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Multiobjective Local Search as an Initialization Procedure for Evolutionary Approaches to Polygonal Approximation

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ABSTRACT

Polygonal approximation is a process based on the division of a closed curve into a shorter set of segments. This problem has been traditionally approached as a single-objective optimization issue where the representation error was minimized according to a set of restrictions and parameters ($min - \#$), or the number of segments minimized keeping the error value below a certain value ($min - \epsilon$). When these approaches try to be subsumed into more recent multi-objective ones, which attempt to provide an algorithm to handle these two objectives jointly, a number of issues arise. Current work successfully adapts two of these traditional approaches, bottom-up and top-down algorithms, and introduces them as initialization procedures for a multiobjective evolutionary algorithm for polygonal approximation, being the results, both for initial and final fronts, analyzed according to their statistical significance over a set of traditional curves from the domain.

Keywords: local search, multiobjective, segmentation, polygonal approximation, evolutionary algorithms.

Mathematics Subject Classification (MSC): 68U05, 68U10, 68T20, 68W40.

Computing Classification System (CCS): I.2.1, I.2.8, I.2.m, I.3.5, I.4.2, I.4.6, I.4.9.

1 Introduction

Segmentation problems are based on the division of a given curve in a set of n segments (being each of these segments represented by a linear model, which points to another common naming convention for this process: piecewise linear representation, PLR) minimizing the representation error. This is an issue faced in several different domains, such as time series (Keogh, Chu, Hart and Pazzani, 2004) and polygonal approximation (Sarfraz, 2008). Polygonal approximation techniques are offline segmentation algorithms which are applied over a closed curve. They can be considered a particular instance of time series, where the timestamp is an implicit ordering value. These techniques can be divided into three different categories: sequential approaches, split and merge approaches and heuristic search approaches.

Sequential approaches (Sklansky and Gonzalez, 1980) are constructive methods which build ever longer segments in their solution from a certainly established initial division. Split and merge approaches (Ramer, 1972) are destructive methods which perform an iterative process where segments are divided at each step until certain stopping criteria are met. Heuristic search approaches are based on the development of heuristic methods in order to avoid the exhaustive search of the optimal *dominant points* (the edges of the segments contained in the segmented solution).

Among heuristic search approaches, it is of special interest for current work to highlight the approaches based on different metaheuristics (Talbi, 2009). Heuristic and metaheuristic based algorithms have been successfully applied to loads of practical domains. Some recent examples may be emerging social structures (Ali, Alkhatib and Tashtoush, 2013), particular swarm optimization (Gao, Huang and Li, 2012), adaptative search algorithms (Asawarungsaengkul, Rattanamanee and Wuttiornpun, 2013), or adaptative gravitational search (Precup, David, Petriu, Preitl and Radac, 2012). Among the different metaheuristic approaches available, we will focus on the different solutions based on evolutionary algorithms for our current domain (Pal, Kundu and Nandi, 2002; Yin, 1999; Tsai, 2006). The idea proposed by these works is to use a genetic algorithm (Goldberg, 1989) to codify the curve or time series as a chromosome with n genes, corresponding each of these genes to one of the points in the original data. If the gene value is a "1", it is considered a *dominant point*, and the algorithm tries to find the ideal codification of the chromosome according to a fitness function which evaluates the quality of the given codified segmentation in the chromosome.

Recently, the multiobjective nature of these processes is being explicitly approached from different perspectives such as the approaches presented in (Kolesnikov, Franti and Wu, 2004; Guerrero, Berlanga and Molina, 2012a). This nature had been already stated in different works (Keogh et al., 2004; Liu, Lin and Wang, 2008; Sarfraz, 2008) but not explicitly approach in a joint way. In polygonal approximation two different problems were traditionally stated: $Min - \#$ and $Min - \epsilon$, where $Min - \#$ optimizes the representation error and $Min - \epsilon$ optimizes the number of segments for a maximum error ϵ .

In (Guerrero et al., 2012a) a multi-objective evolutionary algorithm (Coello, Lamont and Van Veldhuizen, 2007) is proposed for the multi-objective solution of the segmentation issue, while in (Guerrero, Berlanga and Molina, 2012c) a comparison between different possible initializations was carried, focusing on the different results between a random initialization aiming at the coverage of the obtained Pareto fronts versus the results from different local search techniques. One of the detailed issues is the single-objective nature of the traditional techniques used, which required different executions with different parameters in order to obtain different individuals from the front, also introducing issues regarding the configuration of these techniques to obtain such different individuals.

Current work presents an extension over the original proposal presented in the conference paper presented in (Guerrero, Berlanga and Molina, 2013b). An initial coverage of traditional local search techniques is introduced, focusing on bottom-up and top-down techniques. That coverage leads to the analysis of the multiobjective nature of the problem, and finally a multi-objective explicit formulation is introduced. This implementation will produce a whole Pareto

front from a single execution without parametrization required from the user. These initializations will be later tested as initial populations for a MOEA approach, in order to determine whether they have successfully created better initial populations than the approach presented in (Guerrero et al., 2012c) and how these initial populations translate into the final results of the algorithm.

The structure of this work is divided into the following sections: section two will present an overview of local search techniques, focusing in top-down and bottom-up approaches. Section three will introduce the analysis of the multiobjective nature of the problem and the proposal for the two analyzed local search techniques. After the new implementation has been presented, section four will present the results when these implementations are used to create the specified initial populations for a MOEA approach to segmentation, analyzing the final results of the chosen algorithm over the initial and final populations. Finally the conclusions which can be extracted from the presented results will be presented in the last section, leading to the future lines of the work.

2 Local search polygonal approximation techniques

Polygonal approximation techniques (Sarfraz, 2008) are based on a dimensionality reduction over an initial closed curve, producing a set of segments as its output. These input data can be formalized according to equation 2.1.

$$t = \{\vec{p}_i\}, \vec{p}_i = (x_i, y_i, i), i = 1, \dots, n \quad (2.1)$$

In fact, polygonal approximation can be considered a specific case of time series segmentation applying Piecewise Linear Representation (PLR) (Keogh et al., 2004), bearing in mind that in this case the timestamp (i in eq 2.1) is the implicit ordering in the series. Three main categories can be stated regarding these techniques: sequential approaches, split and merge approaches and heuristic search approaches.

