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Stable Takagi-Sugeno Fuzzy Control Designed by Optimization

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ABSTRACT

This paper presents the design of Takagi-Sugeno fuzzy controllers in state feedback form using swarm intelligence optimization algorithms. Three such algorithms are used: Particle Swarm Optimization, Simulated Annealing and Gravitational Search Algorithms. Sufficient stability conditions are expressed in terms of linear matrix inequalities considered as constraints in the optimization problem solved by swarm intelligence algorithms. Simulation results concerning an inverted pendulum system are given for illustration of the proposed design.

Keywords: fuzzy control, Gravitational Search Algorithms, inverted pendulum system, Particle Swarm Optimization, Simulated Annealing, stability conditions.

Mathematics Subject Classification: 82C21, 93A30, 37N35, 34H10, 03E72, 34A34

Computing Classification System: I.2.3, I.2.8, I.2.9

1. INTRODUCTION

Stability is one of the most important problems in the analysis and design of nonlinear control systems. The stability issues related to fuzzy control systems have been considered extensively in the recent years. Such examples are presented in (Idrissi et al., 2013), (Lin and Chen, 2015), (Mellouli and

Boumhidi, 2016), (Precup et al., 2003), (Precup et al., 2009), (Škrjanc et al., 2002), (Tomescu et al., 2007).

The optimal tuning of the parameters of fuzzy controllers is a convenient approach to the model-based design of fuzzy controllers to meet systematic performance specifications. A good overview on swarm intelligence algorithms applied to controllers including Takagi-Sugeno and Mamdani fuzzy ones is given in (Precup et al., 2015a). Some of the latest swarm intelligence algorithms to tune the parameters of fuzzy controllers include genetic algorithms (Das et al., 2013), (Pérez et al., 2013), Simulated Annealing (SA) (Jain et al, 2011), Particle Swarm Optimization (PSO) (Oh et al., 2011), Gravitational Search Algorithms (GSAs) (Precup et al.. 2011), Ant Colony Optimization (Chang et al., 2012), (Lu and Liu, 2013), (Castillo et al., 2015), chemical optimization (Melin et al., 2013), Charged System Search (Precup et al., 2014), harmony search (Wang et al., 2013) or Grey Wolf Optimizers (Noshadi et al., 2016), (Precup et al., 2017a), (Precup et al., 2017b).

This paper proposes the combination of swarm intelligence algorithms and stability by the optimal tuning of the parameters of Takagi-Sugeno fuzzy controllers using stability conditions expressed as linear matrix inequalities (LMIs) to play the role of constraints in the optimization problems. Three swarm intelligence algorithms are considered: PSO, SA and GSA.

The rest of the paper is structured as follows: Section 2 is dedicated to the Takagi-Sugeno fuzzy control system models and stability analysis. The optimal tuning of fuzzy controllers is presented in Section 3. Section 4 offers a numerical example of an inverted pendulum on a cart to show the effectiveness of the results. The conclusions are highlighted in Section 5.

2. TAKAGI-SUGENO FUZZY CONTROL SYSTEM MODELS AND STABILITY ANALYSIS

The i^{th} rule of the fuzzy model for the nonlinear process with parametric uncertainties is

$$\begin{aligned} &\text{Rule}\, i: \ \text{IF}\, z_1(t)\, \text{IS}\, M_{ij} \ \text{AND...AND}\, z_p(t)\, \text{IS}\, M_{ip} \\ &\text{THEN}\begin{cases} \dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) \\ y(t) = C_i x(t) \end{cases} \end{aligned} \tag{1}$$

where M_{ij} is a fuzzy set (linguistic term), $x(t) \in R^n$ is the system state vector, $u(t) \in R^m$ is the control signal vector, $y(t) \in R^p$ is the (controlled) process output, A_i, B_i, C_i are known constant matrices that describe the nominal process, and $z_1(t), z_2(t),..., z_p(t)$ are the premise or scheduling variables, which belong to the input or scheduling vector $z(t) = [z_1(t)\ z_2(t) \ldots z_p(t)]^T$, $i = 1...r$ and r is the number of rules.

