

# Fuzzy Approaches in Failure Mode and Effect Analysis

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## ABSTRACT

*Nowadays, global and local safety and security are gaining increased importance, in terms of product and process safety as well. The most important and widespread method for risk analysis are Failure Mode and Effect Analysis (FMEA), and its predecessor Failure Mode Effects and Criticality Analysis (FMECA), which are used for a wide range of purposes. However, traditional FMEA and FMECA have shortcomings as well, which need to be avoided to gain a complex, correct analysis. In our work, our goal is to give an overview about the non-conventional Failure Mode and Effect methods, which are related to fuzzy logic. Three wide fields are taken into consideration in our study: Multi-Criteria Decision Making methods, Mathematical Programming and Artificial Intelligence approaches. These approaches tend to solve the major shortcomings of FMEA in handling risk analysis. In our work, we sort out the advantages and possible disadvantages of the mentioned analysis types.*

**Keywords:** Failure Mode and Effect Analysis, Multiple Criteria Decision Making, Mathematical programming, Rule-based system

**Mathematics Subject Classification:** 03B52, 62P30

**Computing Classification System:** Applied computing - Physical sciences and engineering - Engineering

## 1 INTRODUCTION

The aim our work is to identify conventional and non-conventional Failure Mode and Effect Analysis methods and to describe the development of the non-conventional types. In the following, we would like to focus on the description of different types (due to purpose, structure and methodological changes) of Failure Mode and Effect Analysis.

Failure Mode and Effect Analysis (FMEA) has approximately 70 years of history since it was invented in the 1940's (Spreafico, C., et al, 2017), (AIAG VDA FMEA Handbook, 2019). At first, the US military developed the method, which was later improved by NASA. The first written process description was MIL-P-1629, a military standard in 1949 (Stamatis, D. H., 2003). During the second half of the 20th century FMEA gained importance in design and process analysis as well.

Although FMEA has become a very popular and widely applied technique it has some shortcomings that led to the development of different non-conventional versions of the original method as well. In this paper, we give an overview and classification of those FMEA types that are related to fuzzy logic. The rest of this paper is organized as follows.

Section 2 describes the conventional FMEA, its types and barriers. In contrast to this, in Section 3 we introduce the main non-conventional FMEA types: FMEA based on Multi-Criteria Decision Method; on Mathematical Programming approaches; on Artificial Intelligence solutions and Integrated approaches.

## 2 CONVENTIONAL FAILURE MODE AND EFFECT ANALYSIS

### 2.1 Basic FMEA types according their purpose

The aim of the Failure Mode and Effect Analysis is to quantify the failure modes of a given system, product or process. FMEA has four basic types, which are the following: System FMEA, Product (Design) FMEA, Process FMEA and Service FMEA (Stamatis, D. H., 2003). System FMEAs are often considered as general analyses, as they obviously cannot contain all sub-FMEAs. Product FMEAs focus on the product itself, divided into parts, which depend on the complexity of the products. Process FMEAs in general are production related. They focus on the production process itself, and they are the basic quality management tools of manufacturers.

The main advantage of FMEA usage is that in case of an individual product a properly conducted FMEA chain (Product-, Design-, Process-FMEA) the failure effects and causes are linked to each other. In the end, this results in a complex analysis of even the most insignificant failure, with links to the effects on system level as well. According to Stamatis (Stamatis, D. H., 2003) System-, Design- and Process FMEAs are linked through failure cause-failure mode connections, as it is shown in Figure 1. According to Stamatis the failure cause of the system analysis is linked to the design failure. This way, the design cause is related to the process failure mode failures (Stamatis, D. H., 2003).

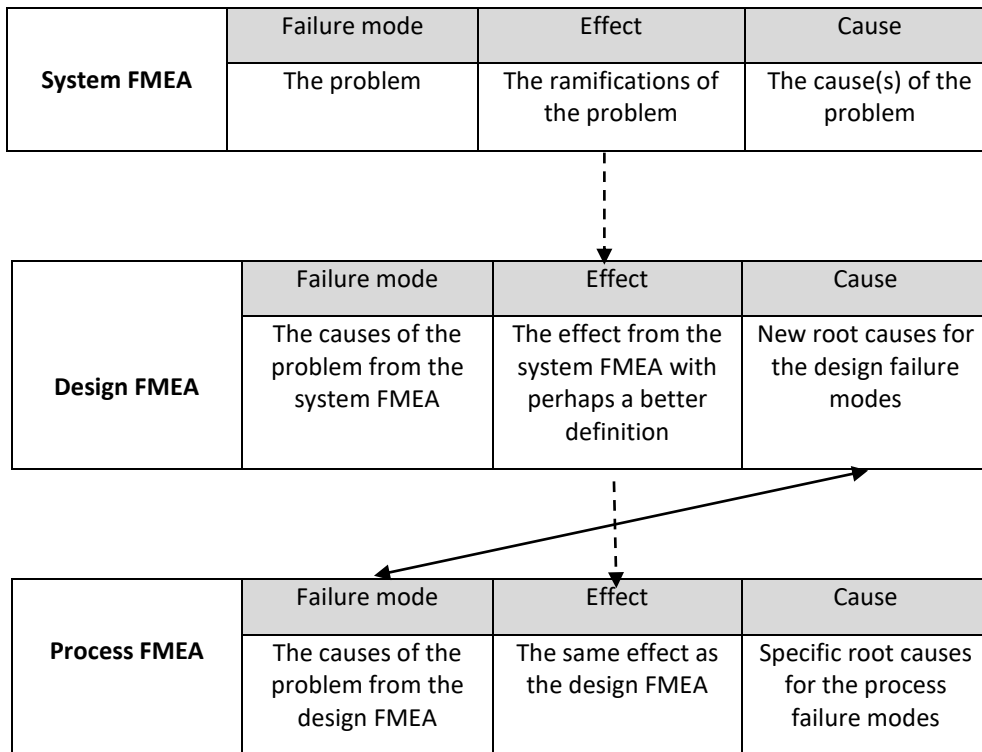
### 2.2 FMEA ratings

The failure modes are ranked according their *Risk Priority Number* (RPN) (Stamatis, D. H., 2003), that is calculated by

$$RPN = S \cdot O \cdot D \quad (1)$$

where, *S* denotes Severity, *O* symbolizes Occurrence, and *D* stands for Detection. Severity measures the seriousness of the failure effect, while occurrence and detection ratings are related to the failure cause or the failure mode. Each factor is rated from 1 to 10 (or from 1 to 5). If all factors are rated with a maximum value of 10, the *RPN* is 1000.

