ABSTRACT

This paper considers a traffic light system optimization within two nearby intersections to deal with the most minimum traffic queue. Including an effort within this scope is to prevent the likelihood of spillover by creating a certain limit for the lane between both intersections. The traffic light system is made adaptive based on a flow model of each cycle obtained from actual conditions in the city of Surabaya, Indonesia. The flow model is grouped into several time clusters based on the similarity of each flow with the Gaussian distribution assumption. The flow model is then used on the traffic model system in order to predict the queue length when the duration of the traffic light is being manipulated. Simulation is carried out in SUMO by combining the max-plus model and stochastic traffic flow data. An approximate optimum solution of traffic light duration is solved by means of the model predictive control framework.

Keywords: Clustering, max-plus model predictive control, urban transportation, intelligent traffic system, stochastic hybrid system.

Mathematics Subject Classification: 15A80, 37N35, 49N90, 90B20

Computing Classification System: J.2, J.7
1. INTRODUCTION

Traffic nowadays has gone all over the place in almost every country, mainly in big cities. This phenomenon causes many losses for many people, some of which are the over-consuming of fuel, carbon emission rising, and productivity reduction. Several efforts to reduce traffic in big cities have been done by the government, such as the three-in-one traffic system in Jakarta, Indonesia (Hanna et al., 2017), open–close road system (Papageorgiou et al., 2003), and the addition of public transportation such as the bus and MRT (Mochtar and Hino, 2006), (Susilo et al., 2007). However, not a single one is effective for the matter at hand. One of the main causes of traffic is the lack of an effective traffic light system for each intersection in big cities, plus the fact that the number of vehicles is constantly rising (Susilo et al., 2007).

In general, a constant time setting of traffic light for one whole day was implemented. However, in reality, the number of vehicles passing through a certain intersection fluctuates from time to time, so that kind of setting will not bring the expected outcome. Inaccurate traffic light duration would cause significant queue length not only in that specific intersection or lane but also on the other intersection that is connected to the previous intersection. That is why there is a need for intelligent traffic light systems that change over time, corresponding to the changes in vehicle queue (Hernández et al., 2002), (Barba et al., 2012).

To develop an intelligent traffic light system, an accurate dynamic model is needed in order to estimate the queue length of a variety of traffic light duration. In order to do that, a method to estimate queue length needs to be acquired (Anusha et al., 2013), (Sutarto and Joelianto, 2015), (Sutarto et al., 2015a), (Sutarto et al., 2015b), (Adzkiya et al., 2015). In addition, max-plus algebra needs to be learned to help formulate the vehicle queue length (Biegler, 2010). Considering the queue length problem as a max-plus system, it can then be solved by means of a model predictive control (MPC) (Farahani et al., 2010), (Van Den Boom and De Schutter, 2006). The framework of MPC has been implemented in many investigations, such as in the following papers (Preitl et al., 2006), (Tan et al., 2007), (Joelianto et al., 2009), (Joelianto et al., 2011), (García et al., 2011), (Salmah et al., 2013), (Yacoub et al., 2014), (Bojan-Dragos et al., 2015) that consider different approaches with successful applications in various fields.

In this traffic light system, MPC will be decentralized and implemented on two intersections. Interaction between these two controllers will be evaluated further in the lane between both intersections. The system will be implemented in a traffic simulator SUMO (Behrisch et al., 2011), (Bajad and Ali, 2016), (Zaky et al., 2017), (Olaverri-Monreal et al., 2018), (Ramadhan et al., 2019), (Kendziorra and Weber, 2019) to synthesize data to be processed further by the controller. After the system finish being implemented, the same kind of controller with its model is implemented for other days. The result gained from this same controller is expected to be still relevant if it is implemented to another day even with the same model.
2. PROBLEM FORMULATION

2.1. Study area overview

In this paper, two traffic lights at two intersections will be optimized; these are Diponegoro–Soetomo and Diponegoro–Musi in Surabaya, Indonesia, as explained further in Figure 1. Both intersections that are regarded as packed up in Surabaya, are only 200 m apart, which makes the traffic queue in both intersections crucial. In both intersections, PUSJATAN (Indonesian local government that manages traffic) has acquired the actual arrival flow data of vehicles for every 15 minutes for one whole day and one whole month in October 2015, Surabaya. In addition, data of the time cycle for every corresponding traffic light have also been acquired.

