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Laguerre Gauss Kernel for COVID-19 Medical Decision Making From Chest Tomography

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ABSTRACT

As March 22nd Colombia had 235 confirmed cases of COVID-19. We already had decided to put together of national mathematicians and engineers along with medical doctors to quickly answer to potential needs on the development of a tool for decision-making support. As well known, several attempts to provide a deep learning solution to early diagnosis tools were not successful firstly for the lack of formalism and medical validity of their findings and secondly, because as known for the mathematics and engineers a learning machine must fulfill bounds for learning depending on its complexity. Consequently with no database fulfilling neither the medical standards nor the guarantees of learning, these attempts are far to be useful. Both medical and mathematical conditions for an artificial intelligence learning machine are going to be satisfied during our work. Looking for the construction of a solid approach to the problem, this paper contains the Laguerre-Gauss kernel applied to computerized tomography images looking for feature extraction in order to provide a preprocessing tool that reduces the complexity of the learning machine, and in consequence reducing the dataset needed to reach the medical doctors standards of sensitivity and specificity. We developed it in Paniagua et al, 2016, 2017, Paniagua, 2018, and proposed in Quintero et al, 2020 as a tool for deep learning in Medical Applications. Our results are promising and will be proved and tested on several deep learning algorithms and the interpretability of the particular structures in lungs generated by the virus.

Keywords: Medical decision making, deep learning, COVID-19, kernels for preprocessing, information extraction.

2010 Mathematics Subject Classification: 44A35, 62H35, 62M20, 62M40, 68T05, 68T10.
2012 ACM Computing Classification System: 10010336, 10010447, 10010383.

1 Introduction

Machine learning and deep learning are wonderful tools and strategies to automate medical practice. Image processing is now a broader field and we have experienced a convergence between mathematics, computer science, and medicine. As striking as the recounted experiences are and success rates increase, we need more efforts regarding human-computer interfaces.

In computer science, learning can be seen as a multi-dimensional process of understanding phenomena or the proficient repetition of the tasks related to those phenomena. In the real world, the measurement of such process is related to *data*. Data become a very important source of *Information* that must be transformed into *Knowledge*, which, when properly formalized is Intelligence.

Supervised learning requires a set of labeled data. A label is a term that specifies the category or the type of object. Learning develops a model \mathcal{M} using the data. The model \mathcal{M} will have afterwards the capability to label the remaining unlabeled data accurately.

In our chapter for the book "Deep Learners and Deep Learner Descriptors for Medical Applications" called "From Artificial intelligence to Deep Learning in Bio-Medical Applications" we addressed: *"The medical and biological fields are one of the most promising fields of progress due to the use of the remarkable world of deep learning and automatic features extraction. The DL applications will help to provide and control treatments that will improve quality of life for humanity. Improving the therapeutic practice and helping physicians and MD to provide an accurate and early diagnosis. ... There is no ambiguity that a machine will never replace an MD expert, but machine intelligence will benefit and its aimed for human decision making.* (Quintero and Paniagua, 2020).

COVID-19 disease is caused by the SARS-CoV-2 virus (Gorbalenya et al., 2020), and the world is currently experiencing a pandemic that has been declared since March. Due to this, the performance of chest tomography has been increased to identify changes in the lung parenchyma suggestive of COVID-19, despite the fact that radiology societies do not recommend its routine, since the gold-standard diagnostic test is PCR (Simpson et al., 2020). The radiological pattern in CT that has been most identified in COVID-19 pneumonia are peripheral ground glass or nodular-type opacities, which have even been seen in infected and asymptomatic patients (Salehi et al., 2020); although other findings that may resemble other infectious or non-infectious processes have also been described (Franquet, 2011). The importance from a medical point of view is to develop a clinical decision support system that allows health personnel to take early measures regarding the treatment of patients suspected of having COVID-19 infection, given that the result of the PCR for COVID-19 takes several days to process in our environment, and the health systems collapsed during the peak of the pandemic.

Now is the time to act. Consequently we joined efforts with the international community to help the practitioners for the heroic effort to treat COVID-19 in our under developed countries. As well known, several attempts to provide a deep learning solution to early diagnosis tools were not successful firstly for the lack of formalism and medical validity of their findings and secondly, because as known for the mathematics and engineers a learning machine must fulfill bounds for learning depending on its complexity. Consequently with no database fulfilling neither the

medical standards nor the guarantees of learning, these attempts are far to be useful.