Sequential approaches attempt to build the output segments from a constructive point of view: at each step, new segments are created following certain specific criteria, providing at each step longer segments (and a fewer number of them in the output of the algorithm). Examples of these criteria might be finding the longest possible segments (Sklansky and Gonzalez, 1980) or a joint approach (which is particularly interesting cause it introduces the multi-objective nature explicitly dealt with in this work) trying to obtain the longest possible segments with the minimum possible error (Ray and Ray, 1992). Bottom-up algorithm is included in this category. Split and merge approaches provide the complementary point of view to the previous techniques, performing a destructive approach where new segments are generated at each iteration by dividing the ones resultant from the previous step. The most extended example of these approaches is the Ramer algorithm (Ramer, 1972), At each iteration, the segment is split at the point that has the farthest distance from the corresponding segment unless the approximation error is no more than the pre-specified error tolerance. This algorithm is also known as Top-down.

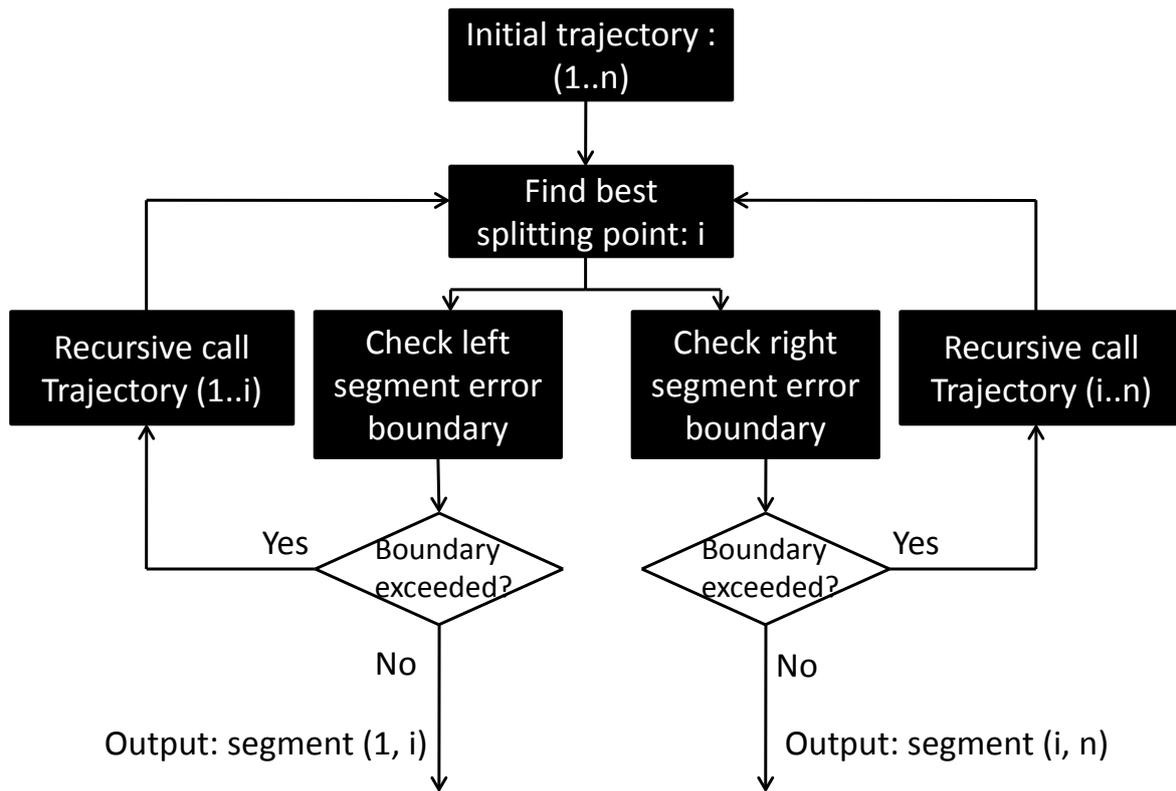


Figure 1: Top Down algorithm overview

Heuristic search approaches try to avoid the exhaustive search of the final segments, providing a wide variety of methods, such as dynamic programming, (Dunham, 1986; Sato, 1992), or metaheuristics, highlighting the use of evolutionary computation in single (Yin, 1999) and multiple objectives (Guerrero, Berlanga, García and Molina, 2010).

Sections 2.1 and 2.2 will detail the top-down and bottom-up algorithms, since these algorithms will be the ones used for their proposal as multi-objective local search in section 3. Finally, these proposal will be tested as initialization process following the metaheuristic evolutionary algorithm proposal in (Guerrero et al., 2012a).

2.1 Top down algorithm

The Top-Down algorithm is an offline approach which considers, recursively, every possible partitioning of a time series, splitting it at the best possible location. Beginning with the whole time series, it finds the best splitting point (where the sum of the errors of both resulting segments has its lowest value) and continues the application of this process to both segments, until both of them reach an error value below the user-defined boundary.

The Top Down algorithm is applied in a wide variety of domains and fields, being also known by different names (Douglas and Peucker, 1973; Ramer, 1972; Duda and Hart, 1973; Li, Yu and Castelli, 1998). There are numerous improvements to the basic top down algorithm. Alternative approaches (Park, Lee and Chu, 1999) perform different initializations based on valleys and