![Figure 1. Study Area Overview](image)

2.2. Queue formulation

In a system of one intersection Figure 2a, queue length can be obtained by using this formula

\[ Q(K+1) = Q(K) + (N_{\text{entry}(K)} - N_{\text{exit}(K)}) \]  

with,

\[ N_{\text{entry}(k)} = \lambda T_C \]
\[ N_{\text{exit}(k)} = \mu T_C \]  

then,

\[ Q(K+1) = Q(K) + (\lambda T_C - \mu T_C) \]  

where

\[ Q(K+1) \] = Number of vehicles queueing in one cycle of traffic light (vehicle)
\[ Q(K) \] = Number of vehicles at Initial Condition (vehicle)
\[ N_{\text{entry}(K)} \] = Number of vehicles entering the roadway
\[ N_{\text{exit}(K)} = \text{Number of vehicles exiting the roadway} \]
\[ \lambda = \text{Vehicle inflow (arrival flow) (vehicle/s)} \]
\[ \mu = \text{Vehicle outflow (departure flow) (vehicle/s)} \]
\[ T_C = \text{Cycle Time (s)} \]
\[ T_G = \text{Green time in one cycle (s)} \]

With the assumption, vehicles can only exit the roadway when the traffic light is green and the value of \( \lambda \) and \( \mu \) are distributed uniformly. The equation is described as follows

\[ Q_{(K+1)} = Q_{(K)} + (\lambda T_C - \mu' T_G) \]  

with, \( \mu' = \frac{T_C}{T_G} \), where \( \mu' \) denotes total outflow within \( T_G \) (vehicles/second). In two-phase systems (Van Den Boom and De Schutter, 2008), the green time can be defined as

\[ T_{G1} = T_{G3} \]
\[ T_{G2} = T_{G4} \]
\[ T_{G2} = T_C - T_{G1} \]

then

\[ Q_{1(K+1)} = Q_{1(K)} + \lambda_1 T_C - \mu'_1 T_{G1} \]
\[ Q_{3(K+1)} = Q_{3(K)} + \lambda_3 T_C - \mu'_3 T_{G1} \]
\[ Q_{2(K+1)} = Q_{2(K)} + \lambda_2 T_C - \mu'_2 (T_C - T_{G1}) \]
\[ Q_{4(K+1)} = Q_{4(K)} + \lambda_4 T_C - \mu'_4 (T_C - T_{G1}) \]  

What needs to be noted is when \( T_G \) is being manipulated, the value of \( Q_{(K)} \) could be negative; hence, there should be max-plus algebra (Van den Boom and De Schutter, 2008) for every queue length equation in every arm.

\[ Q_{1(K+1)} = (Q_{1(K)} + \lambda_1 T_C - \mu'_1 T_{G1}) \oplus Z \]
\[ Q_{3(K+1)} = (Q_{3(K)} + \lambda_3 T_C - \mu'_3 T_{G1}) \oplus Z \]
\[ Q_{2(K+1)} = (Q_{2(K)} + \lambda_2 T_C - \mu'_2 (T_C - T_{G1})) \oplus Z \]
\[ Q_{4(K+1)} = (Q_{4(K)} + \lambda_4 T_C - \mu'_4 (T_C - T_{G1})) \oplus Z \]  

where the max-plus-algebraic addition \( \oplus \) is defined as follows \[ A \oplus B \] = \( a_y \oplus b_y = \max(a_y, b_y) \).

In (6), we define \( Z = 0 \). With the assumption that every arm has the same weight, the total queue length equation in one intersection is then defined as:

\[ Q_{\text{length}(K+1)} = Q_{1(K+1)} + Q_{2(K+1)} + Q_{3(K+1)} + Q_{4(K+1)} \]  

\[ (7) \]
Figure 2b shows the queue length that can have a negative value described in (5). By using switching with linear max-plus algebra given in (6), the value of the queue length with max function has no negative value (red line). Hence, it provides positive values for the $Q_{\text{length}(K+1)}$ in (7).