Proper ratings are the basis of a precise FMEA. Therefore, a common rating catalogue is necessary for a consequent evaluation. Rating catalogues give a common understanding for the FMEA team when it comes to failure evaluation. In the following (Tables 1-3) a widely used FMEA catalogue is described for the three different risk criteria (Chang, K.H., et al., 2010) (Liu, H.C. et al., 2013).



**Figure 1.** Connections between System-, Design- and Process FMEAs (Stamatis, D. H., 2003)

*Table 1: Rating catalogue for Severity criteria (Chang, K.H. et al., 2010).*

Effect	Criteria: severity of effect	Rank
Hazardous	Failure is hazardous and occurs without warning. It suspends operation of the system and/or involves noncompliance with government regulations.	10
Serious	Failure involves hazardous outcomes and/or noncompliance with government regulations or standards.	9
Extreme	Product is inoperable with loss of primary function. The system is inoperable.	8
Major	Product performance is severely affected but functions. The system may not operate.	7
Significant	Product performance is degraded. Comfort or convince functions may not operate	6
Moderate	Moderate effect on product performance. The product requires repair.	5
Low	Small effect on product performance. The product does not require repair.	4
Minor	Minor effect on product or system performance	3
Very minor	Very minor effect on product or system performance.	2
None	No effect	1

*Table 2: Rating catalogue for Occurrence criteria (Chang, K.H. et al., 2010).*

Effect	Criteria: occurrence of failure cause	Rank
Extremely high: failure almost inevitable	$\geq 1$ in 2	10
Very high	1 in 3	9
Repeated failures	1 in 8	8
High	1 in 20	7
Moderately high	1 in 80	6
Moderate	1 in 400	5
Relatively low	1 in 2000	4
Low	1 in 15000	3
Remote	1 in 150000	2
Nearly impossible	$\leq 1$ in 1500000	1

Table 3: Rating catalogue for Occurrence criteria (Liu, H.C. et al., 2013).

Detection	Criteria: likelihood of detection by design control	Rank
Absolute uncertainty	Design control does not detect a potential cause of failure or subsequent failure mode; or there is no design control	10
Very remote	Very remote chance the design control will detect a potential cause of failure or subsequent failure mode	9
Remote	Remote chance the design control will detect a potential cause of failure or subsequent failure mode	8
Very low	Very low chance the design control will detect a potential cause of failure or subsequent failure mode	7
Low	Low chance the design control will detect a potential cause of failure or subsequent failure mode	6
Moderate	Moderate chance the design control will detect a potential cause of failure or subsequent failure mode	5
Moderately high	Moderately high chance the design control will detect a potential cause of failure or subsequent failure mode	4
High	High chance the design control will detect a potential cause of failure or subsequent failure mode	3
Very high	Very high chance the design control will detect a potential cause of failure or subsequent failure mode	2
Almost certain	Design control will almost certainly detect a potential cause of failure or subsequent failure mode	1

The risk assessment process is described in Figure 2.

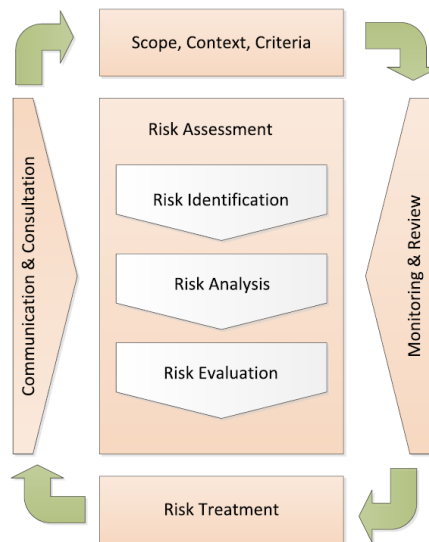


Figure 2. Description of Risk Assessment process (ISO/IEC 31010:2009)

### 2.3 Shortcomings of FMEA

FMEA is a traditional method for risk analysis, which takes the aforementioned three factors into consideration during the analysis. Equation (1) is a simple multiplication of these factors, which is often criticized by researchers (e.g. Liu, H.C. et al, 2013). In the following, the main shortcomings are listed and described.

If the relative importance of *S*, *O*, *D* factors are considered equal, it might cause that some combination of them results in lower *RPN*, but higher risk (Chang, K.H., et al, 2010).

For example:

$$RPN_1 = 8 \cdot 4 \cdot 3 = 96$$

$$RPN_2 = 3 \cdot 4 \cdot 9 = 108$$

In this case *Severity* is 8 (hazardous effect), *Occurrence* is 4 (relatively low rate of occurrence) and *Detection* is 3 (high detection). This  $RPN_1$  value is lower than the result of the following risk analysis.

The second case results in  $RPN_2$ , which is a multiplication of *Severity* 3 (minor severity), *Occurrence* 4 (relatively low rate of occurrence) and *Detection* 9 (very remoted detection) (Chang, K.H. et al, 2010). Having  $RPN_1$  lower than  $RPN_2$  means that the seriousness of the failure is not consequent. The same problem occurs if different combinations of *O*, *S* and *D* may produce the same *RPN* value Liu, H.C. et al, 2013).

Concerning the rating catalogues, the following issues might occur: the three risk factors are difficult to be precisely evaluated; the conversion of scores is different for the three risk factors; the *RPN* cannot be used to measure the effectiveness of corrective actions and *RPNs* are not continuous with many holes (Liu, H.C. et al, 2013).

The method itself has the following shortcomings: the value of *RPN* might be the same, but their hidden risk implications may be totally different and the interdependencies among various failure modes and effects are not taken into consideration (Liu, H.C. et al, 2013).

The above-mentioned concerns have led to the development of several non-conventional FMEA variants that are presented and described in the following sections.

