This article can be cited as R.-E. Precup, S. Preitl, M. Balas, V. Balas, Fuzzy controllers for tire slip control in anti-lock braking systems, Proceedings of IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2004), Budapest, Hungary, doi: 10.1109/FUZZY.2004.1375359, 2004.

Fuzzy Controllers for Tire Slip Control in Anti-lock Braking Systems

Radu-Emil Precup, Stefan Preitl
Department of Automation and Industrial Informatics
"Politehnica" University of Timisoara
Bd. V. Parvan 2, RO-300223 Timisoara, Romania
E-mail: rprecup@aut.ut.ro, spreitl@aut.utt.ro

Marius Bălaş, Valentina Bălaş
Faculty of Engineering
"Aurel Vlaicu" University of Arad
Bd. Revolutiei 81, RO-310025 Arad, Romania
E-mail: balas@inext.ro

Abstract– **A Takagi-Sugeno fuzzy controller and an interpolative fuzzy controller for tire slip control in Anti-lock braking Systems are proposed. Both fuzzy controllers are developed using a benchmark consisting of a simplified nonlinear model describing the slip dynamics for a wheel, and perform the merge between 64 local PI or PID controllers. By employing local linearized models of the controlled plant, the local controllers are developed in the frequency domain. Development methods for the two fuzzy controllers are also offered. Simulation results show the control system performance enhancement ensured by the fuzzy controllers in comparison with the conventional PI ones.**

I. INTRODUCTION

The Anti-lock Braking System (ABS) represents a significant subsystem of the complex steering system for the modern cars. The main task of the ABS control system (CS) is the prevention of wheel-lock during braking to ensure high friction and maintain the steerability of the car.

In the conditions of using several actuator types, the existing ABS control algorithms can be divided in two categories, wheel acceleration control and tire slip control. The first category performs the tire slip control indirectly by either controlling the wheel deceleration / acceleration [1] or controlling the braking torque commanded by the driver [2]. The current approaches belonging to the second category – that deals with tire slip control – include reaching the maximum friction point by measuring the angular velocity of the wheel and the brake pressure [3], the use of feedback linearization [4], the development of model based hybrid controllers [5], of constrained LQ controllers [5], of robust PID controllers [6], [7], of fuzzy logic controllers [8], and the use the of gain scheduling [7].

Since the development of the controllers based on these approaches is rather complex due to the complexity of the nonlinear model of the controlled plant (CP), it is necessary to simplify the controller development aiming a relatively simple further implementation. This comes to the paper goal, to offer simple fuzzy controllers dedicated to tire slip control.

Because of the linear dependence of each rule on the input variables of the fuzzy controller (FC), the Takagi-Sugeno fuzzy controllers (TS-FCs) [9] are ideal for acting as interpolating supervisors of multiple linear controllers applied to different operating conditions of the nonlinear dynamic CPs. The TS-FCs are extremely well suited to play the role of a bumpless interpolators between the linear controllers applied across the input space; therefore, these controllers are natural and efficient gain schedulers [10].

These are the reasons why there is developed firstly a TS-FC. The TS-FC performs the merge of 64 local controllers of PI or PID type. For the development of the local controllers there is employed a benchmark which involves a simplified nonlinear model that describes the slip dynamics for a wheel [11]. In the first step, by linearization of the friction curves around important steady-state operating points there are obtained 64 first order lag plus time delay models of the CP representing local linear models. Then, the local controllers meant for controlling these local linear models are developed in terms of a frequency domain approach.

Secondly, by starting with the TS-FC there is developed an interpolative fuzzy controller (IFC), which can be considered as a quite simple way to implement the TS-FC.

This paper is organized as follows. In the following Section there is analyzed the nonlinear dynamic model used in the tire slip control as part of an ABS, and there are derived the local linear models of the CP, of first order lag plus time delay type. Section III is dedicated to the presentation of the development method for the TS-FC. In addition, the development method of an IFC to approximate the TS-FC is also proposed. Section IV provides simulation results that support the theoretic developments. Finally, Section V contains the conclusions.

II. MODELS OF WHEEL DYNAMICS

A crucial physical parameter that influences the braking process is the adherence of the tire-road system, characterized by the tire-road friction coefficient μ. This coefficient depends mainly on the state of the road, (dry, wet, etc.) but also on the longitudinal slip of the tire against the road referred to as the wheel slip, λ.

When the braking force exceeds the adherence offered by the wheel-road contact, the wheel of the car begins to slide. The tire slip, λ, is defined in terms of (1):

$$\lambda = (v - v_{wheel}) / v = (v - \omega \cdot r) / v \,, \tag{1}$$

where v is the velocity of the vehicle (car), v_{wheel} the tangential velocity of the wheel, ω the angular velocity of the wheel and r the radius of the wheel.

The locking of the wheel (λ=1) must be strictly avoided because it produces notable weariness of the tires, and it causes a very dangerous extension of the braking distance. Still in the '80s it has been discovered that the best adherence occurs when the slip had non-zero values (Fig. 1). The so-called optimal slip is generally between 0.1 and 0.2. This is the reason why modern ABS controllers try to keep λ in this domain, but this is not easy to be fulfilled since λ has significant and fast random variations, and nonlinear behavior.