![Figure 2a. Intersection with 4 directions in Urban Traffic](image)

![Figure 2b. Max-Plus Queue Length Estimation Plot](image)

### 2.2. Constraints

By implementing an intelligent traffic light system, the green–red light duration needs to be constrained in order to maintain balance in every arm. The range for every duration allowed for each traffic light is 60 s in the middle value for one cycle. The range is $46 < T_G < 106$ seconds for the Musi intersection and $43 < T_G < 103$ seconds for the Soetomo intersection. Formally, one can be expressed as:

$$T_{G,\text{min}} \leq T_G \leq T_{G,\text{max}}$$

(8)
Since the gap between both intersections is only 200 m, there should be a constraint for the queue length in the lane between both intersections. This value can be defined as 60% of the capacity of vehicles on each lane based on the recommendation of the Indonesian Road Capacity Manual (Manual Kapasitas Jalan Indonesia (MKJI) in Indonesian) (Sweroad, 1997). The queue length constraints are then defined as follow:

\[ Q_{\text{Soetomo}} \leq Q_{cr} \quad \text{and} \quad Q_{\text{Musi}} \leq Q_{cr} \]  

(9)

In two-intersection systems, as shown in Figure 3, in this paper, the values of \( Q_{\text{Musi}(K)} \) and \( Q_{\text{Soetomo}(K)} \) are considered as 60% from the capacity of each road in order to prevent building up of vehicles in the respected road. In \( Q_{\text{Soetomo}(K)} \), the maximum capacity of vehicles from the observed data is 225 vehicles and \( Q_{\text{Musi}(K)} \) is 202 vehicles. Hence, the 60% limitation of the two intersections is 135 and 121 vehicles. The limitations lead to the bounding value for the condition in (9) as \( Q_{\text{Soetomo}(K)} \leq 135 \) vehicles for the lane toward Soetomo and \( Q_{\text{Musi}(K)} \leq 121 \) vehicles for the lane toward Musi.

3. TRAFFIC CONTROL SYSTEM DESIGN

Before going further about MPC (Rawlings and Mayne, 2009), first, we need to understand the schematic of the controller being used, as explained in Figure 4. The MPC will be designed from the
model and then implemented to the model in SUMO, as a representation of the real condition.

![Figure 4. Block Diagram of the Traffic Control System](image)

3.1. MPC configuration

Firstly, it needs to determine the MPC configuration being used (Findeisen et al., 2006), (Mathwork, 2012) such as the predicted and control horizons. In the data of arrival flow being used, which is for every 15 minutes, there are about 6 traffic light cycles. Due to that, an effort to find the optimum result was held by restricting the predicted and control horizon at $\leq 6$.

For the combination of control and predicted horizons, which results in different values, a simulation cannot be held. This is due to the adjustment of $T_G$ (control horizon) value is not inline with $Q$ length prediction (predicted horizon), which potentially creating a too large $Q$ length in a certain period mainly when the simulation has been undergone for half a day. This could happen because the error from the adjustment of $T_G$ constantly accumulated to the upcoming cycle and prediction. Therefore, only a combination of the same control and predicted horizon value could be undergone.

Subsequently, the queue model and several constraints associated to the traffic signal control problem are merged in the MPC formulation to compute the green time signals at the beginning of the $k$-th cycle, with (7) as an objective function:

\[
\text{Min equation (7)}
\]

\[
\text{s.t.}
\]

\[
\text{Queue length dynamic or equation (6)}
\]

\[
\text{with constraints:}
\]

\[
\text{Equations (8) and (9)}
\]

From the resulted gained in the experiment, the value of 3 for the control and predicted horizons has been chosen since it results in the most stable and minimum queue length, as shown in Figure 5 – Figure 8 mainly in the red color graph.
Figure 5. Prediction Horizon:2 - Control Horizon:2