### 3 NON-CONVENTIONAL FAILURE MODE AND EFFECT ANALYSIS TYPES

In terms of FMEA, there are multiple non-conventional approaches. According to Hu-Chen Liu et al (Liu, H.C. et al, 2013) the following sub-groups can be identified: Multiple Criteria Decision Making applications, Mathematical Programming methods, Artificial Intelligence applications, Integrated approaches and Other (mixed) approaches. In our work, we focus on MCDM applications, Mathematical programming approaches and Artificial Intelligence solutions.

#### 3.1 Multiple Criteria Decision Making applications

According to Massam (Massam, B.H., 1988) Multiple Criteria Decision Making applications (MCDM) are related to several decision making applications, as the following: Multi-Attribute Decision Making (MADM), Multi-Attribute Utility Theory (MAUT), Multi-Objective Decision Making (MODM) and Public Choice Theory (PCT).

They can be used for planning processes, if multiple decision alternatives are applicable (Massam, B.H., 1988), or at FMEA processes if multiple choices are applicable for each factor categories. MADM is applied if there are finite feasible sets of alternatives and the aim is to choose the best solution, in case of planning problems.

MCDM is used if the objective is to define finite number of possible alternatives for a given problem (the problem is typically solved with mathematical programming). MADM and MODM are applied in case of single decision makers or unified opinions (Massam, B.H., 1988).

In case of MAUT approaches the task is to evaluate the utilities of the given alternatives. As a result, the highest utility value is considered as the best possibility (in planning processes) (Massam, B.H., 1988). PCT is applied if consensus is needed in a certain decision situation, as well in a case of a risk category selection.

In general, it can be stated that the MCDM method consists of three areas, which were previously isolated. These are the following: Solution generation via search, Solution selection via preference aggregation and trade-off, and Interactive visualization (Massam, B.H., 1988).

According to the tree fields mentioned, the MCDM methods cover these main solutions of planning problems: well-distributed Pareto sets (Solution generation via search), Bayesian and Fuzzy decision-making techniques (Solution selection via preference aggregation and trade-off, and Interactive visualization) (Massam, B.H., 1988).

Table 4: Example of Fuzzy MCDM related applications used for FMEA and other approaches (Liu, H.C, et al, 2013), (Boral, S., et al, 2020).

Method	Author/year	Practical approaches/Practical FMEA applications
<b>Fuzzy ME-MCDM</b>	Franceschini and Galetto, 2001	risk analysis/Several design and manufacturing purposes
<b>Fuzzy evidence theory</b>	Guo et al.,2007 Li and Liao,2007 Wang et al., 2006 Xu et al., 2006 Yang et al.,2006	comparison of technical products (cars) corporate risk analysis environmental impact assessment personal performance assessment car ranking
<b>Fuzzy AHP/ANP</b>	Hu et al., 2009  Boral et al.,2009	component risk analysis / Fuzzy FMEA of components manufacturing risk analysis / Fuzzy Process FMEA
Fuzzy TOPSIS	Boran et al.,2009 Taylan et al.,2015 Dagdeviren et al.,2009 Braglia et al.,2003	supplier selection (automotive, etc.) risk assessment of construction projects  weapon selection production risk analysis / Fuzzy Production FMEA
<b>Fuzzy Grey theory</b>	Zhou and Thai, 2016  Shi and Fei,2019  Geum et al.,2011	failure analysis / Fuzzy FMEA for tanker equipment failure prediction  failure analysis / Combined Fuzzy FMEA method for medical service process failure analysis / Service specific Fuzzy FMEA (hospital service)
Fuzzy DEMATEL	Seyed et al. ,2006  Govindan and Chaudhuri,2016	failure analysis / Product specific Fuzzy FMEA (turbocharger product FMEA)  risk analysis of third-party logistics service
VIKOR	Liu et al.,2012  Metel et al, 2019	failure analysis / Fuzzy FMEA for medical processes occupational risk assessment of a natural gas pipeline construction
COPRAS	Roobahani et al.,2020	water transfer planning
SWARA/COPRAS	Zarbakhshnia et al., 2018	risk analysis of third-party logistics service

Method	Author/year	Practical approaches/Practical FMEA applications
ELECTRE(-TRI)	Certa et al., 2017 Liu and Ming (2019)	Fuzzy FMEA / Alternative failure mode classification Fuzzy FMECA / Fuzzy FMECA for smart product service
MULTIMOORA	Liu et al. (2014)	Evaluation of failure modes / Fuzzy MULTIMOORA FMEA

### 3.1.1 Integrated FMEA and FAHP (Fuzzy Analytic Hierarchy Process) for risk analysis

The integrated FAHP (Fuzzy Analytic Hierarchy Process) method is derived from the AHP (Analytic Hierarchy Process). AHP is a tool for determining the priority and relative importance of alternatives in a MCDM situation (Hu, A.H. et al, 2009). AHP was first introduced by Saaty (Saaty, 1980). In the following the integrated FAHP method will be introduced, which is closely linked to the traditional AHP method. According to Hu et al. integrated FMEA and FAHP is an effective method for risk analysis. Their proposed solution corrects the disabilities of the traditional AHOP method, which handles uncertainty and imprecision of decision makers less effective. In their study they have analysed the risk of green components and hazardous materials. According to Hu et al.'s approach the integrated method consists of three sub-processes: definition of criteria and risk assessment with FMEA, definition of relative importance of factors, utilization of integrated approach (Hu, A.H., et al, 2009).

The outcome equation of Hu et al.'s approach is

$$RPN = W_{(O_1)} \times S_{(O_1)} + W_{(D_1)} \times S_{(D_1)} + W_{(S_1)} \times S_{(S_1)} + W_{(S_2)} \times S_{(S_2)} \quad (2)$$

where  $W$  is the weight of criteria of  $RPN$ , and  $S$  is the score of criteria of  $RPN$ .