Besides, each Colombian Health institution defined their protocols for triage and attention of patients under suspicious if COVID-19, then the use of X-ray and Computerized Tomography are part of the protocol but they are not going to expose the entire facilities to contamination and will use images in very special cases. Our approach to the problem is holistic. Consequently, the first step is to develop a feasibility study testing several learning machines for X-ray and CT while acquiring images under a standardized protocol for images acquisition. Secondly, a retrospective study of the performance of the learning machines and finally, the prospective study validated on line by the medical experts. Developing solutions is not just training a preconceived convolutional/deep network, neither machine learning is just a matter of a toolbox (Quintero and Paniagua, 2020). It requires interdisciplinary work and mathematical background to imagine and formalize a proper feature extractor closest to the human perception of the world.

This paper addresses the effort of the local community to build a tomography based model for early detection of COVID19, in advance to the pandemic taking all over the country. We aim to develop a very suitable tool to be used in radiology services across the country and build jointly the database to be used on underdeveloped countries.

Looking for the construction of a solid approach to the problem, this paper contains the Laguerre Gauss kernel applied to computerized tomography images looking for feature extraction in order to provide a preprocessing tool that reduces the complexity of the learning machine, and in consequence reducing the dataset needed to reach the medical doctors standards of sensitivity and specificity. We developed this kernel in Paniagua et al, 2016 (Paniagua and Sierra-Sosa, 2016), Paniagua 2017 (Paniagua et al., 2017), Paniagua et al, 2018 (Paniagua, 2018) and proposed in Quintero et al, 2020 (Quintero and Paniagua, 2020) as a tool for deep learning in Medical Applications. Our results are promising and will be proved and tested on several deep learning algorithms and the interpretability of the particular structures in lungs generated by the virus.

Basically we quickly implemented a solution based on pre- processing and define the main structures of the lung on the CT scan. This allows for the location of several features of the development of COVID-19 pathology from day 1 to day 14 as recommended by the medical doctors. As known by the medical community, the patterns are not all the same and progressively modifies the lung structure.

We plan to use this results to trained deep learning models, then we keep transferring learning from real databases collected in several hospitals in Medellín, Colombia.

This paper is organized as follows: section 2 presents the image collection, section 3 introduces the suitable architectures and section 4 will define the architecture of the radiology solution. Finally, section 5 will provide concluding remarks.

2 COVID-19 image data collection

2.1 COVNet-19 Detector from CT

Li et al, 2020 developed a deep learning method was able to identify COVID-19 on chest CT exams (area under the receiver operating characteristic curve, 0.96). To develop a fully automatic framework to detect COVID-19 using chest CT and evaluate its performances.

In this retrospective and multi-center study, a deep learning model, COVID-19 detection neural network (COVNet), was developed to extract visual features from volumetric chest CT exams for the detection of COVID-19. Community acquired pneumonia (CAP) and other non-pneumonia CT exams were included to test the robustness of the model. The datasets were collected from 6 hospitals between August 2016 and February 2020. Diagnostic performance was assessed by the area under the receiver operating characteristic curve (AUC), sensitivity and specificity.(Li et al., 2020)

- The collected dataset consisted of 4356 chest CT exams from 3,322 patients.
- The average age is 49 ± 15 years and there were slightly more male patients than female (1838 vs 1484; p-value=0.29).
- The per-exam sensitivity and specificity for detecting COVID-19 in the independent test set was 114 of 127 (90% [95% CI: 83%, 94%]) and 294 of 307 (96% [95% CI: 93%, 98%]), respectively, with an AUC of 0.96 (p-value \leq 0.001).
- The per-exam sensitivity and specificity for detecting CAP in the independent test set was 87% (152 of 175) and 92% (239 of 259), respectively with an AUC of 0.95 (95% CI:0.93, 0.97).
- The website of the repository <https://github.com/bkong999/COVNet>

2.2 CT Colombian-MED dataset

As of March 23, 2020, Colombia has 235 registered cases of COVID-19. In underdeveloped countries, access to PCR tests for diagnosis requires the use of scientific and technical capabilities in the development of tools that can support medical personnel in this contingency. The Colombian health system and that of several Latin American countries may collapse due to the potential massive assistance to the emergency services.