The matrices $\Delta A_i, \Delta B_i$ in (10) are parametric uncertainties of the process, and have the following bounded structure:

$$\begin{aligned}&[\Delta A_i(t), \Delta B_i(t)] = H_i F_i(t)[E_{1i}, E_{2i}], i = 1...r\\&F_i^T(t) F_i(t) \le I\end{aligned} \tag{2}$$

where H_i, E_{1i}, E_{2i} are known real constant matrices, $F_i(t)$ is an unknown matrix function with Lebesgue-measurable element, and I is the identity matrix.

The Takagi-Sugeno fuzzy model related to (1) is expressed using the following state equation:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} \mu_i(z(t))[(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)]}{\sum_{i=1}^{r} \mu_i(z(t))} \tag{3}$$

Using the notations:

$$\begin{aligned}&\mu_i = \prod_{i=1}^{n} M_{ij} z(t)\\&h_i = (z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{r} \mu_i(z(t))}\end{aligned} \tag{4}$$

the Takagi-Sugeno fuzzy model of the process with parametric uncertainties is:

$$\begin{cases}\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)]\\ y(t) = \sum_{i=1}^{r} h_i(z(t)) C_i x(t)\end{cases} \tag{5}$$

where $h_i(z(t)) \ge 0$, $i = 1...r$, and $\sum_{i=1}^{r} h_i(z(t)) > 0$.

The Takagi-Sugeno fuzzy controller in state feedback form has the classical expression specific to parallel distributed compensation:

$$\text{IF } z_1(t) \text{ IS } M_{ij} \text{ AND... AND } z_p(t) \text{ IS } M_{ip} \text{ THEN } u(t) = -k_i x(t) \tag{6}$$

where $k_i = [k_{i\omega\eta}]_{\omega=1...n, \eta=1...m} \in R^{n \times m}$ are the constant state feedback gain matrices, and they will be computed using swarm optimization algorithms.

The fuzzy controller model is:

$$u(t) = -\sum_{j=1}^{r} h_j(z(t)) k_j x(t) = -\sum_{j=1}^{r} h_j k_j x(t) \tag{7}$$

The combination of (5) and (6) leads to the state-space model of the Takagi-Sugeno fuzzy control system with parametric uncertainties:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \sum_{j=1}^{r} h_j(z(t))[(A_i + \Delta A_i) - (B_i + \Delta B_i)k_j]x(t) \\ y(t) = \sum_{i=1}^{r} h_i(z(t)) C_i x(t) \end{cases} \tag{8}$$

The main results on the stable design of the Takagi-Sugeno fuzzy control system with parametric uncertainties (of the process) are given by the following theorem:

Theorem 1. The Takagi-Sugeno fuzzy system modelled in (8) is globally asymptotically stable if there exists a symmetric and positive definite matrix P and some scalars β that fulfil the following two sets of LMIs:

$$\begin{bmatrix} PA_i + A_i^T P - PB_i k_i - k_i^T B_i^T P & E_{1i} - E_{2i} k_i & PH_i^T \\ * & -\beta^{-1} I & 0 \\ * & * & -\beta I \end{bmatrix} < 0, i = 1...r \tag{9}$$

$$\begin{bmatrix} \begin{matrix} PA_i + A_i^T P + PA_j + A_j^T P - PB_i k_j \\ -k_j^T B_i^T P - PB_j k_i - k_i^T B_j^T P \end{matrix} & E_{1i} - E_{2i} k_j & E_{1j} - E_{2j} k_i & PH_i^T & PH_j^T \\ * & -\beta^{-1} I & 0 & 0 & 0 \\ * & * & -\beta^{-1} I & 0 & 0 \\ * & * & * & -\beta I & 0 \\ * & * & * & * & -\beta I \end{bmatrix} < 0, \tag{10}$$

$i, j = 1...r, i < j$

Proof. The sufficient conditions (9) and (10) that guarantee the global asymptotic stability of the Takagi-Sugeno fuzzy control system with parametric uncertainties are derived as follows using Lyapunov-Krasovskii's method.