### 3.1.2 Franceschini and Galetto's Fuzzy ME-MCDM method

Bellman and Zadeh (Bellman and Zadeh, 1970) introduced fuzzy sets within MCDM, which resulted later in the establishment of FCDM (Fuzzy Multicriteria Decision Making). Due to the usage of linguistic variables FN (Fuzzy Numbers) are implemented. FN can be either Gaussian, trapezoidal or triangular (Franceschini, F. and Galetto, M., 2001).

$$RPC(a_i) = \text{Min}_j [\text{Max} \{ \text{Neg}(I(g_j)), g_j(a_i) \} ], \quad (3)$$

Where:

$RPC(a_i)$  : Risk Priority Code for the failure mode  $a_i$

$I(g_j)$  : the importance associated with each criteria  $g_j$ ;  $g_j$  is the evaluation criteria (S, O, D factors),  $j=1, \dots, n$

$\text{Neg}(I(g_j))$  : the negation of the importance assigned to each decision-making criterion.

With the usage of fuzzy MCDM FMEA method the failure mode with the maximum risk priority code is defined as follows (Franceschini, F. and Galetto, M., 2001):

$$RPC(a^*) = \text{Max}_{a_i \in A} \{RPC(a_i)\}, \tag{4}$$

where  $a$  is the set of failure modes,  $RPC(a_i)$  is defined on a new 10-point ordinal scale as those values utilized for expressing index evaluations.

With the usage of Franceschini and Galetto's method a different level of importance of  $S, O, D$  factors can be defined as follows (Franceschini, F. and Galetto, M., 2001)

$$RPN(a^*) = \text{Max}_{a_i \in A} [RPC(a_1), RPC(a_2), RPC(a_3), \dots, RPC(a_n)] \tag{5}$$

The most important advantage of this method is that different importance levels can be given to each FMEA factors (Severity, Occurrence, Detection). This is important in terms of the FMEA's purpose as well. In case of Design FMEA (Product FMEA) the severity values can have more importance, whilst in case of Process FMEA, the same applies for the Occurrence factor.

$$\overline{(ERS)}(FM_n) = \frac{1}{2} (ERS^L(FM_n) + ERS^U(FM_n)), n = 1, \dots, NERS(FM_n) \tag{6}$$

### 3.1.3 Grey theory used for Fuzzy FMEA

The Fuzzy FMEA based on grey theory proposed by Zhou and Thai (Zhou and Thai, 2016) is based on the assumption that with the fuzzification each risk criteria can be weighted (in contrast to the traditional method).

In the following, linguistic terms for each risk criteria are mentioned. In Table 5, the linguistic terms of Occurrence are presented. In this proposed example 5 levels are mentioned, i.e.  $VH$  (Very High),  $H$  (High),  $M$  (Moderate),  $L$  (Low), and  $R$  (Remote) (Zhou and Thai, 2016):

Table 5: Linguistic terms of Occurrence (factor O) (Zhou and Thai, 2016).

Rating	Probability of occurrence	Fuzzy number
Very high (VH)	Failure is almost inevitable	(8, 9, 10, 10)
High (H)	Repeated failures	(6, 7, 8, 9)
Moderate (M)	Occasional failures	(3, 4, 6, 7)
Low (L)	Relatively few failures	(1, 2, 3, 4)
Remote (R)	Failure is unlikely	(1, 1, 1, 2)

In Table 6, the linguistic terms of Severity are presented. 10 different levels are differentiated in this example ( $HWOW, HWW, VH, H, M, L, VL, MR, VMR, N$ ):

Table 6: Linguistic terms of Severity (factor S) (Zhou and Thai, 2016).

Rating	Severity of occurrence	Fuzzy number
Hazardous without warning (HWOW)	Very high severity ranking without warning	(9, 10, 10)
Hazardous with warning (HWW)	Very high severity ranking with warning	(8, 9, 10)
Very high (VH)	System inoperable with destructive failure	(7, 8, 9)
High (H)	System inoperable with equipment damage	(6, 7, 8)
Moderate (M)	System inoperable with minor damage	(5, 6, 7)



Rating	Severity of occurrence	Fuzzy number
Low (L)	System inoperable without damage	(4, 5,6)
Very low (VL)	System operable with significant degradation of performance	(3, 4,5)
Minor (MR)	System operable with some degradation of performance	(2, 3,4)
Very minor (VMR)	System operable with minimal interference	(1, 2,3)
None (N)	No effect	(1, 1,2)

In Table 7, the linguistic terms of Detection are defined (AU, VR, R, VL, L, M, MH, H, VH, AC).

Table 7: Linguistic terms of Detection (factor D) (Zhou and Thai, 2016).

Rating	Severity of effect	Fuzzy number
Absolute uncertain (AU)	No chance	(9,10,10)
Very remote (VR)	Very remote chance	(8, 9,10)
Remote (R)	Remote chance	(7, 8, 9)
Very low (VL)	Very low chance	(6, 7, 8)
Low (L)	Low chance	(5, 6, 7)
Moderate (M)	Moderate chance	(4, 5, 6)
Moderately high (MH)	Moderately high chance	(3, 4, 5)
High (H)	High chance	(2, 3, 4)
Very high (VH)	Very high chance	(1, 2, 3)
Almost certain (AC)	Almost certainty	(1, 1, 2)

Finally, the S, O, D factors are de-fuzzified according to their membership functions:

$$K(x) = \sum_{i=0}^n (b_i - c) / \left[ \sum_{i=0}^n (b_i - c) - \sum_{i=0}^n (a_i - d) \right] \quad (7)$$

where  $K(x)$  is the defuzzified crisp number,  $n$  is the number of alpha levels. In case of the grey coefficient calculation, there is a correlation measure between  $x_i, y_i$ .

$$\text{For the set } X = \{x_i \mid i \in I, i = 0, 1, 2, \dots, m\}, x_{i,s}, y_i \in X \gamma(x_i, y_i) \quad (8)$$

where  $\Delta_0j(k)$  is the absolute difference between  $x_0(k)$  and  $x_j(k)$ ,  $x_0$  contains standard series,  $x_i$  contains comparative series. In this case, according to the above mentioned definitions, the grey coefficient is calculated as follows:

$$\gamma(x_0(k), x_i(k)) = [x(\min) + \zeta x(\max)] / [\Delta_0i(k) + \zeta x(\max)] \quad (9)$$