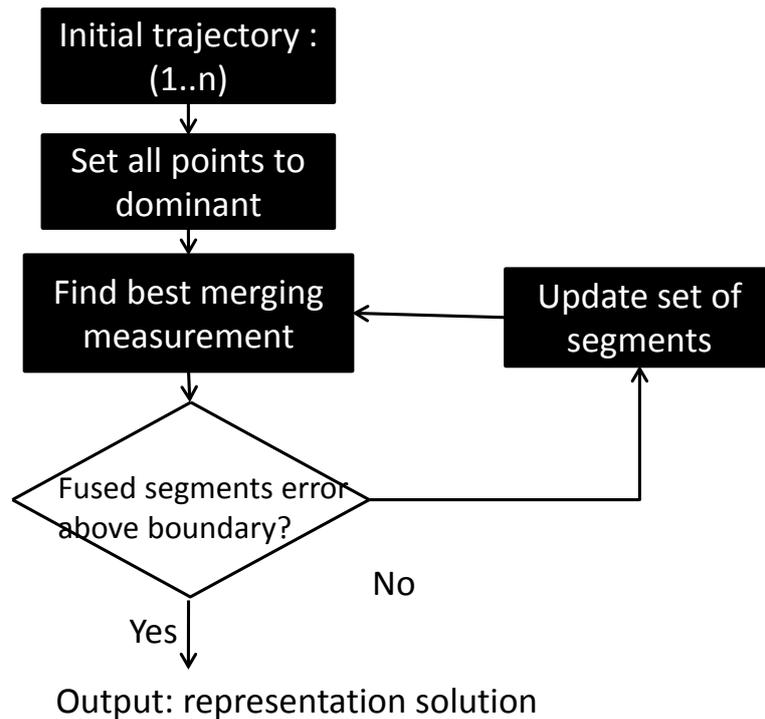


Figure 2: Bottom Up algorithm overview

peaks, which is reported to perform poorly on noisy datasets. The need for alternative stopping criteria was also faced in (Lavrenko, Schmill, Lawrie, Ogilvie, Jensen and Allan, 2000), where they introduced a t-test for that purpose.

2.2 Bottom-up algorithm

The complement to the segmentation analysis proposed in the Top Down algorithm is the synthesis in the Bottom Up proposal. This algorithm creates the finest possible approximation of the figure, dividing it into $n - 1$ segments (where n is the number of points in the time series) of length value 2. Afterwards, the cost of merging each pair of adjacent segments is calculated and, if the merge with the lowest cost has an error below the user defined value, the segments are merged. The process continues until no pair of adjacent segments can be merged with an acceptable error value. It is important to notice that in every step of the algorithm the costs of the adjacent segments to the merged one in the previous step must be updated.

Depending on whether each point belongs to one or two segments (providing a continuous or discontinuous final segmentation) two different versions of this algorithm can be presented. A detailed continuous version can be looked up in (Guerrero, García and Molina, 2011), while the discontinuous version is explained in (Keogh et al., 2004). Current approach will follow the continuous version of this algorithm. This continuous version has been chosen due to the fact that, as it was pointed out in (Keogh et al., 2004), the discontinuous version of the bottom up algorithm imposes restrictions over the size of the segments (having no shared measurement, the resultant merged segments will always have an even number of measurements). This

restriction is not present in the continuous version.

The bottom up algorithm, as well, has spread to different fields and research areas using different names, such as the computer graphics domain and decimation methods (Heckbert and Garland, 1997).

3 A multiobjective perspective to local search polygonal approximation techniques

The traditional criteria in the time series domain to determine the quality of a segmentation process (Keogh et al., 2004; Liu et al., 2008) are the following:

1. Minimizing the overall representation error (*total_error*)
2. Minimizing the number of segments such that the representation error is less than a certain value (*max_segment_error*)
3. Minimizing the number of segments so that the total representation error does not exceed *total_error*

where *total_error* and *max_segment_error* are user defined parameters for the algorithm.

In the polygonal approximation domain, two different issues can be stated (Sarfraz, 2008): *Min - #* and *Min - ϵ* . *Min - #* is based on the optimization of the representation error for a previously set number of segments. *Min - ϵ* , on the other hand, tries to find the minimum number of segments such that the final representation error does not exceed a previously established error ϵ . In (Guerrero et al., 2012a) it was stated that, according to these two different perspectives, the segmentation issue is in fact a multi-objective problem, and also analyzed, according to different techniques available in the literature, how this nature had been faced. It was also shown that, given that some key dominant points are shared by different solutions with different resolutions, the solutions for *Min - #* and *Min - ϵ* problems can be closely related and share information among them. This multi-objective nature is faced with a Multi-objective evolutionary algorithm.

Local search algorithms may be introduced to enhance this approach, leading to several issues: the configuration to obtain the different individuals is hard to establish, and each of these individuals requires an independent execution of the local search algorithm, providing disappointing results (Guerrero et al., 2012c). This section will present alternative, multi-objective parameter-free versions of the two previously covered local search algorithms for polygonal approximation, in order to provide a whole Pareto front of solutions: Top-Down and Bottom-up algorithms.

The multi-objective version of the Top Down algorithm suppresses the two issues available in the traditional implementation: the recursive calls (which may prevent the application of the algorithm to figures with a large number of points) and the user configuration (which introduces the issues previously described in the obtaining of a whole Pareto front). At each step, the best splitting point is located (the one which provides with the smallest representation error), a new individual is generated adding that new dominant point and the costs of the possible