The interdisciplinary team of medical doctors from several institutions Hospital Pablo Tobón Uribe, Fundación Universitaria San Vicente de Paul, Universidad CES, Clinica CES, Universidad de Antioquia IPS univertaria will build the database of patients under the defined protocol. CT protocol: KvP: 88 - 140, Exposure (mAs): 50 - 360, Cutting thickness: 0.6 - 3.0, Reconstruction matrix: 512x512

Scanner models: Siemens, GE, Phillips, Toshiba

The regions of the lung parenchyma will be extracted using a U-net based segmentation method. And other preprocessing schemes will be used on this regard, the Laguerre Gauss filters is one of them.

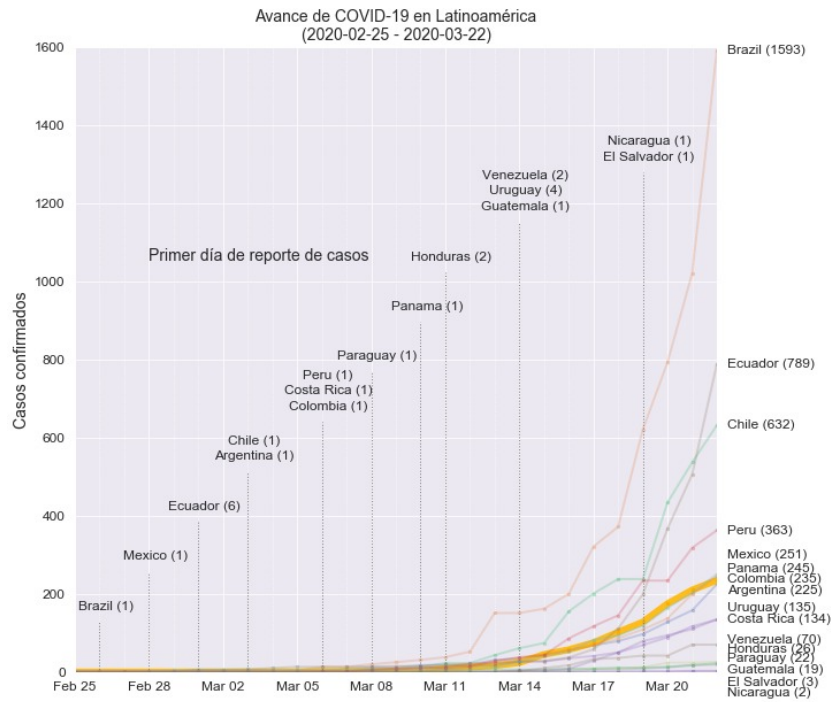


Figure 1: Following the COVID-19 Cases in Colombia. Authored by Nicolas Pinel Associated Prof. Universidad EAFIT

The lung parenchyma will be resampled in the same space (1 mm) in the z direction. To decrease the dimensionality of the tomography it will be re-sampled on the Z axis 5 times and scaled to $S \times 224 \times 224$ pixels. S represents the same number of sections of the tomography. The voxel intensity values were established at window and level values of 1500 / -500 UH and will be normalized in a range of [0, 1].

To reduce the influence of vascular structures and increase the conspicuousness of the lesions, preliminary a Maximum Intensity Projection (MIP) algorithm will be applied to each of the cuts. We will also explore several methods developed by our team.

3 The Learning Machines

The main subfields of machine learning are supervised and unsupervised learning. In supervised learning, the subject is on accurate prediction whereas in unsupervised learning the aim is to find compact descriptions of the data.

We are interested in methods that generalize well to previous unseen data in both cases; so, we can think about learning as the capability to perform a task, as much as possible, in interaction with the environment.

When the task is to learn the relationships between the input and the output -we will momentarily call this the pair (*input, output*)- such that, given a new input, we can classify the input as accurately as possible; it can be called *supervised learning*. Our interest is describing the output conditioned on what we know about the inputs.

But, what if the intelligent system inhabits the environment and also takes some actions? Its

capability can lead it to "know" if its actions are eventually beneficial or disastrous (ha!). Accumulative experience can lead the intelligent system to learn which action will lead to a given situation and a goal can be to maximize its opportunity to reach a long term objective. *Reinforcement learning* is more close to Control Theory and Markov decision processes.