Where:

$$x(\min) = \min_i \min_k \Delta_{0i}(k) \quad (10)$$

$$x(\max) = \max_i \max_k \Delta_{0i}(k) \quad (11)$$

According to the principle of minimum  $\zeta \in [0, 1]$  is generally  $\zeta = 0.5$ . The degree of relation is defined as the value of grey relation coefficient:

$$\Delta_{0i}(k) = x_0(k) - x_i(k) \quad (12)$$

$$\gamma(x_0(k), x_i(k)), k = 1, 2, \dots, n \tag{13}$$

$$\gamma(x_0, x_i) = \frac{1}{n} \sum_{k=1}^n \gamma(x_0(k), x_i(k)) \tag{14}$$

As a conclusion, the following equation stands for the FMEA calculation:

$$\gamma(x_0, x_i) = \omega_0 \cdot \gamma(x_0(O), x_i(O)) + \omega_s \cdot \gamma(x_0(S), x_i(S)) + \omega_D \cdot \gamma(x_0(D), x_i(D)) \tag{15}$$

Finally, Zhou and Thai propose a joint method of fuzzy and grey theory. Their method is separated into three main parts: the establishment of fuzzy rules, determination of linguistic terms and fuzzy membership function; the calculation of FRPN (Fuzzy RPN) by weighted geometric mean method and the defuzzification of S, O, D for obtaining a crisp number (Zhou and Thai, 2016).

The advantage of the joint method is that the advantage of grey theory usage can be applied as well. Grey theory reflects on the nature of relative ranking, which is fortunate, if the evaluation information is not reliable, or incomplete (Zhou and Thai, 2016).

### 3.2 Mathematical programming applications

Mathematical programming applications are relevant parts of the non-conventional FMEA methodology. There are three main of the mentioned applications, summarized in *Table 8*: Fuzzy RPN method, Fuzzy DEA FMEA and Fuzzy Interval DEA FMEA. Fuzzy RPN method is used in cases of process and product level risk analyses, fuzzy DEA FMEA is used mainly for specific purposes (nuclear system risk analysis), as fuzzy interval DEA FMEA (system FMEA for fishing vessel construction).

*Table 8*: Applications of Fuzzy Mathematical programming related to FMEA (Liu, H.C, et al., 2013), (Boral, S. et al, 2020).

Method	Author/year	Practical approaches/Practical FMEA applications
Fuzzy RPN	Wang et al. (2006), Gargama and Chaturvedi (2011), Chen and Ko (2009)	wide usage for both Design-, and Process FMEA
Fuzzy DEA FMEA	Garcia et al.(2013)	example of nuclear system risk analysis
Fuzzy Interval DEA FMEA	Chin et al (2009)	example of System FMEA for fishing vessel

In the following (3.2.1, 3.2.2 and 3.2.3) we introduce the above mentioned methods in detail.

#### 3.2.1 Usage of fuzzy risk priority numbers (FRPNs)

According to Wang et al. (Wang et al, 2006) Risk Priority Numbers (RPNs) can be fuzzified and considered as FRPNs (Fuzzy Risk Priority Numbers). *FRPNs* are calculated as fuzzy weighted geometric means of Severity (S), Occurrence (O), and Detection (D) ratings. FRPNs can be defined with  $\alpha$ -level sets and with linear programming. Defuzzification is done with centroid defuzzification method (Liu, H.C, et al, 2013).

Gargama and Chaturvedi (Gargama and Chaturvedi, 2011) calculated FRPNs as well, but with using benchmark adjustment instead of linear programming (Liu, H.C, et al, 2013),

Chen and Ko's (Chen and Ko, 2007) approach is a FRPN definition which is based on fuzzy ordered weighted geometric averaging of  $S, O, D$  factors (Liu, H.C, et al, 2013). They have defined fuzzy FMEA as the following:

$$(R\tilde{P}N)_j = \max(\tilde{S}_r \times \tilde{O}_s \times \tilde{D}_t)_j, \quad j = 1, 2, \dots, J, \quad (16)$$

where  $\tilde{S}_r, \tilde{O}_s, \tilde{D}_t$  are fuzzy subsets [0, 1]. Chen and Ko (Chen and Ko, 2007) introduced a FOWGA (fuzzy ordered weighted geometric averaging) operator. The FOWGA operator is used to aggregate  $m$  ( $>1$ ) fuzzy sets.

$$f(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_m) = \prod_{i=1}^m (\tilde{b}_i)^{w_i} \quad (17)$$

where  $\tilde{b}_i$  is the  $i$ th largest set of the  $(\tilde{S}, \tilde{O}, \tilde{D})$ ,  $w_i$  is the weight of the  $\tilde{b}_i$  and  $\sum_{i=1}^m w_i = 1, w_i \in [0,1]$  FOWGA can be formulated as the following:

$$(R\tilde{P}N)_j = f(\tilde{S}, \tilde{O}, \tilde{D})_j = \max_{r,s,t} \prod_{i=1}^3 (b_i)_j^{w_i} \quad (18)$$

where  $w$  is the weighting vector,  $w = (w_1, w_2, w_3)^T$ .  $RPN$  is defined with its membership function. The membership function is defined by deriving the lower and upper bounds of the  $\alpha$ -cuts of  $(RPN)_j$ :

$$(RPN_j)_\alpha^L = \max_{r,s,t} \prod_{i=1}^3 [(b_i)_j^{w_i}]_\alpha^L \quad (19)$$

$$[(RPN_j)_\alpha^U] = \max_{r,s,t} \prod_{i=1}^3 [(b_i)_j^{w_i}]_\alpha^U \quad (20)$$

After defining the membership function, the defuzzification is the following:

$$(RPN_j)' = \frac{\sum_i i \left\{ \frac{1}{2[(RPN_j)_\alpha^L + (RPN_j)_\alpha^U]} \right\} \cdot \overline{\mu_i'}}{\sum_i i \overline{\mu_i'}} \quad (21)$$

Where  $\mu_i'$  is the membership degree of  $\frac{1}{2}[(RPN_j)_\alpha^L + (RPN_j)_\alpha^U]$

$$(\tilde{R}i_k)_\alpha = [(Ri_k)_\alpha^L, (Ri_k)_\alpha^U] = \left[ \sum_{j=1}^J (RPN_j)' \cdot m(R'_{2,jk})_\alpha^L, \sum_{j=1}^J (RPN_j)' \cdot m(R'_{2,jk})_\alpha^L \right] \quad (22)$$

The advantages of the method are that different combinations of Severity, Occurrence and Detection factors result in different FRPNs (unless the relative weights used are the same), and more risk factors can be used during the analysis (Chen and Ko, 2007).