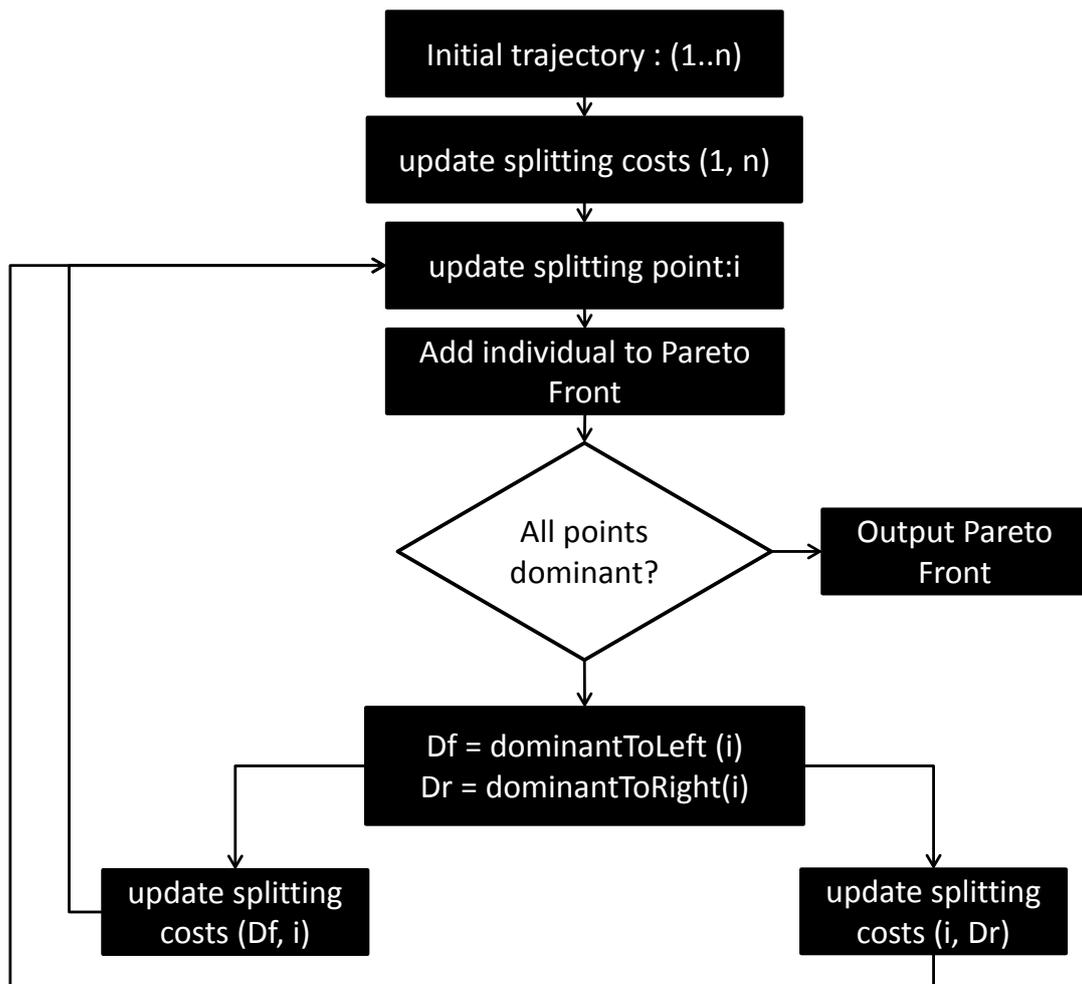


Figure 3: Top Down algorithm multi-objective implementation

segments are updated (implying the recomputation of the costs of the segments from the dominant point immediately to the left of the new splitting point and those from the splitting point to the dominant one immediately to its right). Therefore, no recursive calls are included, and each split point choice has a global view of the representation error (as opposed to the partial one available in the traditional implementation). Figure 3 represents the multi-objective version implementation of this algorithm.

The multi-objective version of bottom-up algorithm removes the user-defined boundaries for the algorithm termination, being this ending triggered once no further merging can be performed. Figure 4 presents the multi-objective version. It must be noted that each update here triggers only one segment update, while every new splitting point in the top down algorithm triggered the recomputation of all the possible new splitting points for the two new segments created in the representation. Since each of these steps are, in fact, mutations over the chromosome guided by a specific heuristic, the principles for an efficient implementation established in (Guerrero, Berlanga and Molina, 2012b) can be applied for the computation of the fitness values of each of the produced individuals.