Let us assume that we have to solve a problem and that we developed two different networks or models \mathcal{M}_1 and \mathcal{M}_2 . The problem can be stated as follows: build a model and code an algorithm able to classify 14 classes of pathology. Another problem to be solved is finding a model to retrieve risk from medical data.

Let \mathcal{M}_1 be a very simple model and \mathcal{M}_2 a very complex model. The complexity of the hypotheses set has to force the simplicity of each network (the number of layers and neurons).

The models may behave with similar training and true error but will vary consistently in their results as:

\mathcal{M}_1 Training Error 30% and True Error 30% and for the \mathcal{M}_2 Training Error 10% and True Error 30%.

One wishes to find a middle complex model \mathcal{M}_3 that may be less successful in training, but better at generalizing the fresh data. The possible model \mathcal{M}_3 should have an intermediate point like Training Error 25% and True Error 25%.

How can we achieve such a thing? In order to vary the size of the model, we should reduce the number of hyperparameters (weights) to be found. It means we should decrease the number of weights. Now, the cost function can not only be defined in terms of the error but it also must be defined in terms of the model/network size.

Deep learning is a NP-hard problem. Some people call it "badly NP hard" because of the architecture of many layers of hidden variables connected by a non linear operations. One imagines that NP-hardness is not a barrier to provable algorithms in Machine Learning because the inputs to the learner are drawn from some simple distribution P and are not worst-case.

Our efforts will be on finding a feasible way to satisfy learning bounds and minimize the complexity of the learning machines.

3.1 Finding structures via Laguerre-Gauss filter (LGF)

The purpose of mathematical modeling and machine learning is to build a model \mathcal{M} that offers a good representation of the phenomenon underlying the task.

On the other hand, several techniques have been proposed to reduce the artifacts occurrence in imaging. Derivative operators as Laplacian are the most frequently used. It is well-known that Laplacian operator boosts the high frequencies relative to the low frequencies. Therefore, the Laplacian is often used to dampen low-frequency artifacts when the background medium contains sharp wave velocity contrasts (Guitton et al., 2006).

Based on this, Paniagua et al. (Paniagua et al., 2017) proposed the use of another method to reduce or eliminate the low-frequency artifacts and not introduce other uncertainties in the scalar field produced in subsurface images. Paniagua's technique is based on a spiral phase function ranging from 0 to 2π and a toroidal amplitude bandpass filter known as Laguerre-Gauss transform.

Through numerical experiments, we presented the application of this particular integral transform on three synthetic data sets. In addition, we presented a comparative spectral study of images obtained by the zero-lag cross-correlation imaging condition, the Laplacian operator, and the Laguerre-Gauss transform, showing their spatial frequency features.

We also presented evidences not only by simulated spatial noisy velocity fields but also by comparison with the velocity field gradients of the dataset that this method improves the Reverse Time Migration (RTM)¹ scalar fields by reducing the artifacts and notably enhance the reflective events (Paniagua and Sierra-Sosa, 2016; Paniagua et al., 2017; Paniagua and Quintero, 2017b; Paniagua and Quintero, 2017a).

Laguerre-Gauss transform of a image $\tilde{I}(x, y)$ is obtained by

$$\tilde{I}(x, y) = I(x, y) * LG(x, y) \quad (3.1)$$

where the $LG(x, y)$ is Laguerre-Gauss kernel given by (Wang et al., 2006)

$$LG(x, y) = (i\pi^2\omega^4)(x + iy)e^{-\pi^2\omega^2(x^2+y^2)} \quad (3.2)$$

Laguerre Gauss transform kernel used a pure-phase function with a vortex structure in spatial frequency domain, defined as

$$B(f_x, f_y) = \tan^{-1} \left(\frac{f_y}{f_x} \right) \quad (3.3)$$

The particular property from this spiral phase function is that it is composed by a heavy-side function with a π gap when crossing the origin in every angular direction. In the amplitude, the kernel includes a gaussian toroidal geometry.

The Laguerre-Gauss transform allows to conduct an isotropic radial Hilbert transform without resolution loss (Guo et al., 2006). In addition to the advantage of spatial isotropy common to the Riesz transform stemming from the spiral phase function with the unique property of a signum function along any section through the origin, the Laguerre-Gauss transform has the favorable characteristics to automatically exclude any DC component (Wang et al., 2006).