Figure 6. Prediction Horizon:3 - Control Horizon:3

Figure 7. Prediction Horizon:5 - Control Horizon:5

Figure 8. Prediction Horizon:6 - Control Horizon:6
3.2. Optimizer

The MPC formulation above is resolved by means of the sequential quadratic programming algorithm, realized by the MATLAB function \textit{fmincon} (Venkarataman, 2009) so that the objective function could be stated at a minimum value (Coleman and Li, 1994). In this case, we approach max-plus model as a smooth nonlinear function, as suggested in (Daganzo et al., 2011). This approximation provides a sub-optimal condition. In the sequential approach, we discretized only the control trajectories. Arbitrarily, we used a discretization by means of 11 parameters (piecewise linear). Furthermore, we need to embed an ODE solver in an SQP-tool. Given the MATLAB environment, the use of a standard function such as the ODE-solver looks straightforward. However, the SQP-tool needs the value of the objective and the constraints along with the sensitivities of these values with respect to parameter variations.

As described above, the optimizer performs interpolation and discretization of $T_G$. Therefore, the optimizer could determine the optimal combination of $T_G$ to be implemented for three upcoming cycles (based on the value of control horizon). Besides, this step also alters the characteristic of the objective function, which is non smooth-nonlinear (max-plus nonlinear) into smooth-nonlinear (Gopal and Biegler, 1999).

4. MODEL OF TRAFFIC FLOW AND SIMULATION

4.1 Model

In the process of counting in order to get the minimum value of the objective function, the arrival and the departure flow values are needed. Since, in reality, both data are random, stochastic modelling of the data is needed. Clustering analysis has been done for freeway (Weijermars and Berkum, 2005). In this paper, we used cluster analysis for the two intersections (Weijermars and Berkum, 2005). Each was divided into 5 clusters based on time: 00:00–04:30, 04:30–05:45, 05:45–08:45, 08:45–18:45, and 18:45–00:00 for Diponegoro–Soetomo and 00:00–04:30, 04:30–06:00, 06:00–17:00, 17:00–21:30, and 21:30–00:00 for Diponegoro–Musi, as shown in Figure 9 and 10.

\textbf{Figure 9.} Clustering the Musi Intersection into 5 Clusters
After that, each cluster is considered as a Gaussian distribution by calculating the mean and variance of each cluster in each arm of both intersections. This value will be used to find standardized values in order to calculate the cumulative distribution function values by assuming that the vehicle flow data is normally distributed. The normalization test is carried out by using the Kolmogorov–Smirnov (K-S) method (Massey Jr, 1951).

\[ D_n = \max_x | F(x) - S_n(x) | \]

\[ S_n(x) = \frac{1}{n} \sum_{i=1}^{n} I_{[x_i \leq x]} \]

(10)

\( S_n(x) \) denotes the empirical distribution function and \( F(x) \) represents the cumulative distribution function that assumes vehicle flow data normally distributed. If \( D_n \) exceeds the \( 1 - \alpha \) quartile, as given by the K-S Table, then we reject the normally distributed assumption. The \( \alpha \)-value that is used in this paper is 0.05.

In general, the max plus-MPC control system has a positive impact to reduce queue length. In a certain time, constraints and the green light duration could cause a rise in the queue length in some arm. A surge in the queue length could also happen when the flow is under cluster changes period/transition period or when the distribution of flow is high. Despite that, the implementation of adaptive traffic light is successful to reduce traffic and decreases the chance of spillover of vehicles on the arm between both intersections.

4.2 Simulation results

Through the usage of data on Friday, October 23, 2015, as the sample, control system with characteristic explained previously is then tested for the optimization of traffic light for one whole day on both intersections. The result could be seen in Figure 11–22.
Figure 11. Vehicle Queue in the South Direction, Musi Intersection

Figure 12. Vehicle Queue in the East Direction, Musi Intersection
Figure 13. Vehicle Queue in the North Direction, Musi Intersection

Figure 14. Vehicle Queue in the West Direction, Musi Intersection

Figure 15. Vehicle Queue in the Musi Intersection
Figure 16. Applied Adaptive Green Time Period in one day in the North–South Direction, Musi Intersection.