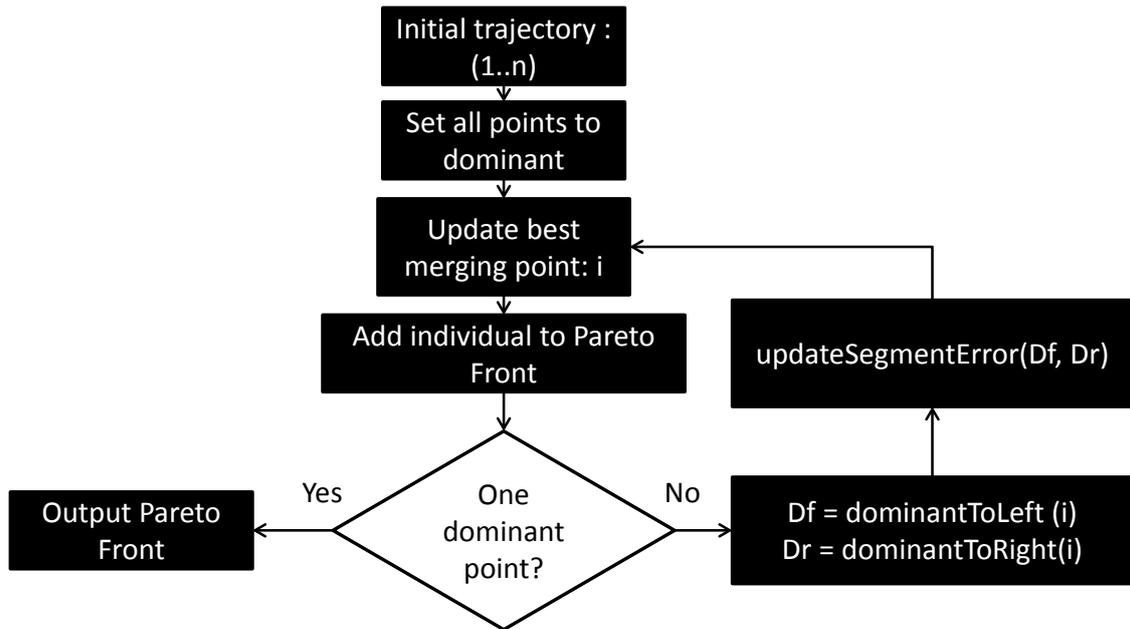


Figure 4: Bottom up algorithm multi-objective implementation

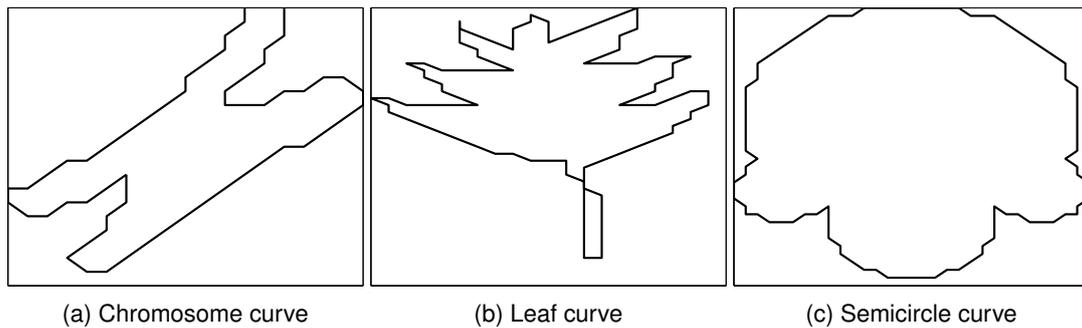


Figure 5: Curves included in the data set

4 Experimental validation: initialization for MOEA polygonal approximation

The experimental validation proposed will include the two detailed multi-objective local search procedures to create the initial populations for a multi-objective evolutionary approach to polygonal approximation. This algorithm is based on the SPEA2 (Zitzler, Laumanns and Thiele, 2001) MOEA, according to the configuration presented in (Guerrero et al., 2012a). The default initialization process creates a uniform Pareto Front in terms of coverage of the objectives, as presented in (Guerrero et al., 2012c). This section will cover the comparison between the initial and final populations of the two techniques presented and the suggested initialization process. The dataset used is composed of three traditional curves, usually named chromosome, leaf and semicircle. Their definition, according to their freeman chain-code representation (Freeman, 1961), can be found in (Guerrero et al., 2012a). Figure 5 represents these figures.

It is interesting to notice, as explained in section 2, the complementary nature of the two multi-

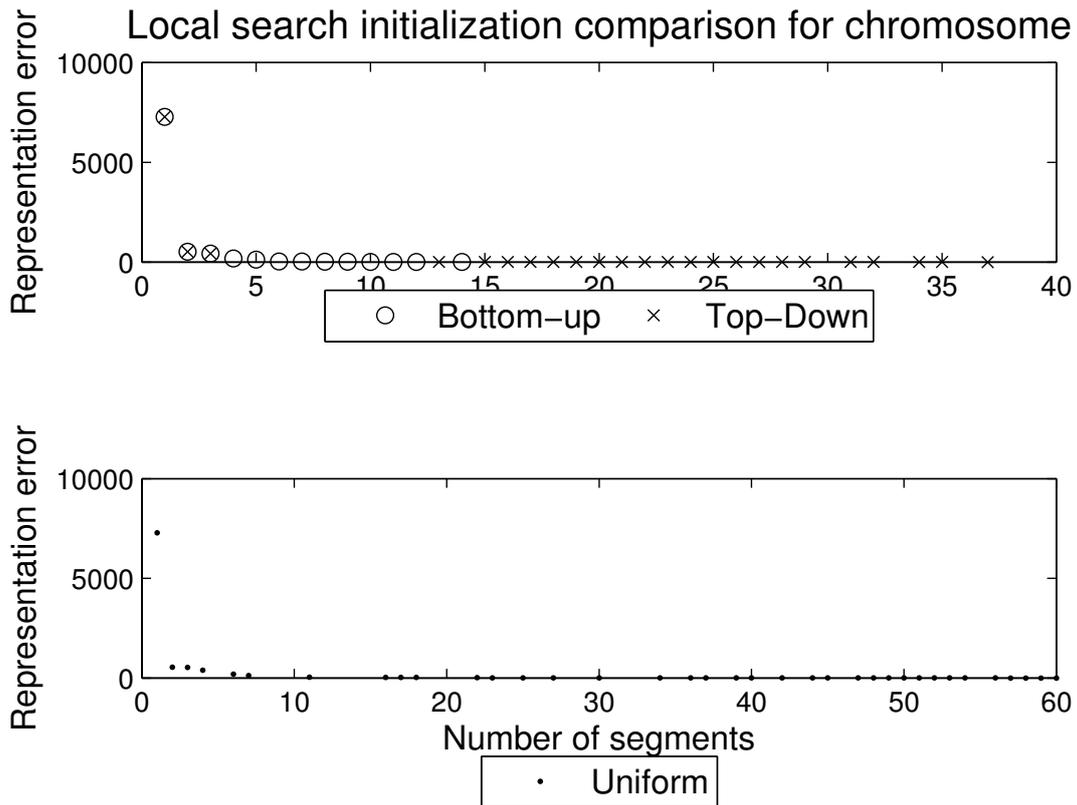


Figure 6: Chromosome initialization comparison

objective techniques presented, since one applies its heuristic with a value of 1 dominant point and applies successive splitting over the figure (Top-Down) and the other begins with a solution with all of its points considered dominant and applies successive merging (Bottom-up). Since the solutions tend to degrade with the successive application of the heuristic, each of them will be more successful at their initial individuals.

Three different comparisons of the two multi-objective local search techniques and the original uniform approach for the three different curves in the dataset are presented in figures 6-8. The only individuals included are those non-dominated (the Pareto fronts for the three techniques). Regarding the previously stated complementary nature of the local search processes, it can be clearly observed in these figures.

The results for the four techniques, including their mean and median values for the hypervolume of the obtained Pareto fronts are included in tables 1 (initial fronts values) and 2 (final fronts values). Also, a best technique column has been added. This value is calculated according to a Wilcoxon test with a 95% confidence performed over 30 different executions, since the values do not follow a normal distribution (according to a Shapiro-Wilk test). If one technique is superior to the remaining ones, its name is included, otherwise the '-' value is included.