An effect associated with the application of the Laguerre-Gauss transform is the phase and amplitude changes in the final image. Changes in amplitude are associated with the topological characteristics of the pseudo complex field and will be analyzed in future works to find its relation with attributes such as amplitude and reflectivity of wave signals.

Properties of the Laguerre-Gauss filter allows feature extraction and artifacts removal, perfect for the enhancement of the previous algorithms of cancer detection (Gallego-Posada et al., 2016), post processing imaging in breast cancer detection (Heinz, 2018), in emotion recognition and solving the fooling problem.

Consequently, the next steps of our research group are to use it as convolution layer within a CNN with transfer learning stages looking for our Convolutional Laguerre Gauss Network (CLG-Net) best performance.

¹There is a direct relation on the physics for RTM and medical imaging check the work of Wang et al. (2016) (Wang et al., 2016) in IEEE TRANSACTIONS ON MEDICAL IMAGING, VOL. 35

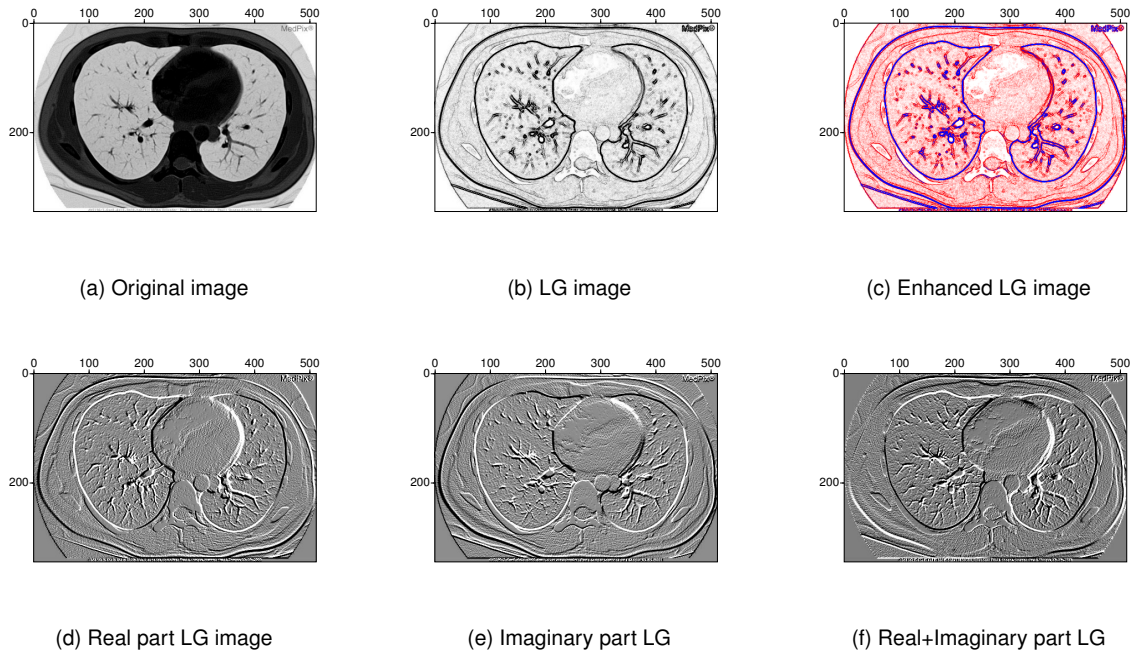


Figure 2: CT pulmonary healthy person 2a Original tomography, 2b tomography plus LGF, 2c color edge enhancement LG image, 2d real part of LG image, 2e imaginary part of LG image, and 2f imaginary part plus real part of LG image.

4 Results

We show the main effects of the Laguerre-Gauss kernel (LG) on the processing of pulmonary tomography images in healthy and unhealthy people with lesions due to COVID-19. Figure 2 shows the processing of a lung tomography of a healthy person using the Laguerre-Gauss kernel (LG). The original tomography is shown in Figure 2a. The processed image is shown in Figure 2b.

We can note that structures are more defined and the edges of the lung are enhanced. Different part of the image are delineated and some low spatial frequency artifacts are reduced, achieving a cleaner image. In Figure 2c, the edges and structures are enhanced using colors to show that, taking into account the isotropic properties of the kernel, the Laguerre-Gauss transform is capable of improving any small change in the tomography image.

