

# Comparison of Input Shaper Based on Genetic Algorithms with Analytical Approach

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## ABSTRACT

*High precision mechanical systems require researchers to solve the problem of flexibility. Input shaping is an effective method for vibration reduction of flexible systems. The most published papers attempted to describe the system residual vibration by mathematical expression. By imposing some constraints impulse parameters in the sequence were solved analytically. In addition to analytical method, input shaper can be designed by numerical approach with the use of genetic algorithms. The appropriate input parameters are selected only when the objective functions are optimized. In this paper, the effectiveness of numerical method was investigated and compared with analytical approach by following the presented square.*

**Keywords:** input shaping, time delay filter, genetic algorithm, vibration, flexible systems

**Mathematics Subject Classification:** 70Q05, 70J35

**Computing Classification System:** G.1.6, I.2.9

## 1. INTRODUCTION

Since input shaping technique was presented by Singer, many papers have been published show that input shaping is the effective method for suppressing vibration of flexible systems. The design strategy of input shaping is based on convoluting a desired command with a sequence of impulses (input shaper) to produce the shaped control signal. In the vector diagram representation, the system has no vibration at the natural frequency  $\omega_o$  when the resultant vector of the impulse sequence is also zero. In other words, a sequence of impulses contains exactly the zeros that cancel the system poles. However, input shaping method is the feed-forward method which requires an exact measurement of the system dynamics including the natural frequency  $\omega_o$  and the damping ratio  $\zeta$ . The simplest possibility of input shaper is a sequence with two impulses which the time-delay is one half period (damped period of vibration). It is commonly referred to as a Zero Vibration (ZV) shaper

(Singer and Seering, 1990; Singhose et al., 1990). For increasing insensitivity to modeling errors, multiple zeros are required to be placed exactly at the system poles. This type of input shaper is called Zero Vibration Derivation (ZVD) shaper. By placing zeros in the neighborhood of the flexible poles Extra-Insensitive (EI) shaper can be derived. In comparison to ZV and ZVD input shaper, the width of EI shaper in the insensitivity curve is widest. All three types of input shaper mentioned above were designed in the s-plane by analytical approach. Based on the same ideas, Murphy proposed an arbitrary rate digital shaping filter (ARDSF) in the z-plane that allows the filter to be adapted by changing the impulse magnitude. The method breaks the impulses that fall in between sample periods into two impulses located at the sample times on either side of the original impulse. Obviously, when the rate between the sample period and one half period of vibration is small enough, impulses can be approximately located at the sample times. Rattan presented a generic method for designing shaped input to control the tip position of single-link flexible arms when using a discrete model. The author stated that input shaper developed by Singer and Seering is the special case of this method. Similarly, Tuttle derived input shaper in the discrete domain by the use of zero placement (ZP) technique. Using impulse amplitude plot as a function of the discrete sampling period  $T$  smallest value of  $T$  is selected to ensure all positive amplitudes. For maximizing system performance a shaper with negative amplitudes can be implemented if shaped signal does not violate actuator limits.

While most of the published papers attempted to solve parameters of input shapers by analytical approach, Duong and Hubinsky concluded that they can be solved numerically by the use of genetic algorithms. Since the constraints on shaper parameters can be negative, the optimization process may lead to some possible combinations of impulses containing negative amplitudes, but with no violation of actuator limits (Rattan, 1992; Tuttle, 1994). Accordingly, the shaper length is reduced remarkably. The disadvantage of the method is the requirement of high computation time. In this paper, input shaper based on genetic algorithm will be presented and compared to the traditional analytical approaches. Some suggestive applications of other useful optimization algorithms are presented in (D'Souza et al., 2015; Ghosn et al., 2016; Martin et al., 2009; Precup et al., 2012, 2013). The effectiveness of the method was investigated by means of the presented two-dimensional trajectory following.

