'_0 = 0$), and by comparing its shape with the Hénon's bifurcation diagram (Fig. 2) we notice that the chaotic regime appears for the same values of parameter a. For a = 1.4, the system is indeed chaotic.

5.3. Chaotic paths with arbitrarily chosen guiding lines

In this section, we depict two demonstrative examples involving more complex guiding lines: a concave polygonal shape and a smooth closed contour obtained using a spline interpolation procedure.

In the first example, the guiding line for the chaotic path is represented by a concave polygon, with five vertices having the following Cartesian coordinates: (−4,−4), (−2,4), (4,4), (2,2) and (4,0). The boundary proximity in which the robot must

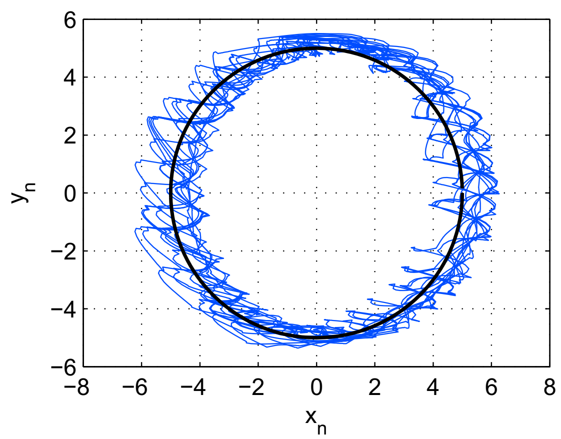

Fig. 6. The circular guiding line and corresponding chaotic path of the differential patrol robot ($k_1 = 1.5,\ k_2 = 4$).

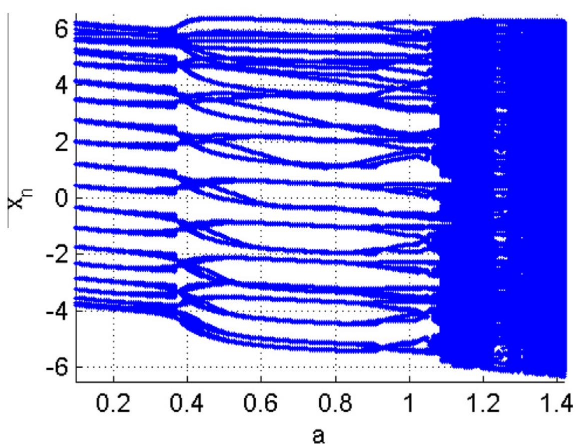

Fig. 7. Bifurcation diagram for the new chaotic system if b = 0.3.

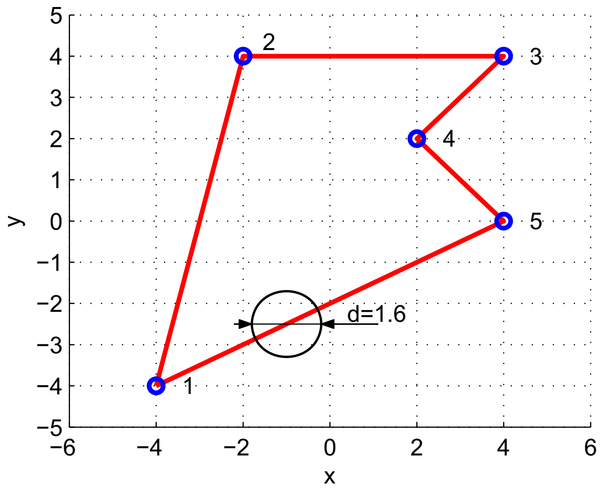

Fig. 8. Concave polygonal contour.

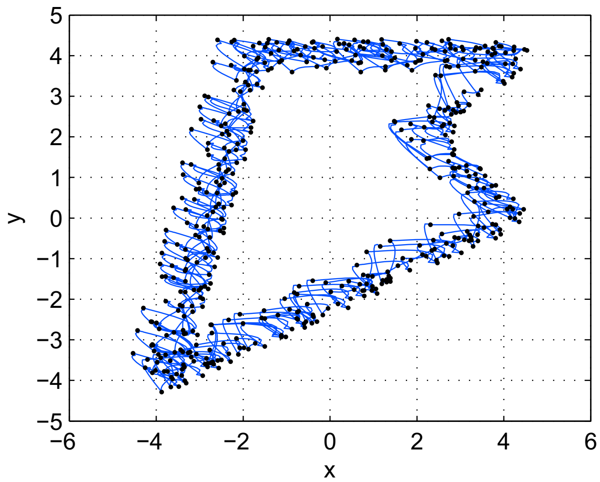

Fig. 9. Chaotic path in the proximity of a polygonal contour (five complete laps along the guiding line).