### 3.2.2 Garcia et. al's fuzzy DEA FMEA

According to Garcia et al. (Garcia, P. et al, 2013) Risk Analysis evaluations are carried as a part of the Probabilistic Safety Analysis (PSA). In their research they have pointed out that that the different combinations of S, O, D factors produce the same value.

As Garcia et al. states (Garcia, P. et al, 2013), that this shortcoming can be solved with the modelling of RPN factors (Severity, Occurrence, Detection) as fuzzy sets. In case of this method, Occurrence and Detection factors are considered to have equal importance and Severity is considered to have more importance than O and D.

$$\text{Max } h_0 = \sum_{j=1}^s u_j y_{j0} \quad (23)$$

$$\sum_{i=1}^r v_i x_{i0} = 1 \quad (24)$$

$$\sum_{j=1}^s u_j y_{j0} - \sum_{i=1}^r v_i x_{ik} \leq 0, \forall k \quad (25)$$

$$v_s - (v_o + v_D) \geq 0 \quad (26)$$

$$u_j, v_i \geq \varepsilon \forall i, j \quad (27)$$

where  $\varepsilon$  is a non-Archimedean figure, which should be as small as possible.  $\varepsilon$  should be defined as a number different from 0, if  $\varepsilon=0$  the model defines one or more factors as not important.

Regarding the disadvantage of the method, according to Chin et al. (Chin, K.S, et al., 2009) Garcia et al.'s method is needs to be corrected, as it does not provide a complete evaluation for the failure modes. Due to Chin et al.'s approach the relative importance weights are taken into consideration, without subjective specification (Liu, H.C. et al., 2013).

### 3.2.3 Chin et. al's fuzzy DEA FMEA

According to Chin et al.'s (Chin, K.S. et al, 2009) model there are  $n$  failure modes, which need to be prioritized. These failure modes are evaluated with the selected  $m$  risk factors. Despite the traditional FMEA method (which equally considers Severity, Occurrence, Detection factors), in this case RPN is calculated as follows (Chin, K.S. et al, 2009):

$$R_i = \sum_{j=1}^m w_j r_{ij}, i = 1, \dots, n, \text{ which defines additive risks} \quad (28)$$

$$R_i = \prod_{j=1}^m r_{ij}^{w_j}, i = 1, \dots, n, \text{ which defines multiplicative risks} \quad (29)$$

If the maximum value of importance ratio is considered as 9, the ratio of maximum weight to minimum weight is defined between the range of 1 and 9.

$$1 \leq \frac{\max\{w_1, \dots, w_m\}}{\min\{w_1, \dots, w_m\}} \leq 9 \quad (30)$$

Chin et al (Chin, K.S. et al, 2009) define Occurrence and Detection ratings on a scale of 1 to 10, whilst Severity is defined on a scale from 1 to 9 (as no importance has no point in this case).

$$\max\left\{\frac{w_j}{w_k} \mid j, k = 1, \dots, m; k \neq j\right\} \leq 9 \quad (31)$$

$$w_j - 9w_k \leq 0, j, k = 1, \dots, m; k \neq j \quad (32)$$

According to the before mentioned, FMEA DEA models are defined as the maximum and minimum risks of each failure mode (additive failure modes), according to the following (Chin, K.S. et al, 2009):

$$R_0^{\max} = \text{Maximize } R_0 \quad (33)$$

Subject to:

$$(R_i \leq 1, \quad i = 1, \dots, n,) \quad (34)$$

$$w_j - 9w_k \leq 0, \quad j, k = 1, \dots, m; k \neq j \quad (35)$$

$$R_0^{\min} = \text{Minimize } R_0 \quad (36)$$

Subject to:

$$R_i \geq 1, \quad i = 1, \dots, n, \quad (38)$$

$$w_j - 9w_k \leq 0, \quad j, k = 1, \dots, m; k \neq j \quad (37)$$

The sum risk of each failure is defined with the following equation, which gives the geometric average of the maximum and minimum risk (Chin, K.S et al., 2009):

$$\bar{R}_i = \sqrt{(R_i^{\max} \cdot R_i^{\min})}, i = 1, \dots, n \quad (38)$$

In case of defining multiple failure modes, the same equation can be used, but transformed to a logarithmic scale:

$$\ln R_0^{\max} = \text{Maximize } \ln R_0 \quad (39)$$

Subject to:

$$\ln R_i \leq 1, \quad i = 1, \dots, n, \quad (40)$$

$$w_j - 9w_k \leq 0, \quad j, k = 1, \dots, m; k \neq j \quad (41)$$

$$\ln R_0^{\min} = \text{Minimize } \ln R_0 \quad (42)$$

Subject to:

$$\ln R_i \geq 1, \quad i = 1, \dots, n, \quad (43)$$

$$w_j - 9w_k \leq 0, \quad j, k = 1, \dots, m; k \neq j \quad (44)$$

The geometric average risk is defined with exponential function:

$$R_i = \sqrt{(\text{EXP}(\ln R_i^{\max}) \cdot \text{EXP}(\ln R_i^{\min}))}, i = 1, \dots, n, \quad (45)$$

The advantages are like Wang et. al's approach (Wang et al, 2006), as more risk factors can be used during the analysis, and there is no need to use if-then rules.

### 3.2.4 Fuzzy Interval DEA FMEA

According to Chin et al. (Chin, K.S. et al, 2009) the idea of an interval DEA FMEA is based on the team approach of the team method of FMEA. If the incomplete evaluation is transformed to an expectation interval, the maximum, minimum and the average risks are stated as intervals as well (Chin, K.S. et al, 2009).