## 2. INPUT SHAPING

### 2.1. Mathematical formulation of input shaper

In terms of mathematical representation, input shaper is a series of Dirac impulses whose amplitudes and time locations meet the following equations:

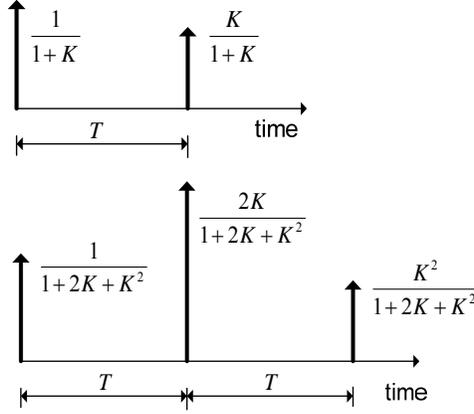
$$\sum_{i=1}^n A_i e^{-j\omega_o T_i} = 0 \quad (1)$$

$$\sum_{i=1}^n A_i = 1 \quad (2)$$

where  $A_i$  and  $T_i$  are the amplitudes and time locations of the impulses respectively,  $\omega_o$  is the undamped natural frequency,  $n$  is the number of impulses in the impulse sequence. Equation (1) describes the resultant residual vibration on the system response. Equation (2) scales the amplitude to sum to unity in order to ensure the shaper has unity gain.

## 2.2. Analytical approach

ZV, ZVD, EI input shapers are the representatives of analytical approach. For a sequence with two impulses satisfying equation (1), Zero Vibration (ZV) shaper was derived in which the time-delay is one half period of vibration  $T_d$ . For increasing the robustness to modeling frequency error, forcing derivative of residual vibration (1) equal to zero results in Zero Derivative Vibration (ZVD) shaper.



**Figure 1:** ZV and ZVD shaper

In Figure 1,  $T$  and  $K$  are given by:

$$T = \frac{\pi}{\omega_o \sqrt{1-\zeta^2}}, \quad K = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \quad (3)$$

where  $\zeta$  is the damping ratio.

By taking z transform, ZV and ZVD shapers are described in the z plane as:

$$IS_{ZV}(z) = \frac{1}{1+K} + \frac{Kz^{-m}}{1+K} \quad (4)$$

$$IS_{ZVD}(z) = \frac{1 + 2Kz^{-m} + K^2 z^{-2m}}{1 + 2K + K^2} \quad (5)$$

where  $m = \frac{T}{T_s}$ . When the sampling time  $T_s$  is equal to  $T$  ( $m = 1$ ), the shaper of (4) is called a first order digital shaping filter and the shaper of (5) is a second order digital shaping filter (Murphy, 1992).

EI shaper is obtained by allowing the percentage vibration to the specified small value  $V$ . Its design is then transformed to the z-plane as:

$$IS_{EI}(z) = A_1 + A_2 z^{-m_2} + A_3 z^{-m_3} \quad (6)$$

where  $m_2 = \frac{T_2}{T_s}, m_3 = \frac{T_3}{T_s}$ ;  $T_2, T_3$  are the time delay of the second and third impulse in an EI shaper;  $A_1, A_2, A_3$  are the first, second and third impulse amplitude magnitude respectively.

Rattan proposed a generic method for designing the controller of shaped input. Assuming that the transfer function describing for flexible systems is given by an under-damped second order system as:

$$G(s) = \frac{\omega_o^2}{s^2 + 2\zeta\omega_o s + \omega_o^2} \quad (7)$$

The discrete transfer function is then expressed in the z-plane as:

$$G(z) = \frac{\omega_o^2(z_1 - z_1^*)}{p_1 - p_1^*} \frac{z}{(z - z_1)(z - z_1^*)} \quad (8)$$

where  $p_1, p_1^* = -\zeta\omega_o \pm j\omega_o\sqrt{1-\zeta^2}$  are the poles of  $G(s)$  and  $z_1 = e^{p_1 T}, z_1^* = e^{p_1^* T}$  are the poles of  $G(z)$ .

The input shapers are generally designed in the discrete time so that to cancel out the poles by its zeros:

$$IS_{zp}(z) = \frac{(z - z_1)(z - z_1^*)}{(1 - z_1)(1 - z_1^*)} z^{-2} \quad (9)$$

Again if we choose a delay time between impulses as given in (3) then  $z_1 = z_1^*$  and the transfer function of the controller is the same as ZVD shaper obtained by Singer and Singhose. Generally, a delay time can be chosen arbitrarily such that the control signal limit is not exceeded.