Figures 2d, 2e and 2f show the real part, the imaginary part and the sum of the real part and the imaginary part of the Laguerre-Gauss complex field obtained in the transformation. These images can be used to complement and contrast relevant details of the processed image. They show different perspectives on the transformation of the tomography. We show the results of the processing of a lung tomography of an unhealthy person with COVID conditions in Figure 3. Figure 3a shows the original lung tomography.

The processed tomography by applying the Laguerre-Gauss filter and the Laguerre-Gauss image with highlighted color borders are shown in Figure 3b and Figure 3c, respectively. We can notice how the edges and structures of the lung are highlighted and illuminated. It can be seen that small changes in the image are delineated and highlighted, improving some details

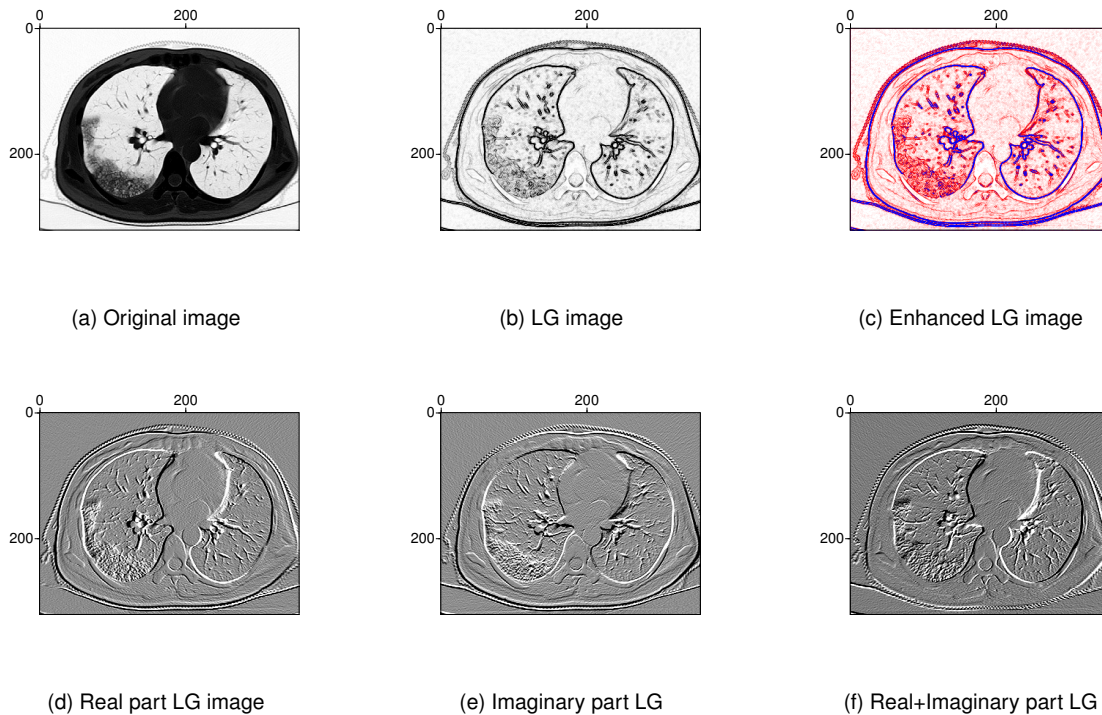


Figure 3: CT Pulmonary unhealthy person 3a Original tomography, 3b tomography plus LGF, 3c color edge enhancement LG image, 3d real part of LG image, 3e imaginary part of LG image, and 3f imaginary part plus real part of LG image.

of the image. We can also see that in the lower left part in Figure 3c, a region with a more intense red shade is highlighted, being a possible injury in that part of the lung. This pattern can also be seen in Figures 3a, 3d, 3e, and 3f in the same sector of the image. Figures 3d, 3e, and 3f show an unusual relief compared to the tomography of a healthy person shown in Figure 2.

Privacy of patients and medical protocols in the medical institutions are the priority of this research. Consequently, none of the former will be altered. Our framework contains an image extraction from DICOM format and the patient will become anonymous. After this, the treatment of the images will be under the most careful standards of security transforming the (Digital imaging communications) DICOM to a vectored file to be used by the learning machines. Our Laguerre Gauss pre-processing tool will be used to simplify the information retrieved from the CT scan images.

The purpose is two fold: a measure to reduce the exhausting hours of the medical journeys and interpretations and information compression measure.