The geometric average risks are calculated as follows

$$\left[ \overline{R}_i^L, \overline{R}_i^U \right] = \left[ \sqrt{\text{EXP}(\ln(R_i^{\max})^L) \cdot \text{EXP}(\ln(R_i^{\min})^L)}, \sqrt{\text{EXP}(\ln(R_i^{\max})^U) \cdot \text{EXP}(\ln(R_i^{\min})^U)} \right], i = 1, \dots, n \quad (46)$$

This method is related to the minimax regret approach (MRA), implemented by Wang et al (Wang et al, 2006) MRA uses the maximum regret value (MRV) for comparing and ranking of interval numbers:

$$R(u_i) = \max[\max_{j \neq i} (u_j^U) - u_i^L, 0], i = 1, \dots, N \quad (47)$$

### 3.3 Artificial intelligence approaches related to FMEA

In the following we would like to give a summary of the artificial intelligence approaches related to FMEA, according to Hu-Chen Liu et al (Liu, H.C. et al 2013). Based on the grouping of Hu-Chen Liu et al (Liu, H.C. et al 2013), there are four major groups of FMEA related solutions. These are the following: rule-base system (Sankar and Prabhu, 2001) fuzzy rule-based system (Sharma ,R.K., Sharma,P,2010),

fuzzy ART (Adaptive Resonance Theory) algorithm (Keskin, G et al.,2010) and fuzzy cognitive map (Peláez, C.E. and Bowles,J.B,1996).

Table 9: Applications of artificial intelligence approaches related to FMEA.

Method	Author/year	Practical approaches/Practical FMEA applications
Rule base system	Sankar and Prabhu (2001)	Process FMEA example of off-shore cooling plant example
Fuzzy rule-base system	Sharma and Sharma (2010)	Process FMEA example for paper mill system
Fuzzy ART algorithm	Keskin, G.,(2010)	Process FMEA for testing purposes
Fuzzy cognitive map	Peláez and Bowles (1996), Gargama and Chaturvedi (2011)	Design FMEA for water tank levelling system

### 3.3.1 Rule base system for FMEA

According to Shankar and Prabhu (Sankar, N.R. and Prabhu, B.S, 2001) the rule-based system for FMEA is carried out according to the following steps:

- (1) Description of the part name, number, and function.
- (2) Listing the possible failure modes
- (3) Estimation of failure severity values
- (4) Listing the potential failure causes
- (5) Estimation of occurrence frequency of failures
- (6) Description of failure detection methods
- (7) Estimation of failure detection
- (8) Evaluate the RPR (Risk Priority Rank)
- (9) Recommendation of corrective actions

Step 8 is an addition to the traditional FMEA process with an implementation of a new risk prioritization scale. The suggested variable, RPR (Risk Priority Rank) can take up values from 1-1000, and is calculated with If-Then relations. In this case the rules are formulated in numerical form (Sankar, N.R. and Prabhu, B.S., 2001). With the usage of the rules, we receive the RPR value, which differs from the traditional RPN, which is the multiplication of the Severity, Occurrence and Detection factors. RPR indicates relative priority. For visualization purposes the outcome of the analysis is represented in an ordering matrix.

Table 10: Example of ordering matrix of a functional FMEA of a centrifugal pump

(Sankar, N.R. and Prabhu, B.S., 2001).

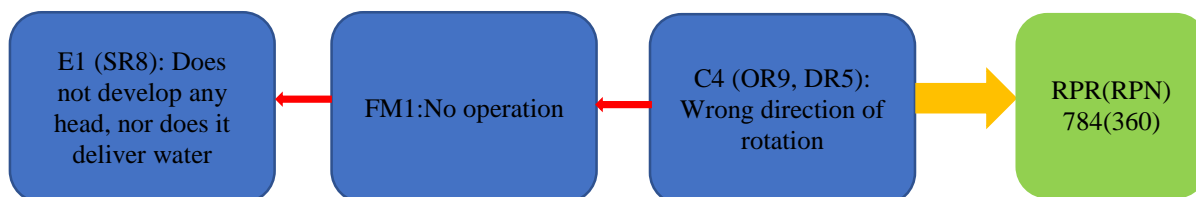
Causes	OR	DR	E1 SR8 FM1	E4 SR7 FM4	E2 SR6 FM2	E3 SR5 FM3	E5 SR5 FM5
C4	9	5	784(360)	0	0	0	0
C6	6	7	759(336)	0	0	0	0
C29	8	3	754(192)	0	0	0	0
C17	8	5	0	754(280)	0	0	0

Causes	OR	DR	E1 SR8 FM1	E4 SR7 FM4	E2 SR6 FM2	E3 SR5 FM3	E5 SR5 FM5
C27	7	8	0	739(392)	0	0	0
C14	7	5	0	732(245)	0	578(175)	0

In the ordering matrix the columns represent the following:

- Causes (Cx): the identified failure causes in the failure net,
- OR (Occurrence Rating): the value of failure cause occurrence (1-10),
- DR (Detection Rating): the value of failure cause detection (1-10),
- Ex (Effect): the identified failure cause effect,
- SRx (Severity): the identified failure effect severity,
- FMx (Failure Mode): the identified failure mode

The ordering matrix can be understood as follows: each failure cause (Cx) is related to an occurrence (OR), detection (DR) and severity (SR) value. The failure net consists of failure effects (Ex), and failure modes (FMx) as well, as in case of the traditional FMEA. RPN value is generated from the multiplication of S, O, D factors. The RPN values are placed in brackets. With the usage of the previously defined rules are placed besides the brackets. If there is no connection between a certain failure cause and a failure mode or failure effect, 0 is placed in the cell. This visual method helps to identify the potential problematic areas of a product or process (Sankar, N.R. and Prabhu, B.S, 2001). According to the before mentioned the first row of Table 10 can be illustrated in a net as well.



**Figure 3.** Example of failure net (Functional FMEA of rotation pump)  
(Sankar, N.R. and Prabhu, B.S, 2001)

The main advantage of this method is that it gives relative importance for each failure, which helps to improve the numerical shortcoming of traditional FMEA, visualization is surplus solution as well, as it gives a good overview of the process or design. The proper definition of rules is essential in this case, as it has major influence of the sequence of failure importance.