In the discrete domain, Tuttle used zero-placement technique to derive the discrete input shaper. Based on the same idea of zero-pole cancellation, a shaper zero must be placed at the location of the pole. For increasing robustness additional zeros can be placed at or near the poles. The shaper transfer function containing a complex conjugate zero pair is given by:

$$H(z) = C \frac{(z - z_1)(z - z_1^*)}{z^2} \quad (10)$$

$C$  is the constant and is used to assure the unity gain of the amplitudes. We can see explicitly that the expressions in (9) and (10) are the same. The difference is only that zero placement technique helps to select the optimal time of  $T$  which meets the actuator constraints by plotting the impulse amplitudes as a function of  $T$ . In comparison to the mentioned previous methods, in the discrete domain zero-placement technique has the advantage in terms of design because impulses are placed exactly at multiple of the sampling time  $T_s$ .

### 2.3. Input shaper based on genetic algorithm

As mentioned above, impulses are arranged in the sequence such that their amplitudes and time locations satisfy two constraints (1), (2). In case when the impulse sequence contains more than two impulses, there would be an infinite number of solutions for amplitudes and time locations. To avoid

the trivial solutions an additional constraint must be added for selecting the optimal input shaper and exclude the other possible ones. Apparently, the constraint should be predefined based on the system response to each specific sequence. Genetic algorithm (GA) is a numerical optimization method that helps to seek the minimum of residual vibration. Objective function can be formulated by means of the system responses such as tracking error, contour error or velocity error. The insensitivity range of input shaper to modeling errors can be improved by adding a number of impulses to the sequence. The more comprised impulses in the sequence, the more system is robust. However, the system has to pay at the expense of the longer shaper length. The problem can be stated that solving for impulse amplitudes and impulse time locations such that the objective function is optimized (minimized) subject to the constraint in (2):

$$\text{ObjectiveFunction} = f(\text{chromosome}), \text{chromosome} = [A_1 \dots A_n, T_1 \dots T_n] \quad (11)$$

$$\text{Subject to the constraint: } \sum_{i=1}^n A_i = 1$$

In machining process, two types of position errors are often used as criteria for evaluating the machine tool accuracy: tracking and contouring error. In this paper, GA based input shaper was designed with the use of contouring error. Because of the difficult computation of contouring error exactness, the estimated contouring error was replaced. The objective function was then formulated as sum of estimated contouring errors along the overall toolpath:

$$\text{ObjectiveFunction} = \sum_{i=1}^n \hat{\varepsilon}_i \quad (12)$$

where  $\hat{\varepsilon}_i$  is the estimated contouring error at the  $i$ -th point. The estimated contouring error  $\hat{\varepsilon}_i$  is defined as the distance from the actual point to the tangential line of the reference position.

In this work a combination of extrapolation method with a crossover method was used to produce new variable values to be the crossover points in the offspring (Haupt and Haupt, 2004). Assume that each chromosome has  $n$  impulse amplitudes and  $n$  impulse time locations. An amplitude variable and a time variable in one pair of parents are selected randomly as:

$$\begin{aligned} \alpha_A &= \text{ceil}\{\text{random} * N_i\} \\ \alpha_T &= \text{ceil}\{\text{random} * N_i\} \end{aligned} \quad (13)$$

New variable values for crossover points are formed by a combination of the selected variables:

$$\begin{aligned} A_{new1} &= A_{mA} - \beta_A (A_{mA} - A_{dA}) \\ A_{new2} &= A_{dA} + \beta_A (A_{mA} - A_{dA}) \\ T_{new1} &= T_{mT} - \beta_T (T_{mT} - T_{dT}) \\ T_{new2} &= T_{dT} + \beta_T (T_{mT} - T_{dT}) \end{aligned} \quad (14)$$

where  $A_{mA}$  is the  $\alpha_A$ -th amplitude variable in the mother chromosome

$A_{dA}$  is the  $\alpha_A$ -th amplitude variable in the father chromosome

$T_{mT}$  is the  $\alpha_T$ -th time variable in the mother chromosome

$T_{dT}$  is the  $\alpha_T$ -th time variable in the father chromosome

$\beta_A, \beta_T$  are random numbers on the interval  $[0, 1]$

In the optimization process, for decreasing the complexity of optimization process and without loss of generality the time location of the first impulse is set equal to zero and its amplitude equal to one. At the end of optimization process the dividing of their amplitudes by their sum is needed to ensure the amplitude unity in (2).