Regarding the initial populations, the local search techniques are able to find the individual with zero error with a much lower number of segments that the uniform approach. This is especially important since finding solutions with a higher number of segments does not provide information to the final solution, and can be considered a waste of computational cost. Also, this information could be used to manage the size of the archive, allowing a reduction of the

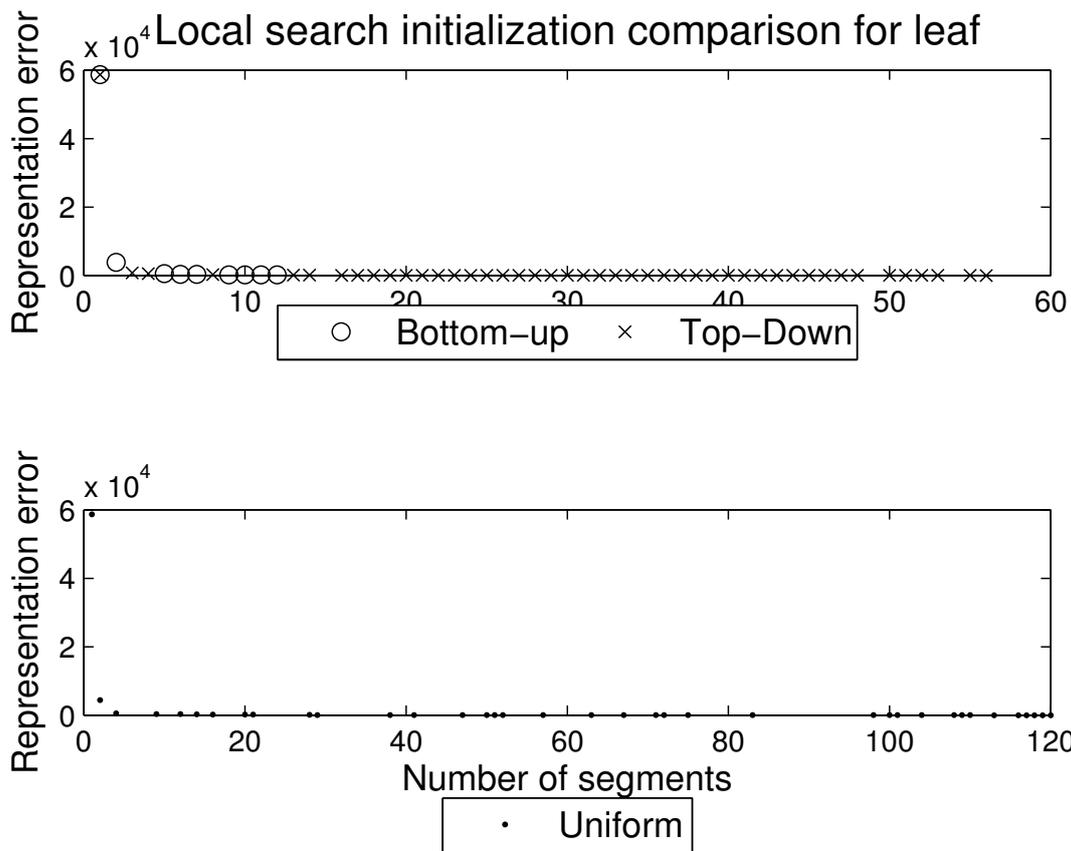


Figure 7: Leaf initialization comparison

Table 1: Initial populations comparison

Figure	Bottom-up		Top-down		Local search		Uniform		Best
	Mean	Median	Mean	Median	Mean	Median	Mean	Median	
Chrom.	0,98647	0,98647	0,98646	0,98646	0,98651	0,98651	0,98436	0,98427	L.S.
Leaf	0,99355	0,99355	0,99322	0,99322	0,99365	0,99365	0,99271	0,99281	L.S.
Semi.	0,99157	0,99157	0,99183	0,99183	0,99218	0,99218	0,99101	0,99111	L.S.

Table 2: Final populations comparison

Figure	Bottom-up		Top-down		Local search		Uniform		Best
	Mean	Median	Mean	Median	Mean	Median	Mean	Median	
Chrom.	0.98665	0.98664	0.98667	0.98671	0.98665	0.98667	0.98671	0.98672	Unif.
Leaf	0.99376	0.99376	0.99374	0.99376	0.99376	0.99376	0.99377	0.99378	-
Semi.	0.99206	0.99219	0.99213	0.99217	0.99219	0.99219	0.99213	0.99217	L.S.