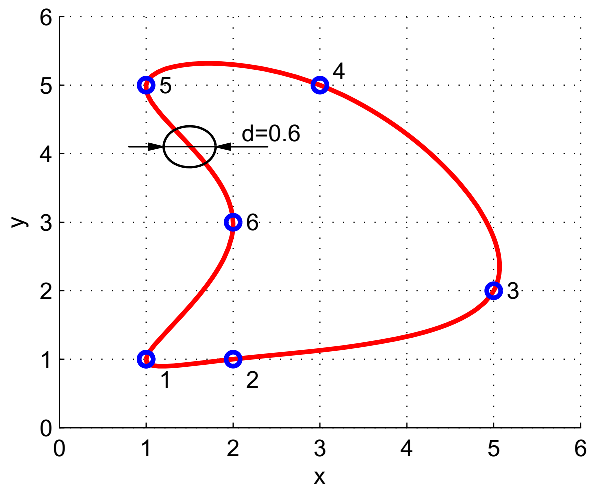

Fig. 10. Smooth closed curve used as a guiding line.

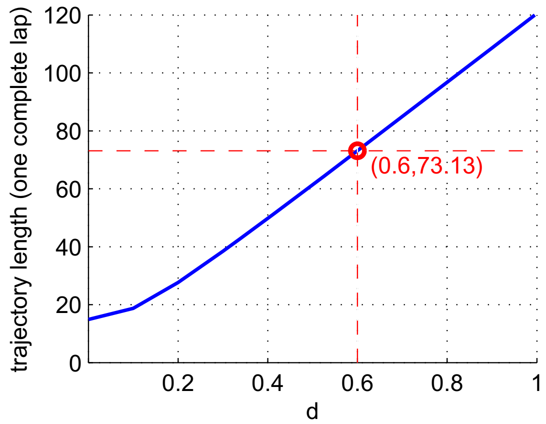

Fig. 11. Trajectory length for diverse diameters of the vicinity.

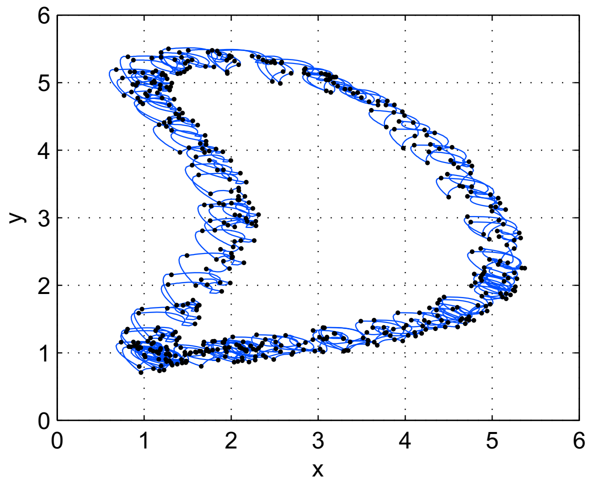

Fig. 12. Chaotic robot path in the proximity of closed curve (two complete laps along the guiding line).

evolve is defined by the diameter $d = 1.6$ of a circular vicinity for an arbitrary point of the contour. As a consequence, we adapted the Hénon system using (10). The guiding line and the resulted chaotic path followed by the mobile robot (for $k_1 = 1.5, k_2 = 4$) are presented in Fig. 8 and Fig. 9.

For the second example we started with the following set of points: (1,1), (2,1), (5,2), (3,5), (1,5) and (2,3). The smooth guiding line (Fig. 10) was constructed using the natural cubic spline interpolation method [32,33]. In choosing the surrounding area of the contour where the robot will evolve, we started with the following assumptions: the terrain is flat with no obstacles; the contour is represented by a sequence of 220 intermediary points; the average velocity of the robot is considered around 0.3 m/s; and the desired time to complete a full lap is around 4 min. By simulating the trajectory length of a complete lap for diverse vicinity diameters d, we obtained the diagram presented in Fig. 11. To meet the desired time to accomplish a full lap, we selected $d = 0.6$. For this value, the length of a complete lap is 73.13 m, which can be accomplished by our robot in 243.18 s.

By applying the change of variables (10) to obtain the adapted Hénon system, the robot's chaotic path (for $k_1 = 1.5, k_2 = 2.5$) will have the shape presented in Fig. 12, which confirms once again the efficiency of our methodology.

6. Conclusions

This paper addresses the problem of obtaining unpredictable paths for mobile robots accomplishing boundary patrol missions in adversarial conditions. We have demonstrated that new chaotic trajectories can be obtained using the relative motion concept in a simple and efficient manner. We started with the periodic motion on a closed contour of a reference frame in which the Hénon chaotic system evolves. We proved that the compound trajectories obtained in the fixed frame are also chaotic and, furthermore, preserve the chaotic properties of the Hénon system. Based on this result, we developed an original method to create chaotic trajectories in the proximity of any arbitrary boundary shape.

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