A simplified model of the braking wheel as subject to the brake torque and the ground contact reaction force was proposed in [11] and will be used further on:

$$\frac{\mathrm{d}\omega(t)}{\mathrm{d}t} = \alpha \cdot \mu(\lambda(t)) - \beta \cdot T_b(t-\tau) \cdot \mu_b(\omega(t)),\ T_b \geq 0, \tag{2}$$

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = -\gamma \cdot \mu(\lambda(t)), \tag{3}$$

where T_b is the brake torque, μ_b the friction coefficient in the brakes, τ the time delay, and α, β and γ are positive constants resulting from the physical parameters of the car.

There will be used in this paper the following particular values of the CP parameters: r=0.3 m, α=1500, β=1and γ=10 [11]. For simplicity it is assumed that μ_b=min(ω/ε,1), for small ε >0, and the signals ω, v and μ_H (the maximum road-tire friction coefficient) can be measured / estimated.

For the development of the TS-FC the nonlinear model (1) … (3) will be linearized around a steady-state operating point A_0 having the coordinates (v_0, ω_0, λ_0, μ_0), where the lower index 0 applied to a certain variable highlights the steady-state value of that variable. Only the nonlinear components will be linearized here excepting the component $\mu_b(\omega(t))$ which will be approximated to 1 in the development phase:

$$\mu_b(\omega(t)) = 1. \tag{4}$$

The equation (1) can be expressed in its equivalent form:

$$v(t) = \lambda(t)v(t) + r \cdot \omega(t). \tag{5}$$

By linearizing the product in (5) around A_0 and differentiating the result (with respect to time), the linearized form of (3) can be expressed as:

$$(1-\lambda_0)\dot{v}(t) = v_0\dot{\lambda}(t) + r \cdot \dot{\omega}(t). \tag{6}$$

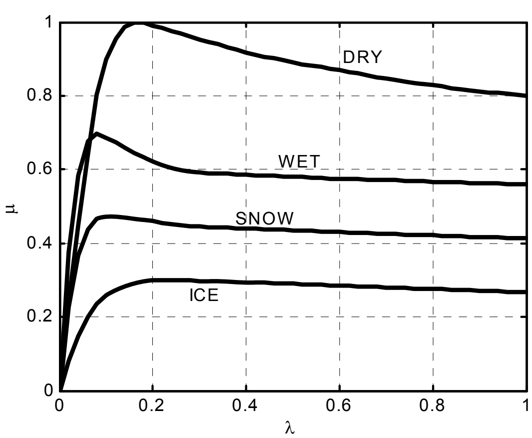

Fig. 1. μ versus λ, extracted from the data presented in [11]

On the other hand, the nonlinear component $\mu(\lambda(t))$ can be linearized around A_0:

$$\mu(\lambda(t)) = m_0(\lambda(t) - \lambda_0) + \mu_0, \tag{7}$$

where m_0 represents the slope of the tire friction curve at the considered operating point A_0:

$$m_0 = (\mathrm{d}\mu / \mathrm{d}\lambda)\big|_{A_0}. \tag{8}$$

By the substitution of the expression (7) of $\mu(\lambda(t))$ in (2) and (3) (in the condition (4)), the modified versions of the equations (2) and (3) can be expressed in terms of (9) and (10), respectively:

$$\dot{\omega}(t) = \alpha m_0 \lambda(t) - \beta \cdot u(t-\tau) + \alpha(\mu_0 - \lambda_0 m_0), \tag{9}$$

$$\dot{v}(t) = -\gamma m_0 \lambda(t) - \gamma(\mu_0 - \lambda_0 m_0). \tag{10}$$

Then, (9) and (10) are substituted in (6) and the linearized model of the wheel dynamics can be expressed as (11):

$$T_1\dot{\lambda}(t) + \lambda(t) = k_1 u(t-\tau) + d(t), \tag{11}$$

with the parameters k_1 and T_1 expressed in (12) and (13):

$$k_1 = r \cdot \beta / m_0 / [\gamma(1-\lambda_0) + \alpha \cdot r], \tag{12}$$

$$T_1 = v_0 / m_0 / [\gamma(1-\lambda_0) + \alpha \cdot r], \tag{13}$$

and the disturbance input $d(t)$ expressed as:

$$d(t) = (m_0\lambda_0 - \mu_0)/(\beta \cdot r \cdot m_0). \tag{14}$$

Note that by the examination of the equations (12) .. (14) it is necessary the choice A_0 to avoid $m_0 = 0$.