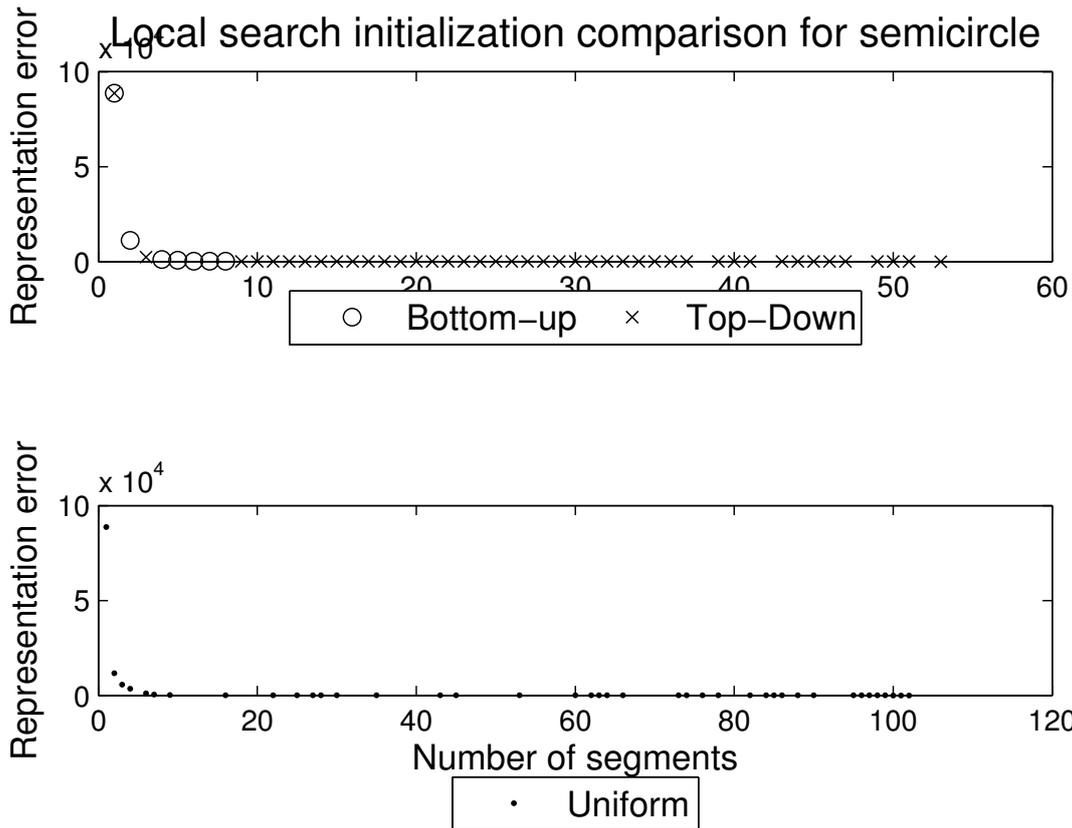


Figure 8: Semicircle initialization comparison

computational cost (a detailed analysis of the impact of the archiving technique can be looked up in (Guerrero, Berlanga and Molina, 2013a)). The representation errors for the individuals for the different number of segments are also clearly better than those obtained by the uniform initialization, which is reflected in the results in table 1.

In the analysis of the final populations results, different cases appear. For easy problems, such as chromosome (fig 5a), the uniform initialization provides better final results, while as the problem difficulty is increased, the statistical difference first disappears in leaf curve (fig 5b) and finally the local search initialization provides better results in the hardest problem, the semicircle (fig 5c).

The analysis of these results can be obtained from the previous remark on initial populations: the repeated application of a heuristic approach provides an ever growing error (as seen in the comparisons of the local search approaches in figures 6-8). Translated to the evolutionary approach, the local search initialization introduces a certain bias to the further search, according to its underlying heuristic. Even though the initial results are clearly improved, the final ones are too guided by this heuristic, and thus, they fall into local minima solutions. To highlight this analysis and provide a further understanding to the presented techniques, figures 9-11 provide a comparison of the evolution of the hypervolume value through the different generations of the algorithm.

The presented results seem to point to a combination of both techniques to provide initial populations that, while benefiting from the enhanced initial populations of local search techniques,