Let us consider the Lyapunov function V that is defined and fulfils:

$$V(x(t)) = x^T(t) P x(t) > 0 \tag{11}$$

because $P = P^T > 0$. The derivative of (11) is:

$$\dot{V}(x(t)) = 2x^T(t) P \dot{x}(t) = x^T(t) P \dot{x}(t) + \dot{x}^T(t) P x(t) \tag{12}$$

Applying Lyapunov-Krasovskii's method to (12) using (8) we have:

$$P[A_a + H_a F_a E_a] + [A_a^T + E_a^T F_a^T H_a^T] P < 0 \tag{13}$$

with the notations:

$$A_a = A_i - B_i k_j, E_a = E_{1i} - E_{2i} k_j, H_a = H_i, F_a = F_i \tag{14}$$

Equation (13) is written as:

$$\varphi + RF_a E_a + E_a^T F_a^T R^T < 0 \tag{15}$$

with the notations:

$$\varphi = PA_a + A_a^T P, R = PH_a \tag{16}$$

and (16) leads to:

$$R^T = H_a^T P \tag{17}$$

Finally, (15) can be transferred into the LMIs (9) and (10) by applying the lemma given in (Peterson and Hollot, 1986) and the Schur complement. The LMIs (9) and (10) are solved numerically.

3. OPTIMAL TUNING OF TAKAGI-SUGENO FUZZY CONTROLLERS

The design of the Takagi-Sugeno fuzzy controller means to obtain the values of the parameters in the constant state feedback matrices $k_i \in R^{n \times m}$, $i = 1...r$. The gains of these matrices are grouped in the parameter vector $\boldsymbol{\rho}$:

$$\begin{aligned} &\boldsymbol{\rho} = [k_{111}\ k_{112}\ ...\ k_{11m}\ k_{121}\ k_{122}\ ...\ k_{12m}\ ...\ k_{1n1}\ k_{1n2}\ ...\ k_{1nm} \\ &k_{211}\ k_{212}\ ...\ k_{21m}\ k_{221}\ k_{222}\ ...\ k_{22m}\ ...\ k_{2n1}\ k_{2n2}\ ...\ k_{2nm} \\ &...\ k_{r111}\ k_{r12}\ ...\ k_{r1m}\ k_{r21}\ k_{r22}\ ...\ k_{r2m}\ ...\ k_{rn1}\ k_{rn2}\ ...\ k_{rnm}]^T \end{aligned} \tag{18}$$

The constrained optimization problem that ensures the stable design of the Takagi-Sugeno fuzzy controllers is defined as follows:

$$\begin{aligned} &\boldsymbol{\rho}^* = \underset{\boldsymbol{\rho} \text{ subject to (9) and (10)}}{\arg\min} J(\boldsymbol{\rho}) \\ &J(\boldsymbol{\rho}) = \int_0^{t_f} e^2(t) dt \end{aligned} \tag{19}$$

where $\boldsymbol{\rho}^*$ is the optimal value of the vector $\boldsymbol{\rho}$, e is the control error, $J(\boldsymbol{\rho})$ is the objective function of ISE type and $[0, t_f]$ is the time horizon. As pointed out in Section 1, the optimization problem (19) is solved in this paper by three swarm optimization algorithms briefly described in the next paragraphs.