By taking into consideration the fact that the slope m_0 can take positive or negative values, the other parameters in (12) and (13) take only positive values, the parameters k_1 and T_1 will take positive or negative values. This is the reason why there will be used in the sequel two model types for the wheel dynamics in tire slip control. A unified expression of these models points out the CP transfer function, $H_P(s)$:

$$\lambda(s) = H_P(s)u(s) + d(s). \tag{15}$$

The expressions of $H_P(s)$ for these two model types are (16) and (17):

$$H_P(s) = \frac{k_P e^{-s\tau}}{1 + sT_P} \quad \text{in the case} \quad m_0 > 0, \tag{16}$$

$$H_P(s) = \frac{k_P e^{-s\tau}}{-1 + sT_P} \quad \text{in the case} \quad m_0 < 0, \tag{17}$$

where k_P, T_P and τ are the CP gain, lag time constant and time delay, respectively. The parameters k_P and T_P of the CP can be derived by calculating the absolute value in (12), (13):

$$k_P = |k_1| = r\beta / |m_0| / [\gamma(1-\lambda_0) + \alpha r] > 0, \tag{18}$$

$$T_P = |T_1| = v_0 / |m_0| / [\gamma(1-\lambda_0) + \alpha r] > 0. \tag{19}$$

The linear models (16) and (17) represent first order lag plus time delay models. A large number of CPs can be characterized by the stable models (16) [12], [13]. This is not the situation for the unstable models (17).

Since the TS-FC consists of the merge between several local PI / PID controllers, for the development of these local controllers devoted to the local models (16) / (17) of the CP it is necessary to obtain the local models valid in the vicinity of significant operating points. For the choice of the operating points it must be highlighted that μ_0 does not appear in the expressions of the CP parameters (18) and (19); therefore, it is sufficient to compute the slope m_0 for several values of μ_H (see Fig. 1).

The operating points are characterized by the following sets of steady-state values: $\mu_H \in \{0.1, 0.3, 0.7, 1\}$, $\lambda_0 \in \{0.05, 0.1, 0.2, 0.3\}$, $v_0 \in \{1, 10, 20, 30\}$. So, there will result 64 local models of the CP, of type (16) or (17). The parameters, types and indexes (marked with i=1…64) of these models are presented in Table I (T_P in seconds).

III. Development of Takagi-Sugeno Fuzzy Controller and of Interpolative Fuzzy Controller

The proposed CS structure, dedicated to tire slip control (Fig. 2), is based on the measurement / estimation of the signals $\{\mu_H, v, \lambda\}$ available from the CP. The other elements in Fig. 2 represent: λ_r – the reference input (tire slip setpoint), $e = \lambda_r - \lambda$ – the control error.

It can be seen that the tire slip λ plays the role of controlled output. For the sake of simplicity the measuring devices and the estimators are considered to be part of the CP.

As already mentioned in Section I, the development of the TS-FC starts with the development of the local controllers to control the local CP models of type (16) or (17). In the first phase there are developed continuous-time controllers.

For controlling the CP model (16) there are widely used several development techniques [13], [14] based on employing a PI controller with the transfer function $H_C(s)$:

$$H_C(s) = k_C[1 + 1/(T_i s)]\,, \tag{20}$$

with k_C – controller gain and T_i – integral time constant.

By applying the frequency domain approach to the CP (16) controlled by the controller (20), the phase margin φ_m can be expressed as:

$$\varphi_m = 0.5\pi + \tan^{-1}\omega_c T_i - \tan^{-1}\omega_c T_P - \omega_c \tau\,, \tag{21}$$

where ω_c stands for the crossover frequency.

T_i can be chosen to cancel the CP lag time constant T_P:

$$T_i = T_P\,. \tag{22}$$

By imposing a desired phase margin φ_m, the equations (21) and (22) result in the expression of ω_c:

$$\omega_c = (0.5\cdot\pi - \varphi_m)/\tau\,. \tag{23}$$

The controller gain, k_C, is obtained in terms of the modulus condition (24):

$$k_C = \omega_c T_i \sqrt{1+(\omega_c T_P)^2}\,/\,k_P\,/\sqrt{1+(\omega_c T_i)^2}\,. \tag{24}$$

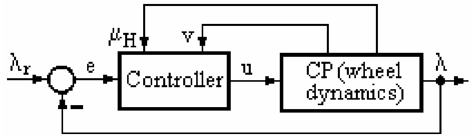

Fig. 2. Structure of tire slip control system

TABLE I

Parameters k_P / T_P and Model Types ((16) or (17)) / Model Indexes (i) for Local Models of Controlled Plant