### 3. SIMULATION RESULTS

In the simulations, the machining tool travelling along the square with the dimension of  $50 \times 50$  [mm $\times$ mm] was simulated. The axis velocity maximum was set to  $f_{x\max} = f_{y\max} = 141.67$  [mm/s] and acceleration maximum was set to  $a_{x\max} = a_{y\max} = 3000$  [mm/s<sup>2</sup>] in both X and Y directions. The control signal was set within the limit  $\pm 8$ [V]. The sampling time for the machining tool controlling was  $T_s = 5 \cdot 10^{-4}$  [s]. Machining process was supposed to be started from the origin (0, 0) with a departure angle  $0^\circ$  in direction +X and then in direction +Y. In the offline part, the trajectory from the one corner to the next one was generated using bang coast bang acceleration profile. The switching times were rounded as a multiple of the sampling control time. In order to achieve the desired length, the acceleration and velocity maximum were then readjusted. Each axis drive was implemented by the position P controller cascaded with the velocity PI controller. Dynamics of the drive in X and Y direction was assumed to be identical. So there is no requirement on nonlinearity compensation due to the difference between the axis drive dynamics. For improving the tracking accuracy, the feed-forward controller ZPETC was added to cancel the phase shift for all frequencies (Tomizuka, 1987).

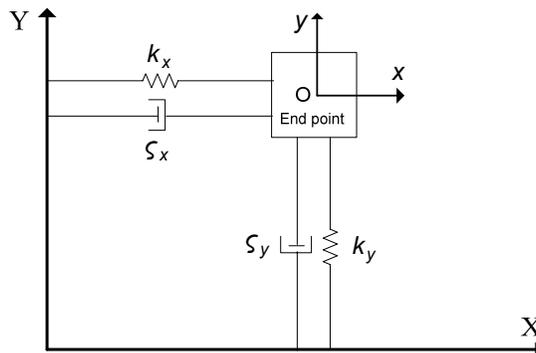
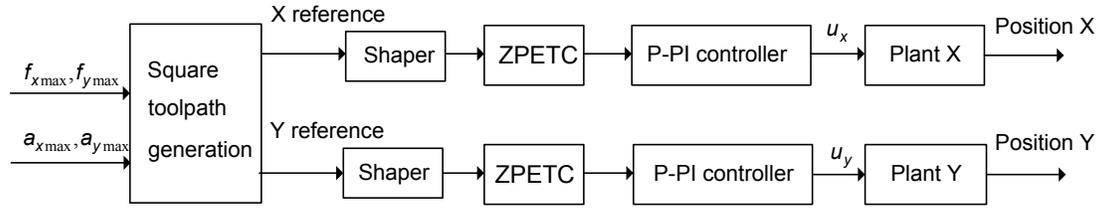


Figure 2: Biaxial model of CNC machine



**Figure 3:** P-PI + ZPECT controller for axis feed drives

As depicted in Figure 3, shapers are used to alter the reference for both X and Y direction before sending them to drive the system. For real mechanical systems ZV shaper is not effective due to the narrow range of insensitivity to frequency error. Therefore, it is rarely applied to drive the system.

Input shaping technique requires the good knowledge of system vibration. Parameters of the machine tool's structural mode were identified using either the high precision of the measuring tool KGM 182 grid encoder or the low-cost MEMS linear acceleration sensor LSM303DLHC. By reading the tracking positions and analyzing them in the frequency domain, both measuring tools showed that the natural frequency of vibration and damping ratio dominantly vibrate at  $f_x = 33[Hz]$ ,  $\zeta_x = 0.075$  for the X axis and  $f_y = 31[Hz]$  for the Y axis. Based on the well-known system parameters, damped period of system vibration is then computed by (3), so  $T_{dx} = 2 \cdot T_x = 0.030[s]$ ,  $T_{dy} = 2 \cdot T_y = 0.032[s]$ . ZVD, EI shapers were designed according to equations (4), (5). Input shaper controller using zero placement technique was also simulated. From plotting of amplitude function with respect to the time,  $T$  was selected equal to  $9 \cdot 10^{-3} [s]$  which ensures that all the impulse amplitudes are positive. This leads to no actuator constraint violation.