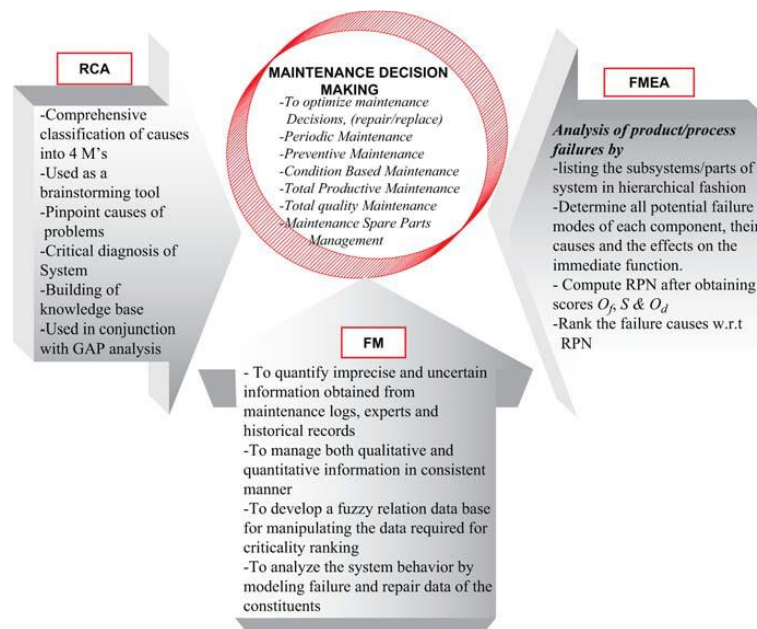
### 3.3.2 Fuzzy rule-base system

Rule-based systems are implemented in fuzzy FMEA methods as well. According to Sharma and Sharma (Sharma and Sharma, 2010), shown on Figure 4, fuzzy methodology (FM), root cause analysis (RCA) and FMEA can be merged in a common approach. RCA is tool for the comprehensive classification of cause into 4M's (4M stands for Machine, Method, Man and Material) (Sharma, R.K. and



Sharma, P, 2010). In this integrated approach, FMEA defines the input variables ( $O_f$ ,  $S$ ,  $O_d$ ) which form RPN.

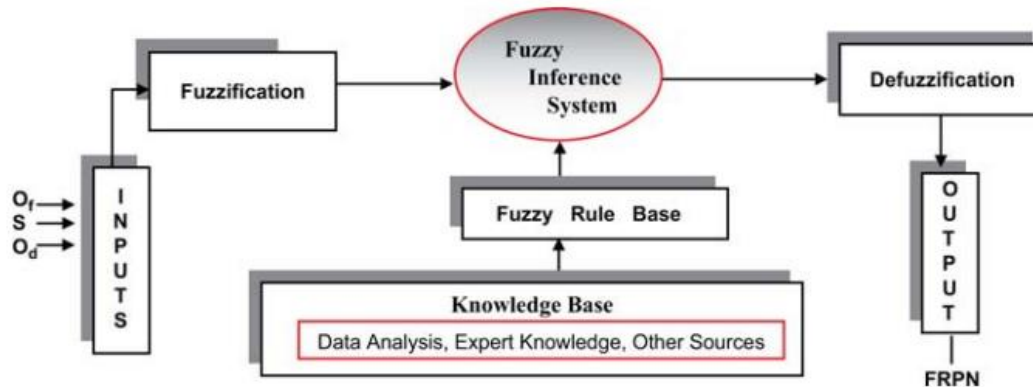
The third so-called tool is *FM*, which is responsible for the quantification of imprecise and uncertain information provided by the experts and their analysis (Sharma, R.K. and Sharma, P, 2010). In Sharma and Sharma's example (Sharma, R.K. and Sharma, P, 2010) maintenance decision making is aided with this merged approach.



**Figure 4.** Merged approach for maintenance decision making (Sharma, R.K. and Sharma, P, 2010)

As shown on Figure 5, the knowledge base is provided by data analysis and expert knowledge, which are evaluated with fuzzy rule-based analysis. The inputs of the integrated approach are  $O_f$  (Probability of occurrence of failure),  $S$  (severity) and  $O_d$  (likelihood of non-detection of failure) factors.  $O_f$  is determined as a function of mean time between failures,  $O_d$  is estimated (for example as 0.5 % in case of visual inspection of operator's),  $S$  is the numerical definition of failure effect on system performance. This way, the fuzzified factors are the inputs of the fuzzy interference systems, which results in FRPNs after defuzzification.

For the determination of the FRPN variable both triangular and trapezoidal membership functions were used (Sharma, R.K. and Sharma, P., 2010). In Sharma and Sharma's example (Sharma, R.K. and Sharma, P, 2010) five fuzzy sets were applied in case of each factor ( $O_f$ ,  $S$ ,  $O_d$ ) and a total of 125 rules were used. For the interference system Petrinet models are used.



**Figure 5.** Definition of fuzzy rule base system (Sharma, R.K. and Sharma, P., 2010)

The main advantage and disadvantage of this solution is related to the same root: information and data are gathered from three sub-systems, which makes the tool complex or even too complex for the analysts. Upon the whole, the usage of this method provides a more realistic overview of industrial systems (modelling, predictions and analysis) (Sharma, R.K. and Sharma, P., 2010).

#### 4 CONCLUSIONS

Failure Mode and Effect Analysis has almost 80 years of history to look back on. In the past, it was implemented to analyse complex problems (e.g. military related) or structures (e.g. aeronautical). As products became more complicated, industry (e.g. automotive, machinery industry) implemented FMEA as well. After decades of usage, it became known that the traditional method has shortcomings, which might cause non-conformities whilst usage. The non-conventional FMEA methods came to life to solve these issues.

In our work, we have summarized the main fuzzy logic related non-conventional FMEA solutions, which are the following: Multi-Criteria Decision Making related, Mathematical Programming related and Artificial Intelligence related approaches. MCDM approaches are effective in risk analysis, as well in terms of supplier selection.

Mathematical programming solution implement the term FRPN, and are basically used for system analysis, whilst Artificial intelligence solutions provide great visual aid (Rule-based solutions) for decision makers. According to our research, artificial intelligence solution might provide the widest aid for engineering problems, but the summarized methods need to be improved, as their complexity might slow down the analysis process of a given problem. Further research plans include investigation of the application possibilities of rule interpolation (Vincze, G. and Kovács, S., 2015), fuzzy control techniques (Guechi, E.H. et al, 2010; Precup, R.E. et al, 2013; Precup, R.E. and Preitl, S., 2003), fuzzy cognitive maps (Mls, K. et al, 2017), and fuzzy signatures (Bukovics et al, 2020).

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