		v_0=1	v_0=10	v_0=20	v_0=30
μ_H= 0.1	λ= 0.05	$7.25\cdot10^{-4}$ / 0.0024 (16) / 1	$7.25\cdot10^{-4}$ / 0.0242 (16) / 2	$7.25\cdot10^{-4}$ / 0.0484 (16) / 3	$7.25\cdot10^{-4}$ / 0.0725 (16) / 4
	λ = 0.1	0.0022 / 0.0073 (16) / 5	0.0022 / 0.0726 (16) / 6	0.0022 / 0.1452 (16) / 7	0.0022 / 0.2179 (16) / 8
	λ = 0.2	0.0328 / 0.1092 (16) / 9	0.0328 / 1.0917 (16) / 10	0.0328 / 2.1834 (16) / 11	0.0328 / 3.2751 (16) / 12
	λ = 0.3	0.0656 / 0.2188 (17) / 13	0.0656 / 2.1882 (17) / 14	0.0656 / 4.3764 (17) / 15	0.0656 / 6.5646 (17) / 16
μ_H= 0.3	λ= 0.05	$2.61\cdot10^{-4}$ / $8.71\cdot10^{-4}$ (16) / 17	$2.61\cdot10^{-4}$ / 0.0087 (16) / 18	$2.61\cdot10^{-4}$ / 0.0174 (16) / 19	$2.61\cdot10^{-4}$ / 0.0261 (16) / 20
	λ = 0.1	$5.23\cdot10^{-4}$ / 0.00173 (16) / 21	$5.23\cdot10^{-4}$ / 0.0174 (16) / 22	$5.23\cdot10^{-4}$ / 0.0349 (16) / 23	$5.23\cdot10^{-4}$ / 0.0523 (16) / 24
	λ = 0.2	0.0655 / 0.2183 (17) / 25	0.0655 / 2.1834 (17) / 26	0.0655 / 4.3668 (17) / 27	0.0655 / 6.5502 (17) / 28
	λ = 0.3	0.0066 / 0.0219 (17) / 29	0.0066 / 0.2188 (17) / 30	0.0066 / 0.4376 (17) / 31	0.0066 / 0.6565 (17) / 32
μ_H= 0.7	λ= 0.05	$9.15\cdot10^{-5}$ / $3.05\cdot10^{-4}$ (16) / 33	$9.15\cdot10^{-5}$ / 0.003 (16) / 34	$9.15\cdot10^{-5}$ / 0.0061 (16) / 35	$9.15\cdot10^{-5}$ / 0.0091 (16) / 36
	λ = 0.1	$8.83\cdot10^{-4}$ / 0.0029 (17) / 37	$8.83\cdot10^{-4}$ / 0.0294 (17) / 38	$8.83\cdot10^{-4}$ / 0.0589 (17) / 39	$8.83\cdot10^{-4}$ / 0.0883 (17) / 40
	λ = 0.2	0.0033 / 0.0109 (17) / 41	0.0033 / 0.1092 (17) / 42	0.0033 / 0.2183 (17) / 43	0.0033 / 0.3275 (17) / 44
	λ = 0.3	0.0047 / 0.0156 (17) / 45	0.0047 / 0.1563 (17) / 46	0.0047 / 0.3126 (17) / 47	0.0047 / 0.4689 (17) / 48
μ_H= 1	λ= 0.05	$7.83\cdot10^{-5}$ / $2.61\cdot10^{-4}$ (16) / 49	$7.83\cdot10^{-5}$ / 0.0026 (16) / 50	$7.83\cdot10^{-5}$ / 0.0052 (16) / 51	$7.83\cdot10^{-5}$ / 0.0078 (16) / 52
	λ = 0.1	$1.63\cdot10^{-4}$ / $5.45\cdot10^{-4}$ (16) / 53	$1.63\cdot10^{-4}$ / 0.0054 (16) / 54	$1.63\cdot10^{-4}$ / 0.0109 (16) / 55	$1.63\cdot10^{-4}$ / 0.0163 (16) / 56
	λ = 0.2	0.0022 / 0.0073 (17) / 57	0.0022 / 0.0728 (17) / 58	0.0022 / 0.1456 (17) / 59	0.0022 / 0.2183 (17) / 60
	λ = 0.3	0.0016 / 0.0055 (17) / 61	0.0016 / 0.0547 (17) / 62	0.0016 / 0.1094 (17) / 63	0.0016 / 0.1641 (17) / 64

The CP model (17) is treated relatively seldom in the literature, a good overview on the development techniques being presented in [15], [16]. Since a PI controller cannot stabilize the CP, (17), an ideal PID controller with the transfer function $H_C(s)$ will be used as follows:

$$H_C(s) = k_C[1 + 1/(T_i s) + T_d s]\,, \tag{25}$$

with T_d – derivative time constant. For applying the frequency domain approach, the series PID form is preferred:

$$H_C(s) = (k_c/s)(1 + T_{c1}s)(1 + T_{c2}s)\,, \tag{26}$$

where the connections between the forms (25) and (26) are:

$$k_C = k_c(T_{c1} + T_{c2}), T_i = T_{c1} + T_{c2}\,, \\ T_d = T_{c1}T_{c2}/(T_{c1} + T_{c2}). \tag{27}$$

This article can be cited as R.-E. Precup, S. Preitl, M. Balas, V. Balas, Fuzzy controllers for tire slip control in anti-lock braking systems, Proceedings of IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2004), Budapest, Hungary, doi: 10.1109/FUZZY.2004.1375359, 2004.

By considering the form (26), φ_m can be expressed as:

$$\varphi_m = 0.5\pi + \tan^{-1}\omega_c T_{c1} + \tan^{-1}\omega_c T_{c2} - \\ - \tan^{-1}\omega_c T_P - \omega_c \tau. \tag{28}$$

An approximate analytical solution of (28), ω_c, may be obtained if the following quite accurate approximation for the arctan=tan^{-1} function is made [17]:

$$\tan^{-1} x \approx (0.25\pi)x,\ |x| < 1. \tag{29}$$

By imposing a desired phase margin φ_m, the equations (28) and (29) lead to (30):

$$\omega_c = (0.5\pi + \varphi_m)/[0.25\pi(T_{c1} + T_{c2} + T_P) - \tau]. \tag{30}$$