The M-file program was written to start the simulation with initial constrained parameters and obtain the system response for evaluating the objective function. Random numbers used for equation (13) and (14) are provided by the uniformly random number generation function in Matlab. Parameters of genetic algorithms based input shaper were optimized after the number of iterations equal to 100. In case of the square, contouring error is simply computed as the distance from the actual point to the edge.

The knowledge of the input shaper length given by analytic method may help us to impose the constraints on the amplitudes and time locations. In addition, this enables to speed up the convergence of the searching process. The M-file program was written to start the simulation with initial constrained parameters and obtain the output data for evaluating the objective function.

GA based input shaping provides the solutions that cover all types of input shapers mentioned by the analytical method. Table 1 indicates that input shaper using zero placement technique help finding the optimal solution which Rattan also mentioned in his work but without any solution. Consequently, optimal time length of input shaper results in energy saving. From mathematical point of view, input shaper based genetic algorithms covers all types of input shaper designed by numerical approach. In the optimization process, obtained results depend either on allowable limit of impulse time location or impulse amplitude except the formulation of objective function. In comparison to ZVD and EI shapers, input shapers using zero placement and genetic algorithm have the shorter shaper length but are

more sensitive to the frequency error. The simulations were verified when the frequency was given by error of 10% of the real value.

Table 1: Impulse amplitude and impulse time delay

ZVD	$A_x$	[0.3115 0.4931 0.1954]	$T_x$	[0 0.015 0.030]
	$A_y$	[0.3031 0.4948 0.2021]	$T_y$	[0 0.016 0.032]
EI	$A_x$	[0.3305 0.4609 0.2087]	$T_x$	[0 0.015 0.030]
	$A_y$	[0.3210 0.4636 0.2154]	$T_y$	[0 0.016 0.032]
ZP	$A_x$	[0.4438 0.2207 0.3355]	$T_x$	[0 0.009 0.018]
	$A_y$	[0.4732 0.1500 0.3768]	$T_y$	[0 0.009 0.018]
GA/Pos. response	$A_x$	[0.46034 0.38886 0.1508]	$T_x$	[0 0.013 0.0220]
	$A_y$	[0.34874 0.3364 0.31486]	$T_y$	[0 0.010 0.0215]
GA/Neg. response	$A_x$	[0.49934 0.23427 0.26639]	$T_x$	[0 0.0115 0.0180]
	$A_y$	[0.39124 0.29069 0.31808]	$T_y$	[0 0.0095 0.0205]

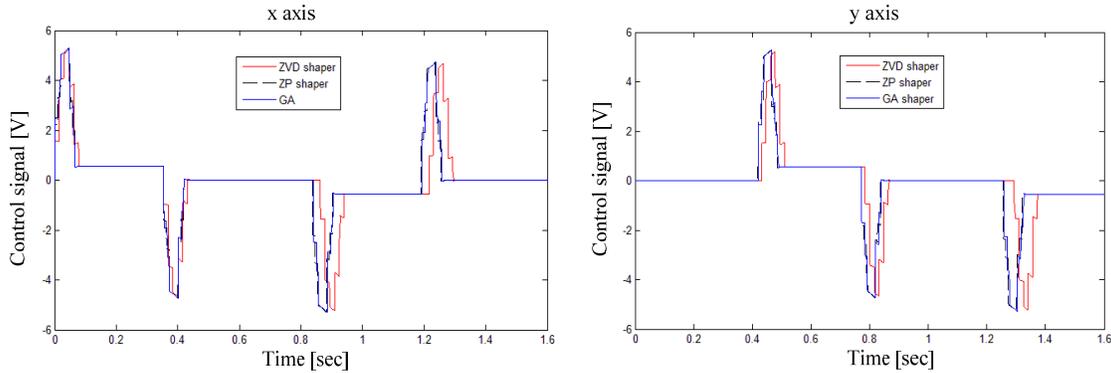
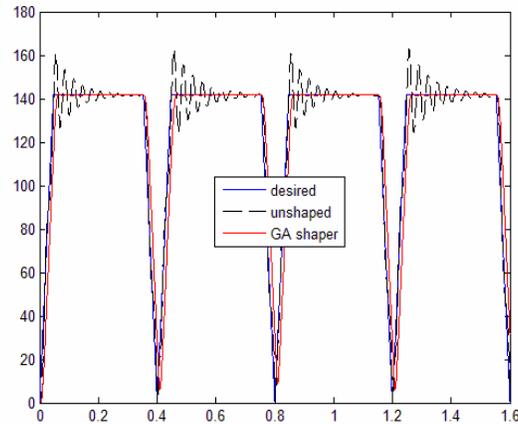