While developing the feasibility stage, we will perform training on our own architecture with the pre-processed images of Laguerre-Gauss and also will re training the COVNet architecture. In a following paper we will detail the technological solution based on InterPlanetary File System IPFS + BlockChain + Encryption + Federated learning for the privacy and confidentiality of our dataset and real deployment scenario.

For instance, it is important to address that the model to be implemented on the local or remote

teleradiology solution must be as generalizable as not complex possible aiming for the least hyperparameters and computing time in FLOPS. This features force us to use the LGF as a method to not only expose the main features of the CT slide but also allow the learning machine to simplify the localization task. To be clear, the problem must not be only focused on classification of each CT slice series per patient, it must help with artificial intelligence the proper localization of structures that will determine the class for each result. Using LG clearly the main structures arise and the possibility to keep with the continuity on the images treatment allows better job on training.

Our contribution can be also used on several modelling approaches such as the following classical and fresh papers, which are successful in various applications (Cai et al., 2007; Preitl et al., 2007; Ahmed et al., 2019).

4.1 The Model COVNet

For the feasibility test on the pre existing models on the CT-Colombian MED data-set we will use the Model COVNet to evaluate the results. COV net is a 3D deep learning framework for the detection of COVID-19, referred to COVNet. It is able to extract both 2D local and 3D global representative features. The COVNet framework consists of a ResNet50 (16) as the backbone, which takes a series of CT slices as input and generates features for the corresponding slices. The extracted features from all slices are then combined by a max-pooling operation. (Li et al., 2020). The model is a PyTorch implementation of the paper "Artificial Intelligence Distinguishes COVID-19 from Community Acquired Pneumonia on Chest CT". It supports training, validation and testing for COVNet. (Li et al., 2020)

On the retrospective study the CT Colombian-MED data-set will be pre-processed and then transferred to the COVNet Architecture. In order to evaluate its performance. After several attempts to retrieve the so called model, it was not possible to test the claimed results. Nevertheless the model was retrained on its dataset and we found a clear overfitting and lack of robustness. Instead, we used our LG kernel on the entire dataset and the obvious improvement on the structures are depicted on the figures. Nowadays our dataset is as follows:

Class	CT Thorax
Healthy	1452
Others	598
Bacterial Pneumonia	797
Viral Pneumonia - Possible COVID	1.640
Total	4.487

Table 1: Description of the database built for the preliminary study

Some parts of our dataset are available on the internet and are freely accessible, in which the identity of the patients is respected. Additionally, we have images obtained from local hospitals, which include images of patients who have a positive PCR test for COVID-19. For databases and real data, a classification of the images was carried out as follows: "Healthy" and "Not-Healthy". This binary classification was subdivided into "Bacterial Pneumonia", "Viral

Pneumonia - Possible COVID” and “Others”. This last category includes other pathologies that affect the lung parenchyma such as nodules, lung mass, pleural effusion, pneumothorax, among others. Preliminary results on our learning machines are close to 0.96 of precision, 0.98 recall and 0.97 in F1 score. Matthews correlation coefficient of 0.94 provide good medical results to be tested on fresh and colombian real data. As described in (Heinz, 2018) our findings suggest the LG and the provided evidence will reduce our learning machine from an architecture as in (Li et al., 2020) to a simpler one.

5 Conclusions

Preprocessing and post processing of a medical image is not a straightforward process. Particularly for CT scan in COVID-19 patients, the development of an artificial based assistance tool for medical decision making requires not only a deep knowledge about the pathology but also a formal statement of the mathematical problem to a learning machine. Due to the lack of data, a joint strategy was developed: a medical formal study with three phases: feasibility, retrospective and prospective studies and a mathematical/engineering solution to reduce the complexity of the learning machines. Here we introduced the mentioned strategy and later on, focused on the Laguerre-Gauss kernel as a structures preprocessing tool that will reduce the complexity of the deep learning algorithms due to its capability to remove artifacts and enhance the spatial domain characteristics. Compared with similar techniques, the proposed kernel will allow better interpretation of the CT scans and easy computability of the algorithms. Future work will be also reported very often, providing advances on the topic. Promising results on the influence of this kernel over the learning machines can be evidenced with our feasibility dataset and will be tested on real colombian data. Our proposal can used in other kinds of modelling approaches based on artificial vision of classification, segmentation and localization.

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