The choice of the controller parameters T_{c1} and T_{c2} according to (31):

$$T_{c1} = 5\tau/\pi,\ T_{c2} = 15\tau/\pi, \tag{31}$$

enables the calculation of ω_c:

$$\omega_c = (0.5\pi + \varphi_m)/(0.25 T_P + 4\tau). \tag{32}$$

The controller parameter k_c is obtained by imposing the modulus condition associated with the form (26):

$$k_C = (\omega_c / k_P)\sqrt{1 + (\omega_c T_P)^2}\ / \\ /\ [\sqrt{1 + (\omega_c T_{c1})^2}\sqrt{1 + (\omega_c T_{c1})^2}]. \tag{33}$$

In the second phase, the development of the TS-FC it is performed the discretization of the continuous-time PI and PID controllers resulting in the incremental version of quasi-continuous digital PI or PID controllers expressed in the unified form (34):

$$\Delta u_k = K_P(\Delta e_k + \alpha_e e_k + \alpha_f f_k), \tag{34}$$

where e_k is the control error, $\Delta e_k = e_k - e_{k-1}$ and $\Delta u_k = u_k - u_{k-1}$ stand for the increment of control error and of the control signal, respectively, and f_k is the derivative term.

The parameters $\{K_P, \alpha_e, \alpha_f\}$ in (34) depend on the discretization method and on the sampling period T_s. By applying Tustin's discretization method to the continuous-time PI and PID controllers (20) and (25), their expressions will be:

$$K_P = 0.5 k_c (2T_i - T_s),\ \alpha_e = 2T_s/(2T_i - T_s), \\ \alpha_f = 4T_i T_d/(2T_i - T_s). \tag{35}$$

In the case of the PI controller α_f=0. In the case of the PID controller, the derivative term f_k is obtained in terms of the discrete-time equation (36):

$$f_k = -f_{k-1} + e_k - 2e_{k-1} + e_{k-2}. \tag{36}$$

For the accepted benchmark, by imposing a phase margin $\varphi_m = 7\pi/18$, the parameters $\{K_P, \alpha_e, \alpha_f\}$ of the 64 local digital PI / PID controllers take the values grouped in Table II, for T_s = 0.005 s. Such a phase margin ensures a relatively good robustness to model uncertainties specific to ABSs.

The structure of the proposed TS-FC is presented in Fig. 3, and it contains the basic FC (B-FC) and the blocks with dynamics. In the initial phase, the fuzzification is done by using the membership functions presented in Fig. 4.

The block B-FC employs the max and min operators in the inference engine, and the weighted sum method for defuzzification [10].

A complete rule base of 64 rules having the general form (37) assists the inference engine of the block B-FC:

TABLE II

PARAMETERS K_P / α_e / α_f / INDEXES (i) OF LOCAL DIGITAL PI AND PID CONTROLLERS

-3.586 / -49.72 / 0 / 1	778 / 0.2288 / 0 / 2	1646 / 0.1081 / 0 / 3	2510 / 0.0709 / 0 /4
56.7 / 1.0406 / 0 / 5	828 / 0.0713 / 0 / 6	1886 / 0.0356 / 0 / 7	2545 / 0.0232 / 0 / 8
84.6 / 0.0469 / 0 / 9	863 / 0.0046 / 0 / 10	1729 / 0.0023 / 0 / 11	2594 / 0.0015 / 0 / 12
34.6 / 0.0577 / 6.87 / 13	7.406 / 0.0577 / 6.87 / 14	3.835 / 0.0577 / 6.87 / 15	2.586 / 0.0577 / 6.87 / 16
-162 / -3.0741 / 0 / 17	616 / 0.8084 / 0 / 18	1484 / 0.3356 / 0 / 19	2351 / 0.2118 / 0 / 20
-39.8 / -6.256 / 0 / 21	741 / 0.3361 / 0 / 22	1611 / 0.1546 / 0 / 23	2476 / 0.1006 / 0 / 24
34.7 / 0.0577 / 6.87 / 25	7.432 / 0.0577 / 6.87 / 26	3.85 / 0.0577 / 6.87 / 27	2.595 / 0.0577 / 6.87 / 28
181 / 0.0577 / 6.87 / 29	344 / 0.0577 / 6.87 / 30	264 / 0.0577 / 6.87 / 31	204 / 0.0577 / 6.87 / 32
-624 / -2.2772 / 0 / 33	142 / 10 / 0 / 34	1029 / 1.381 / 0 / 35	1876 / 0.7575 / 0 / 36
967 / 0.0577 / 6.87 / 37	1531 / 0.0577 / 6.87 / 38	2083 / 0.0577 / 6.87 / 39	2408 / 0.0577 / 6.87 / 40
295 / 0.0577 / 6.87 / 41	679 / 0.0577 / 6.87 / 42	687 / 0.0577 / 6.87 / 43	609 / 0.0577 / 6.87 / 44
226 / 0.0577 / 6.87 / 45	496 / 0.0577 / 6.87 / 46	435 / 0.0577 / 6.87 / 47	356 / 0.0577 / 6.87 / 48
-743 / -2.234 / 0 / 49	33.2 / 50 / 0 / 50	897 / 1.8506 / 0 / 51	1769 / 0.9384 / 0 / 52
-312 / -2.5545 / 0 / 53	463 / 1.7214 / 0 / 54	1340 / 0.5948 / 0 / 55	2201 / 0.3621 / 0 / 56
415 / 0.0577 / 6.87 / 57	907 / 0.0577 / 6.87 / 58	1058 / 0.0577 / 6.87 / 59	1031 / 0.0577 / 6.87 / 60
554 / 0.0577 / 6.87 / 61	1114 / 0.0577 / 6.87 / 62	1402 / 0.0577 / 6.87 / 63	1458 / 0.0577 / 6.87 / 64