As shown in (Precup et al., 2015b), the operating mechanism of PSO algorithms uses swarm particles, which are characterized by two vectors, namely the particle position vector $\mathbf{X}_i$ and the particle velocity vector $\mathbf{V}_i$:

$$\begin{aligned} &\mathbf{X}_i = [x_i^1\ \ ...\ \ x_i^d\ \ ...\ \ x_i^q]^T \\ &\mathbf{V}_i = [v_i^1\ \ ...\ \ v_i^d\ \ ...\ \ v_i^q]^T, i = 1...N_p \end{aligned} \tag{20}$$

where $i, i = 1...N_p$ is the index of the current particle in the swarm, and N_p is the number of particles in the swarm. Using the notations $\mathbf{P}_{i,Best}$ for the best particle position vector of a specific particle with the index $i, i = 1...N_p$, and $\mathbf{P}_{g,Best}$ for the best swarm position vector:

$$\begin{aligned} \mathbf{P}_{i,Best} &= [p_i^1 \quad \dots \quad p_i^d \quad \dots \quad p_i^q]^T \\ \mathbf{P}_{g,Best} &= [p_g^1 \quad \dots \quad p_g^d \quad \dots \quad p_g^q]^T, i = 1...N_p \end{aligned} \tag{21}$$

the next particle velocity $v_i^d(\mu+1)$ and the next particle position $x_i^d(\mu+1)$ are obtained by the state-space equations:

$$\begin{aligned} v_i^d(\mu+1) &= w(\mu)\, v_i^d(\mu) + c_1\, r_1\, [p_g^d(\mu) - x_i^d(\mu)] + c_2\, r_2\, [p_i^d(\mu) - x_i^d(\mu)] \\ x_i^d(\mu+1) &= x_i^d(\mu) + v_i^d(\mu+1), d = 1...q, i = 1...N_p \end{aligned} \tag{22}$$

where μ is the current iteration index, and the values of the parameters of the PSO algorithm indicated in (22) are taken from (Precup et al., 2015c).

The PSO algorithm is mapped onto the optimization problem using the following relations:

- between the agents' position vector $\mathbf{X}_i$ in the PSO algorithm and the parameter vector $\boldsymbol{\rho}$ in the optimization problem:

$$\mathbf{X}_i = \boldsymbol{\rho}, i = 1...N_p \tag{23}$$

- between the fitness function *g* in the PSO algorithm and the objective function *J* in the optimization problem:

$$g(\mathbf{X}_i) = J(\boldsymbol{\rho}), i = 1...N_p \tag{24}$$

The PSO algorithm stops when the maximum number of iterations $\mu_{\max}$ is reached. The vector solution to the optimization problem (19) is:

$$\boldsymbol{\rho}^* = \mathbf{P}_{g,Best} \tag{25}$$

where $\mathbf{P}_{g,Best}$ is the best swarm position vector obtained so far.

The steps of the SA algorithm are (Precup et al., 2015b):

- *Step 1*. Set $\mu = 0$, $s_r = 0$ and the minimum temperature $\theta_{\min}$. Choose the initial temperature θ_0.
- *Step 2*. Generate a random initial solution φ and compute its fitness value $g(\varphi)$.
- *Step 3*. Generate a probable solution $\boldsymbol{\psi}$ by disturbing φ and evaluate the fitness value $g(\boldsymbol{\psi})$.

- *Step 4*. Compute $\Delta g_{\varphi\psi} = g(\varphi) - g(\boldsymbol{\psi})$. If $\Delta g_{\varphi\psi} \le 0$, then accept $\boldsymbol{\psi}$ as the new solution. Otherwise, set the value of the random parameter r_n, $0 \le r_n \le 1$, and compute the probability of $\boldsymbol{\psi}$ to be the next solution: $p_{\psi} = \begin{cases} 1 & \text{if } \Delta g_{\varphi\psi} > 0 \\ \exp(\Delta g_{\varphi\psi} / \theta) & \text{otherwise} \end{cases}$. If $p_{\psi} > r_n$, then $\boldsymbol{\psi}$ is the new solution.
- *Step 5*. If the new solution is accepted, then update the new solution, increment s_r and set $r_r = 0$. Otherwise, increment r_r. If r_r has reached its maximum value $r_{r\max}$, the algorithm is stopped; otherwise, continue with step 6.
- *Step 6*. Increment s_r. If s_r has reached its maximum value s_r, go to step 7; otherwise increment μ. If μ has reached its maximum value $\mu_{\max}$, go to step 7; otherwise, go to step 2.
- *Step 7.* Alleviate the temperature according to the temperature decrement rule:

$$\theta_{\mu+1} = \alpha_{cs}\theta_{\mu},\ \alpha_{cs} = \text{const},\ \alpha_{cs} \approx 1 \tag{26}$$

- *Step 8.* If $\theta_{\mu} > \theta_{\min}$ then go to step 3, otherwise the algorithm is stopped.