Figure 17. Vehicle Queue in the South Direction, Soetomo Intersection

Figure 18. Vehicle Queue in the East Direction, Soetomo Intersection
Figure 19. Vehicle Queue in the North Direction, Soetomo Intersection

Figure 20. Vehicle Queue in the West Direction, Soetomo Intersection

Figure 21. Vehicle Queue in the Soetomo Intersection
In the result obtained from the implementation of the control system, a decrease of queue length in each intersection mainly happen. However, if we examine every single arm, there is queue increase for almost the whole day in the east arm of the Musi intersection, which is the same with the east and west arm of the Soetomo intersection. This could happen since the north arm of the Musi intersection and the south arm of the Soetomo intersection was given a queue length constraint. Therefore, the green time duration on the north–south arm will be longer than the east–west arm to force spillover does not happen in the south arm of Soetomo or the north arm of Musi. Alteration of the green time at the north–south arm could be seen in Figures 12 and 22. It is clear that the green time duration in the north–south arm is more likely to be above the middle value of one cycle, which also shows the urge from controller to keep the queue length match with the constraint given. This value is always within the range, which was defined previously in the constraints.

There are several surges in some ranges, which are mainly seen at the east arm of Musi at about 16.00–17.00 or in the west arm of Soetomo at about 8.00–10.00. One of the causes is the alteration of cluster, in which generally a differentiation between flows occur between both clusters. Besides that, the standard deviation, which is big enough in a cluster, could cause a rise in the queue length on several other ranges. This standard deviation value represents the variety of flow value in a cluster, which also means the accuracy of modeling flow is lower than the reality.

4.3 Simulation in the other days

In the implementation on Thursday, October 22 and Sunday, October 25, the results are shown in Figure 25–26 and Figure 29–30. It is clear that generally a decrease in the queue length occur, despite for the flow at Thursday and Sunday are different than the one at Friday as can be seen in Figure 23–24 and Figure 27–28. This control system could still be working on Thursday and Sunday since vehicle flow on both days could still be grouped upon on a relatively similar cluster as the one in Friday. On Thursday and Sunday, at about 6 AM was the start of significant flow changes, while on Friday it was about 5:45 AM.

In the examination on every arm, the same result derived on both days. In the north–south arm, there was a decrease in queue length thanks to the constraints, whereas there as a surge that happened several times on the east–west arm.
Figure 23. Arrival and Departure Flows in the Soetomo Intersection, Thursday, October 22

Figure 24. Arrival and Departure Flow in the Musi Intersection, Thursday, October 22

Figure 25. Vehicle Queue in the Musi Intersection, Thursday, October 22
Figure 26. Vehicle Queue in the Soetomo Intersection, Thursday, October 22

Figure 27. Arrival and Departure Flow in the Soetomo Intersection, Sunday, October 25

Figure 28. Arrival and Departure Flow in the Musi Intersection, Sunday, October 25

Figure 29. Vehicle Queue in the Musi Intersection, Sunday, October 25
5. DISCUSSION AND CONCLUSION

The paper introduced a max-plus MPC technique with the flow model obtained by grouping into several time clusters based on the similarity of each flow and by considering a Gaussian distribution. The proposed controller enabled the successful implementation of the feedback control for avoiding queue spillback in a certain intersection. From the perspective of the paper of Daganzo (Daganzo et al., 2011) that a network’s density is never allowed to approach the critical value, the proposed control system is a potential technique to decrease the likelihood of gridlock on the network of the urban traffic.

ACKNOWLEDGEMENT

This research was supported in part by the Ministry of Research, Technology and Higher Education of the Republic of Indonesia under the Decentralized Research Program on Research Development of Science, Technology and Art, National Strategic Research, and Competency Based Research at Institut Teknologi Bandung, Bandung, Indonesia 2016-2019. This research was partially funded by USAID through the SHERA program – Centre for Collaborative (CCR) National Center for Sustainable Transportation Technology (NCSTT) – ITB with Contract No. IIE00000078-ITB-1.

The authors would like to gratefully acknowledge Taufik Sumardi S.T., MSc. from Research and Development Center of Roads and Bridges (PUSJATAN), the Ministry of Public Works of Indonesia for his helpful discussion and assistance of the measurement data. The authors would also like to acknowledge Singgih Wibowo S.T., M.T. from Politeknik Negeri Bandung for helpful discussion of SUMO traffic simulator.

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