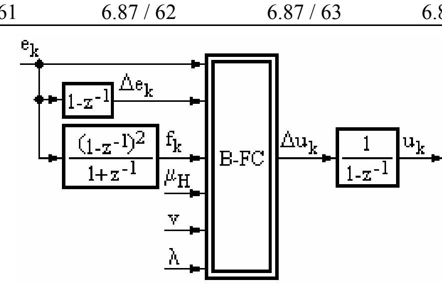

Fig. 3. Structure of Takagi-Sugeno fuzzy controller

This article can be cited as R.-E. Precup, S. Preitl, M. Balas, V. Balas, Fuzzy controllers for tire slip control in anti-lock braking systems, Proceedings of IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2004), Budapest, Hungary, doi: 10.1109/FUZZY.2004.1375359, 2004.

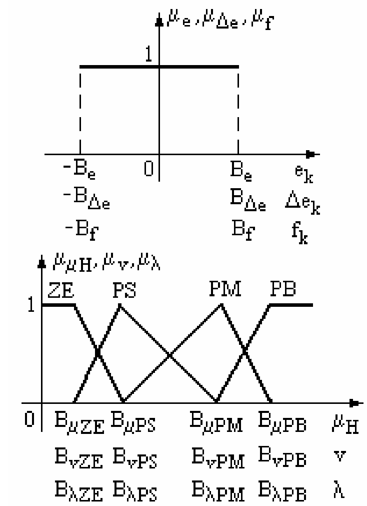

Fig. 4. Initial input membership functions

$$\begin{aligned}&\text{IF } (e_k = LT_e) \text{ AND } (\Delta e_k = LT_{\Delta e}) \text{ AND}\\&(f_k = LT_f) \text{ AND } (\mu_H = LT_\mu) \text{ AND } (v = LT_v)\\&\text{AND } (\lambda = LT_\lambda) \text{ THEN}\\&(\Delta u_k^i = K_P(\Delta e_k + \alpha_e e_k + \alpha_f f_k))\end{aligned}, \qquad (37)$$

where LT stands for linguistic term (Fig. 4), and the upper index i (i=1…64) corresponds to the index of the local model of the CP for which the digital PI / PID controller was developed (shown in Table I and Table II).

Fig. 4 illustrates the specific parameters of the TS-FC, tuned by the modal equivalence principle [18] in connection with the coordinates of the significant operating points. For the accepted benchmark and the operating points mentioned in the previous Section, the following values of TS-FC parameters were obtained: $B_{\mu ZE}$=0.1, $B_{\mu ZE}$=0.3, $B_{\mu ZE}$=0.7, $B_{\mu ZE}$=0.9, B_{vZE}=1, B_{vZE}=10, B_{vZE}=20, B_{vZE}=29, $B_{\lambda ZE}$=0.05, $B_{\lambda ZE}$=0.1, $B_{\lambda ZE}$=0.2, $B_{\lambda ZE}$=0.25, B_e=0.2, $B_{\Delta e}$=0.2, B_f=0.2.The parameters {B_e, $B_{\Delta e}$, B_f} depend on the universes of the variables e_k, Δe_k and f_k, respectively, and were obtained for ensuring the scaling factors which are not shown in Fig. 3.

By summarizing the results presented in this Section, the development method for the TS-FC consists of the following steps:
- choose the significant operating points of the CP, linearize the mathematical model around these points and express the local linearized models in the forms (16) or (17);
- by using a conventional development technique (the frequency domain approach is exemplified in this paper), tune the continuous-time local PI controllers (20) and PID controllers (25) to control the local CP models (16) and (17), respectively;
- choose a sufficiently small sampling period T_s accepted by quasi-continuous digital control and take into account the presence of a zero-order hold;
- discretize the continuous-time local PI / PID controllers and compute the parameters {K_P, α_e, α_f} in (35);
- by applying the modal equivalence principle determine the parameters of the TS-FC (defined in Fig. 4).

The structure of the IFC is presented in Fig. 5 [19].

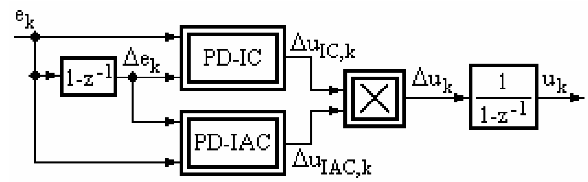

Fig. 5. Block diagram of the interpolative fuzzy controller

It contains the nonlinear blocks PD-IC – PD interpolative controller and PD-IAC – PD interpolative adaptive corrector. Both blocks can be developed in a heuristic (linguistic) manner, thanks to the fact that TS-FCs with triangular input membership functions, prod and sum operators in the inference engine and weighted area method for defuzzification have equivalent interpolative variants. This result is applied also for the considered TS-FC. The two nonlinear PD controllers are implemented by look-up-tables with linear interpolations, allowing fine tunings of the ABS performance, in a smooth manner [19].