The SA algorithm is mapped onto the optimization problem using the following relations:

- between the parameter vectors:

$$\begin{aligned} \boldsymbol{\rho} &= \boldsymbol{\psi} \\ \boldsymbol{\rho} &= \varphi \end{aligned} \tag{27}$$

- between the fitness function *g* in the Simulated Annealing algorithm and the objective function *J* in the optimization problem:

$$\begin{aligned} J(\boldsymbol{\rho}) &= g(\boldsymbol{\psi}) \\ J(\boldsymbol{\rho}) &= g(\varphi) \end{aligned} \tag{28}$$

The operating mechanism of GSA makes use of *N* agents and a *q*-dimensional search space, and the position of i^{th} agent is defined by the vector:

$$\mathbf{X}_i = [x_i^1 \quad \ldots \quad x_i^d \quad \ldots \quad x_i^q]^T, i = 1 \ldots N_p \tag{29}$$

The force acting on i^{th} agent from j^{th} agent is defined as follows at the current iteration index μ:

$$F_{ij}^d(\mu) = g(k) \frac{m_{Pi}(\mu) m_{Aj}(\mu)}{r_{ij}(\mu) + \varepsilon\, x_j^d(\mu)} [x_j^d(\mu) - x_i^d(\mu)] \tag{30}$$

where $m_{Pi}(\mu)$ is the active gravitational mass related to i^{th} agent, $m_{Aj}(\mu)$ is the passive gravitational mass related to j^{th} agent, $\varepsilon > 0$ is a small constant, and $r_{ij}(\mu)$ is the Euclidian distance between i^{th} and j^{th} agents. The position and velocity of an agent are updated in terms of the following state-space equations:

$$\begin{aligned} v_i^d(\mu+1) &= \rho_i v_i^d(\mu) + a_i^d(\mu) \\ x_i^d(\mu+1) &= x_i^d(\mu) + v_i^d(\mu+1) \end{aligned} \tag{31}$$

where $\rho_i, 0 \le \rho_i \le 1$ is a uniform random variable, $a_i^d(\mu)$ is the acceleration of i^{th} agent in d^{th} dimension, and the values of the GSA parameters are taken from (Precup et al., 2011). The GSA is mapped onto the optimization problem (19) using once more (23) and (24).

4. NUMERICAL EXAMPLE

The problem of balancing of an inverted pendulum on a cart is considered in this section in order to validate the controller design. The state equations of this process are (Ma et al., 1998):

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1}{[(M+m)(J+ml^2) - m^2l^2\cos^2 x_1]}[-f_1(M+m)x_2 - m^2l^2x_2^2 \sin x_1 \cos x_1 \\ + f_0 mlx_4 \cos x_1 + (M+m)mgl \sin x_1 - ml\cos x_1 u] \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{1}{[(M+m)(J+ml^2) - m^2l^2\cos^2 x_1]}[-f_1 mlx_2 \cos x_1 + (J+ml^2)mlx_2^2 \sin x_1 \\ - f_0(J+ml^2)x_4 - m^2gl^2 \sin x_1 \cos x_1 + (J+ml^2)u] \end{cases} \tag{32}$$

where x_1 is the angle (rad) of the pendulum from the vertical, x_2 is the angular velocity (rad/s), x_3 is the displacement (m) of the cart, x_4 is the velocity (m/s) of the cart, $g = 9.8 m/s^2$ is the gravity constant, m is the mass (kg) of the pendulum, M is the mass (kg) of the cart, f_0 is the friction factor (N/m/s) of the cart, f_1 is the friction factor (N/rad/s) of the pendulum, l is the length (m) from the center of mass of the pendulum to the shaft axis, J is the moment of inertia (kg m^2) of the pendulum round its center of mass and u is the force (N) applied to the cart, playing the role of control signal.