Figure 4: Control signal for axis drive in X and Y direction

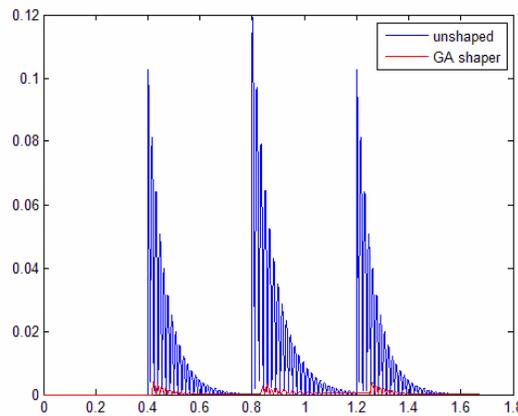
Figure 4 plots the control signals for the drive axis in x and y direction. Simulated results shows that the control signals do not violate the saturation limits when using ZVD shaper, zero placement based shaper and GA based shaper. Control signals have the similar profile, especially for ZVD and GA shaper due to the nearly equal parameters.

The main problem with the use of input shaping is adding a delay to the overall system process. For the square, this would lead to the rounding of the corners. For solving the problem, Singhose proposed an improvement by adding a delay time equal to the shaper length to the process when the system moves from one corner to the next one. This requires zero valued velocity at the corner. Another solution is proposed in this paper by employing genetic algorithms based input shaper. This type of input shaper helps to reduce system vibration amplitudes while attempting to minimize the contouring error. The method does not require any improvement at each corner so the total time of the machining process is reduced in three times of the shaper length. On the second hand, the corners are punished qualitatively at the corner. As the typical problem of input shaping, a delay time

must be added at the end to complete the machining process. As seen in Figure 5 and 6, the maximum peak of the feed amplitude was reduced in percentage from 12.4 down to 0.7 and the maximum contouring error was remarked from 0.1191 [mm] to 0.023 [mm].



**Figure 5:** Feed response to unshaped and GA shaped position commands



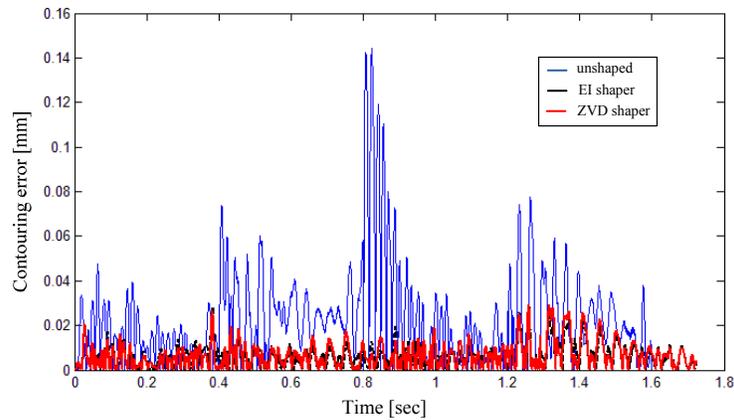
**Figure 6:** Contouring error response to unshaped and GA shaped position commands