The development of the IFC is performed by applying the development steps devoted to the TS-FC presented before followed by the implementation of the input-output static map of the TS-FC by means of interpolative blocks.

With respect to the structure presented in Fig. 5, the look-up tables of the blocks PD-IC and PD-IAC have the composition presented in (38) and (39), respectively:

$$\begin{aligned}&e_k : [-0.25\ -0.03\ 0\ 0.03\ 0.25],\\&\Delta e_k : [-0.04\ 0\ 0.04],\\&\Delta u_{IC,k} = [-1\ -1\ 0;-1\ -1\ 0;-1\ 0\ 1;0\ 1\ 1;0\ 1\ 1],\end{aligned} \qquad (38)$$

$$\begin{aligned}&e_k : [-0.25\ -0.1\ 0\ 0.1\ 0.25],\\&\Delta e_k : [-0.03\ 0\ 0.03],\\&\Delta u_{IAC,k} = [1\ 1\ 1;2\ 1.5\ 1;2\ 2\ 2;1.5\ 1\ 1;1\ 1\ 1].\end{aligned} \qquad (39)$$

IV. Digital Simulation Results

The simulation results are obtained by accepting that the simplified dynamic model (1) … (3) characterizes well the CP involved in tire slip control, as part of an ABS, and the CS structure is that presented in Fig. 2. The parameter values that correspond to the considered model are those presented in Section II, together with τ=0.014 s and ε=0.001.

The TS-FC and the IFC are developed in terms of the methodology presented in the previous Section. For the accepted case study, the simulation scenario consists of feeding a reference input λ_r=0.015 (acceptable for several surfaces), and the braking is started on a high friction surface (μ_H=0.9 for 0.9 s), then a low friction surface is encountered (μ_H=0.1 for the next 1.5 s), and the braking is finished on a medium friction surface (μ_H=0.5 for the final 0.6 s). The initial value of v is considered v_0=25 m/s.

In these simulation conditions, the synthesis of the digital simulation results is presented in Figs. 6 … 8 for three CSs, the CS with conventional PI controller developed for the local model i=51 (close to the initial operating point) of the CP (Fig. 6), the CS with TS-FC (Fig. 7), and the CS with IFC (Fig. 8).

This article can be cited as R.-E. Precup, S. Preitl, M. Balas, V. Balas, Fuzzy controllers for tire slip control in anti-lock braking systems, Proceedings of IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2004), Budapest, Hungary, doi: 10.1109/FUZZY.2004.1375359, 2004.

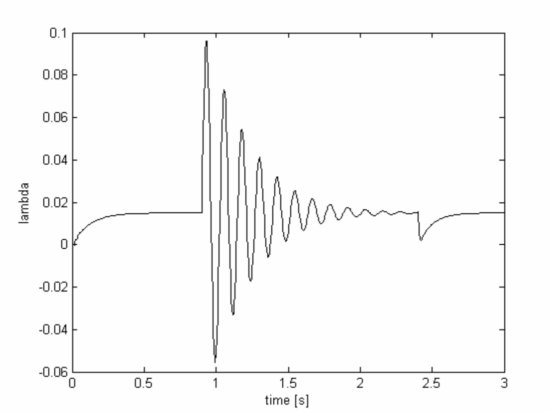

Fig. 6. λ versus time for CS with conventional PI controller

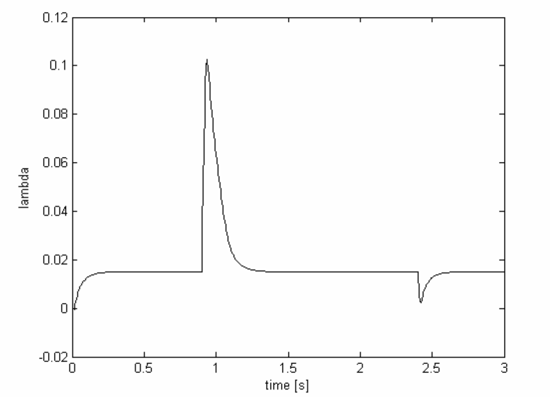

Fig. 7. λ versus time for CS with TS-FC

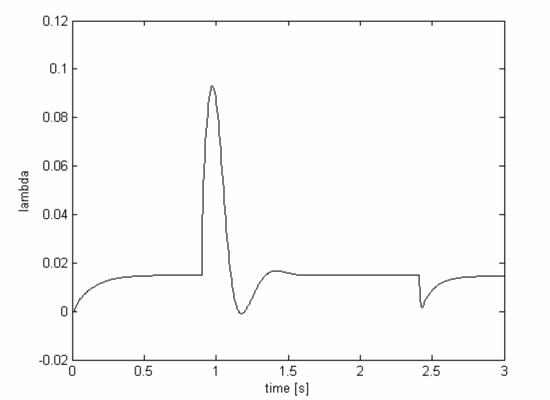

Fig. 8. λ versus time for CS with IFC

V. CONCLUSIONS

The paper proposes a CS structure together with two FCs, a TS-FC and an IFC, dedicated to tire slip control in ABSs. There are derived development methods for the FCs.