The values of the parameters in (32) are:

$$\begin{aligned} &M = 1.3282kg, m = 0.22kg, f_0 = 22.915N/m/s, f_1 = 0.00756N/rad/s, \\ &l = 0.304m, J = 0.0044963kgm^2 \end{aligned} \tag{33}$$

The process (32) can be approximated by a simple Takagi-Sugeno fuzzy model with only two rules:

$$\begin{aligned}&\text{Rule1: IF } x_1(t) \text{ IS ABOUT 0 THEN } \begin{cases}\dot{x}(t) = A_1 x(t) + B_1 u(t) \\ y(t) = C_1 x(t)\end{cases} \\ &\text{Rule2: IF } x_1(t) \text{ IS ABOUT } \pi/3 \text{ THEN } \begin{cases}\dot{x}(t) = A_2 x(t) + B_2 u(t) \\ y(t) = C_2 x(t)\end{cases}\end{aligned} \tag{34}$$

with the matrices:

$$\begin{aligned}&A_1 = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 \\ a_{21} & a_{22} & 0.0 & a_{24} \\ 0.0 & 0.0 & 0.0 & 1.0 \\ a_{41} & a_{42} & 0.0 & a_{44} \end{bmatrix}, A_2 = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 \\ a'_{21} & a'_{22} & 0.0 & a'_{24} \\ 0.0 & 0.0 & 0.0 & 1.0 \\ a'_{41} & a'_{42} & 0.0 & a'_{44} \end{bmatrix} \\ &B_1 = \begin{bmatrix} 0.0 \\ b_2 \\ 0.0 \\ b_4 \end{bmatrix}, B_2 = \begin{bmatrix} 0.0 \\ b'_2 \\ 0.0 \\ b'_4 \end{bmatrix} \\ &C_1 = C_2 = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}\end{aligned} \tag{35}$$

The expressions of the elements of the matrices in (35) are:

$$\begin{aligned}&a_{21} = (M+m)mgl/a, a_{22} = -f_1(M+m)/a, a_{24} = f_0 ml/a \\ &a_{41} = -m^2 gl^2/a, a_{42} = f_1 ml/a, a_{44} = -f_0(J+ml^2)/a \\ &b_2 = -ml/a, b_4 = (J+ml^2)/a \\ &a \ = (M+m)(J+ml^2) - m^2 l^2 \\ &a'_{21} = \frac{3\sqrt{3}}{2\pi} f_1(M+m)mgl/a', a'_{22} = -f_1(M+m)/a', a'_{24} = f_0 ml(\cos\pi/3)/a' \\ &a'_{41} = -\frac{3\sqrt{3}}{2\pi} m^2 gl^2 \cos(\pi/3)/a', a'_{42} = f_1 ml\cos(\pi/3)/a', a'_{44} = -f_0(J+ml^2)/a' \\ &b'_2 = -ml\cos(\pi/3)/a', b'_4 = (J+ml^2)/a' \\ &a' = (M+m)(J+ml^2) - m^2 l^2 \cos^2(\pi/3)\end{aligned} \tag{36}$$

The uncertainty matrices in (2) are (Zhang et al., 2006):

$$E_{11} = E_{12} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 23 & -0.246 & 0 & 12.8 \\ 0 & 0 & 0 & 1 \\ 0.38 & 0.0051 & 0 & -6.2 \end{bmatrix}$$
$$H_1 = H_2 = \begin{bmatrix} -0.2 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix} \quad (37)$$

The expressions of the membership functions of the linguistic terms related to (34) are:

$$h_1(x_1(t)) = \left\{1 - \frac{1}{1.0 + e^{-7[x_1(t)-\pi/6]}}\right\} \frac{1}{1 + e^{-7[x_1(t)+\pi/6]}}$$
$$h_2(x_1(t)) = 1 - h_1(x_1(t)) \quad (38)$$