#### 4. EXPERIMENTAL RESULTS

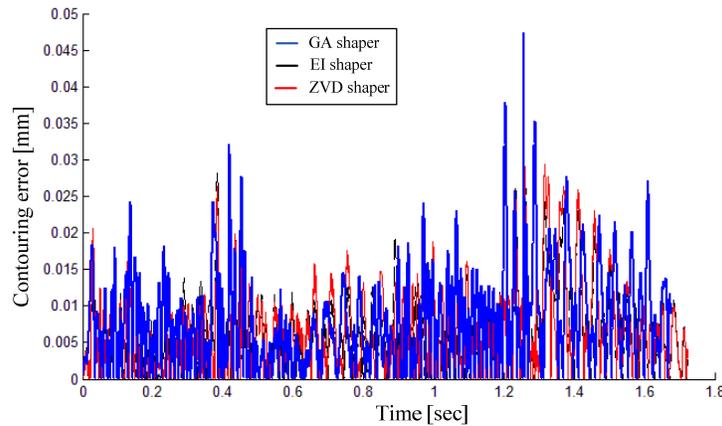
The real time implementation of the square toolpaths was conducted on a high speed biaxial CNC machine using laser MSF1863 as shown in Figure 7. The axial position feedbacks are sampled through two encoders with a resolution of 32500[im/rev] and used for servo position control. Measured positions used for machining evaluation are obtained by the KGM 182 grid encoder with a resolution of 2 [ $\mu m$ ]. The scanning head moving over the grid plate is vertically mounted at the tool tip and the mounting based is fixed on the X-Y table. The shaped positions generated by Matlab/Simulink were used to send as input position commands for the CNC feed drives. Since the position controller of servo system is implemented to receive the command with an accuracy of 1 [ $\mu m$ ], the position commands were rounded up to 1 [ $\mu m$ ].



**Figure 7:** Implementation of biaxial CNC machine



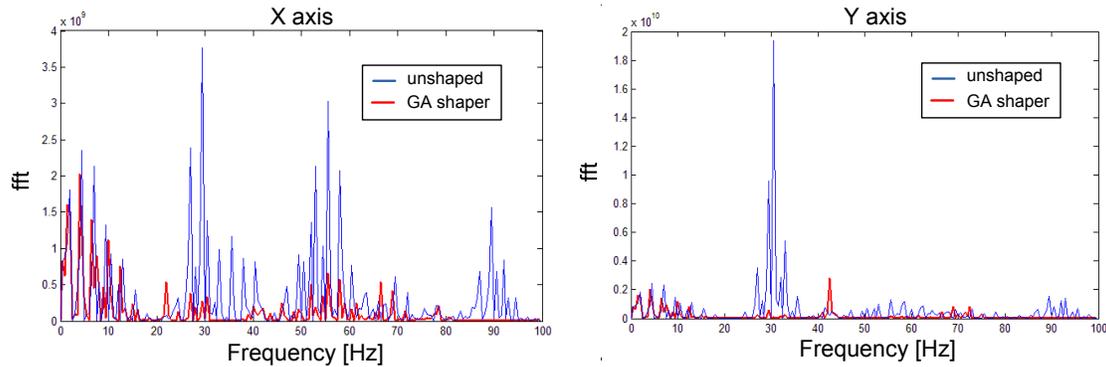
**Figure 8:** Contouring error response of the square to unshaped, EI and ZVD shaped positions



**Figure 9:** Contouring error response of the square toolpath to EI, ZVD and GA shaped positions

Figure 8 and 9 show that input shaping performs as well as the results obtained by simulations. Position commands were altered to eliminate the possibility of the structural mode. The reduction of the frequency content prevents the system to excite during the machining process along the square toolpath (Figure 10). The maximum contouring errors were reduced from 0.144 [mm] of unshaped to 0.02 [mm] of ZVD shaped positions. Different type of input shapers were compared to investigate the

efficiency of input shaping on vibration reduction. Moreover, the S-curve velocity generation was also used for the purpose of transient phase smoothing. The experimental results indicated the similar response of the square to ZVD, EI and ZP shaped positions. The same result was also tested for GA based shaper. At the end of machining process, the endpoint still oscillates but within a circle with the radius of 20 [ $\mu\text{m}$ ]. This may be influenced by the first low mode of vibration. The results can be even improved if we take into consideration the low mode.



**Figure 10:** Power spectral density of unshaped and GA shaped square toolpath

## 5. CONCLUSIONS

The effectiveness of input shapers designed by analytical approach was compared to shapers based on genetic algorithms. Firstly, shapers were tested in the simulations for suppressing the system vibration. Then the experimental results were validated on a high speed biaxial CNC machine using laser MSF1863. Both simulation and experimental results have proven the effectiveness of input shaping on trajectory following. The improved contouring error performance with the use of genetic algorithm based input shaper is also remarked. GA based input shaping provides the solutions that cover all types of input shapers mentioned by the analytical method. Therefore, GA based shapers performs the same results as obtained by the mentioned analytical methods.

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