The controllers and development methods are validated by considering a case study based on an accepted benchmark, and prove their potential in tire slip control.

Digital simulation results prove that the two FCs ensure the CS performance enhancement in comparison with a conventional PI controller (including the negative values of λ that cannot be accepted due to the physical interpretation).

In the first phase, the TS-FC structure is relatively simple due to the reduced number of input LTs of the block B-FC aiming a simple implementation. By defining at least three LTs for the input variables e_k, Δe_k and f_k, the CS performance with either the TS-FC or the IFC can be further improved.

The proposed FCs can be extended with supplementary features to more complex dynamic models used in ABS tire slip control. But, the rigorous analysis, similar to the case of Mamdani fuzzy controllers [20], must be performed in all applications.

ACKNOWLEDGEMENT

The authors thank Mr. Stefan Solyom, Lund Institute of Technology, for offering a copy of his licentiate thesis and of the benchmark for control of ABSs.

REFERENCES

[1] U. Kiencke and L. Nielsen, *Automotive Control Systems*, Berlin, Heidelberg, New York: Springer-Verlag, 2000.

[2] W. K. Lennon and K. M. Passino, “Intelligent control for brake systems,” *IEEE Trans. Control Systems Technology*, vol. 7, pp. 188-202, March 1999.

[3] S. Drakunov, Ü. Özgüner, P. Dix, and B. Ashrafi, “ABS control using optimum search via sliding mode,” *IEEE Trans. Control Systems Technology*, vol. 3, pp. 79-85, March 1995.

[4] Y. Liu and J. Sun, “Target slip tracking using gain-scheduling for braking systems,” *Proceedings of American Control Conference*, Seattle, Washington, 1998, pp. 1178-1182.

[5] T. A. Johansen, J. Kalkkuhl, J. Lüdemann, and I. Petersen, “Hybrid control strategies in ABS,” *Proceedings of American Control Conference*, Arlington, TX, 2001.

[6] F. Jiang, *A novel control approach to a class of anti-lock brake problems*, PhD thesis, Cleveland State University, 2000.

[7] S. Solyom, *Synthesis of a model-based tire slip controller*, Licentiate thesis, Lund Institute of Technology, 2002.

[8] J. R. Layne, K. M. Passino, and S. Yurkovich, “Fuzzy learning control for antiskid braking systems,” *IEEE Trans. Control Systems Technology*, vol. 1, pp. 122-129, June 1993.

[9] T. Takagi and M. Sugeno, “Fuzzy identification of systems and its applications to modeling and control,” *IEEE Trans. Systems, Man, and Cybernetics*, vol. 15, pp. 116-132, Jan./Feb. 1985.

[10] R. Babuška and H. B. Verbruggen, “An overview on fuzzy modeling for control,” *Control Eng. Practice*, vol. 4, pp. 1593-1606, Nov. 1996.

[11] S. Solyom, A. Rantzer, and J. Kalkkuhl, *A benchmark for control of Anti-lock Braking Systems*, Res. Report, Lund Inst. of Tech., 2003.

[12] C. C. Hang and D. Chin, “Reduced order process modeling in self-tuning control,” *Automatica*, vol. 27, pp. 529-534, May 1991.

[13] C. C. Hang, K. J. Åström, and Q. G. Wang, “Relay feedback auto-tuning of process controllers – a tutorial review,” *Journal of Process Control*, vol. 12, pp. 143-162, Febr. 2002.

[14] K. J. Åström and T. Hägglund, *PID Controllers Theory: Design and Tuning*, Research Triangle Park, NC: Inst. Society of America, 1995.

[15] A. O’Dwyer, “A summary of PI and PID controller tuning rules for processes with time delay. Part 1: PI controller tuning rules,” *Proc. of IFAC Workshop PID’00*, Terrassa, Spain, 2000, pp. 175-180.

[16] A. O’Dwyer, “A summary of PI and PID controller tuning rules for processes with time delay. Part 2: PID controller tuning rules,” *Proc. of IFAC Workshop PID’00*, Terrassa, Spain, 2000, pp. 242-247.

[17] W. K. Ho, C. C. Hang, and J. H. Zhou, “Performance and gain and phase margins of well-known PI tuning formulas,” *IEEE Trans. Control Systems Technology*, vol. 3, pp. 245-248, June 1995.

[18] S. Galichet and L. Foulloy, “Fuzzy controllers: synthesis and equivalences,” *IEEE Trans. Fuzzy Systems*, vol. 3, pp. 140-148, May 1995.

[19] M. Balas, V. Balas, S. Galichet and L. Foulloy, “A model of the sliding wheel during the braking,” *Proc. 5th Int. Conf. Railway Bogies and Running Gears BOGIE2001*, Budapest, Hungary, 2001, 8 pp.

[20] R. -E. Precup, S. Preitl, and G. Faur, “PI predictive fuzzy controllers for electrical drive speed control: methods and software for stable development,” *Computers in Industry*, vol. 52, pp. 253-270, Dec. 2003.