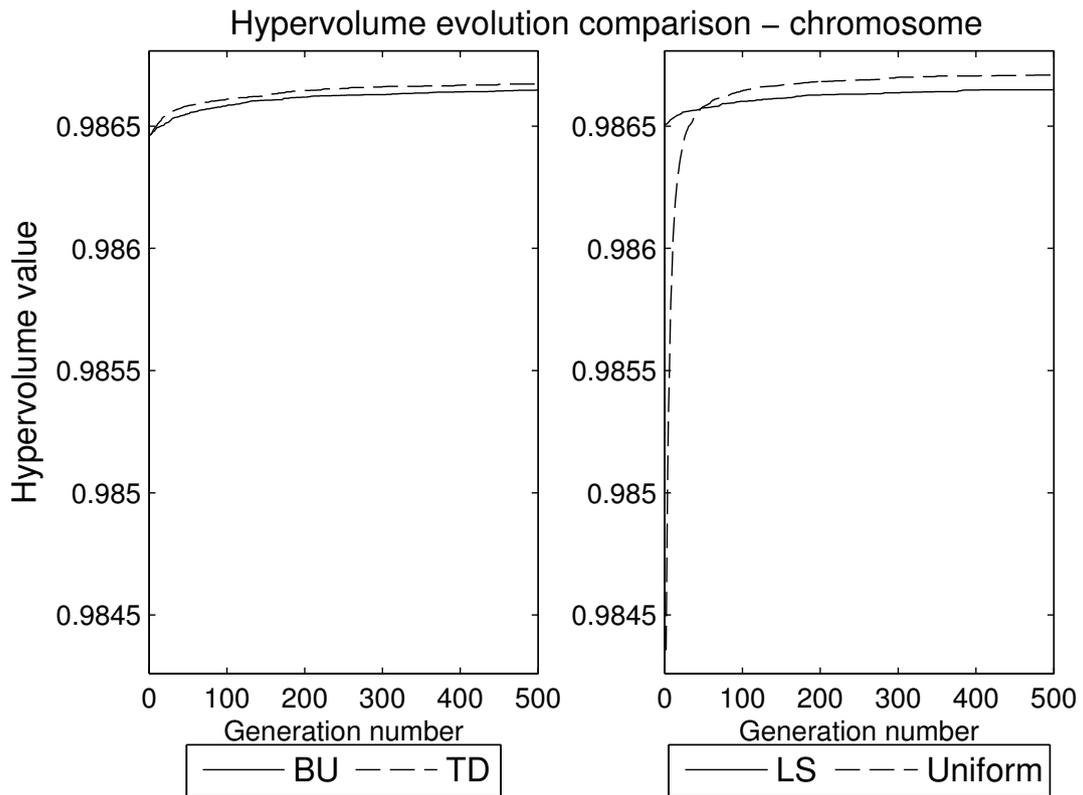


Figure 9: Hypervolume evolution comparison

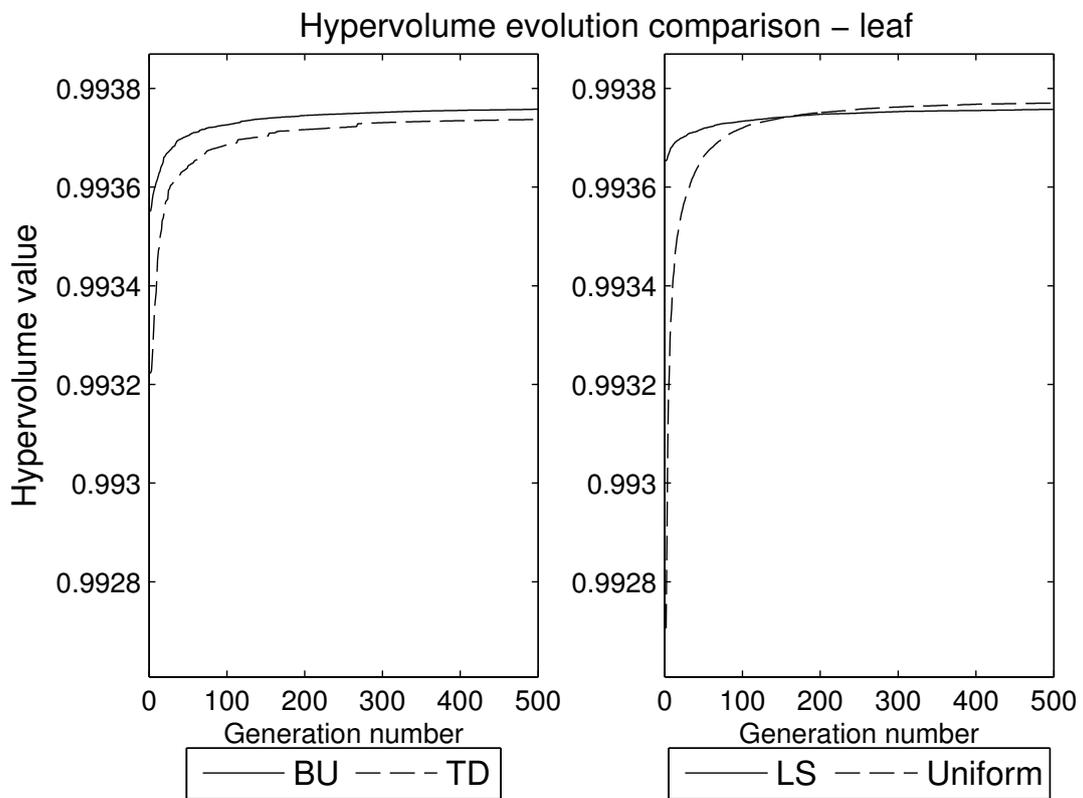


Figure 10: Leaf evolution comparison

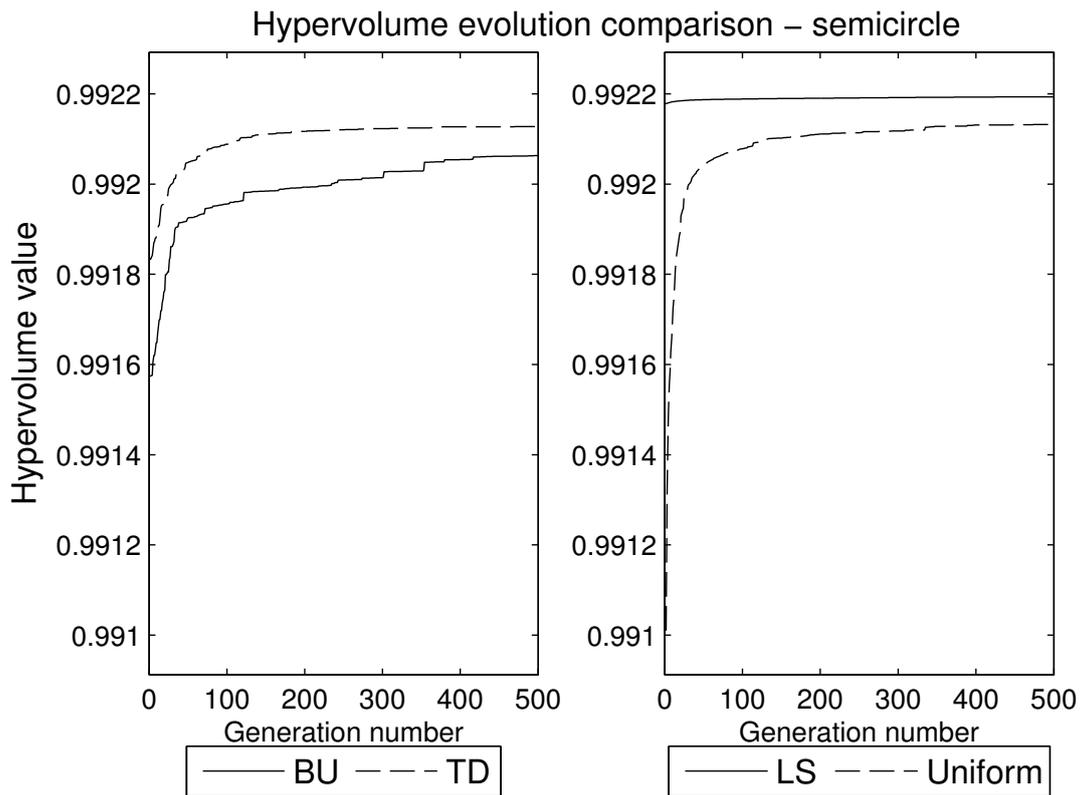


Figure 11: Semicircle evolution comparison

can be not hampered by the heuristic focus. Also, an initial run of constructive techniques such as bottom-up can be used for the configuration of some algorithm parameters like archive size.

5 Conclusions

Local search techniques have been the focus of polygonal approximation, developing different techniques based on specific heuristics for this issue. The nature of this problem is multi-objective, minimizing the representation error and the number of segments of this representation jointly. To provide a proper multi-objective approach for this topic based on available local search techniques, a number of modifications have to be performed over these techniques in order to efficiently obtain the required Pareto Fronts. For parametric techniques, these individuals could be obtained with different runs of the algorithm with different parameters, but this is a computational costly process, requiring also a difficult configuration to obtain a well-spread Pareto front.

This work has modified two representative constructive and destructive local search techniques, namely Bottom-up and Top-down, to provide a multi-objective approach with the required characteristics presented. Once this definition has been presented, these techniques are embedded as the initialization procedure of a MOEA algorithm to solve the segmentation issue (the final technique would benefit from the fast heuristic approach and the thorough metaheuristic search). These results show that the multi-objective techniques are successful in providing statistically better initial populations, however the final results may be too focused

on the heuristic used in these techniques, which makes the evolutionary search performed afterwards less effective, making the results fall into local minima.

Future lines imply the research of the combination which may be performed over local-search and uniform initialization in order to provide initial populations taking advantage of local search improved initial populations without their excessive focus on their underlying heuristic. Also, the design of these multiobjective local search techniques allow the introduction of a fully multiobjective memetic algorithm for segmentation.

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