The dynamic regime considered in the evaluation of the objective function in (19) and the simulation of the Takagi-Sugeno fuzzy control system is characterized by the initial conditions:

$$x_1(0) = 20^{\circ}, x_2(0) = x_3(0) = x_4(0) = 0 \quad (39)$$

and $t_f = 20\,\mathrm{s}$. The application of the three swarm intelligence algorithms in the conditions described in (Precup et al., 2015b) leads to the following optimal values of the state feedback matrices:

$$k_1^* = [-69.1679 \;\; -12.8245 \;\; -6.6685 \;\; -33.9422]$$
$$k_2^* = [-145.2253 \;\; -30.0898 \;\; -8.8434 \;\; -37.5585] \quad (40)$$

The values of the parameters of PSO, SA and GSA are given in (Precup et al., 2015b). The LMIs (9) and (10) have been solved at each iteration and each solution has been next used as a feasible solution to the optimization problem (19).

The simulation results are presented in Figure 1 and Figure 2 as the Takagi-Sugeno fuzzy control system responses. These responses correspond to the control system with optimized fuzzy controller parameters presented in (40).

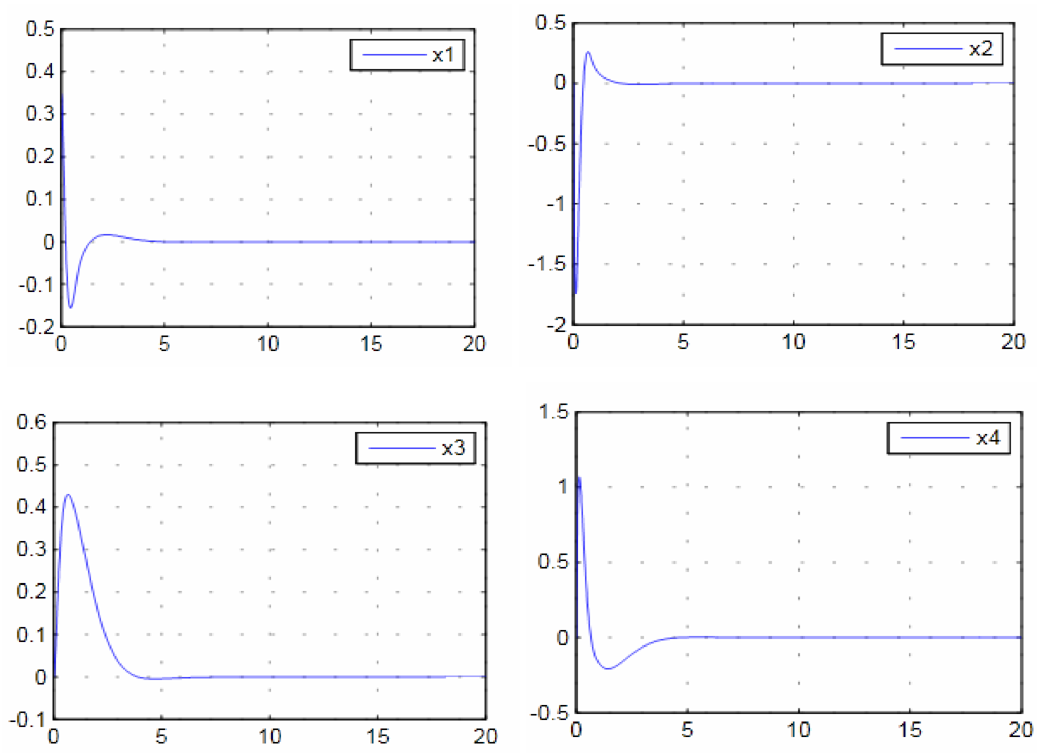

Figure 1. State variables x_1 (rad), x_2 (rad/s), x_3 (m) and x_4 (m/s) versus time (t(s)).

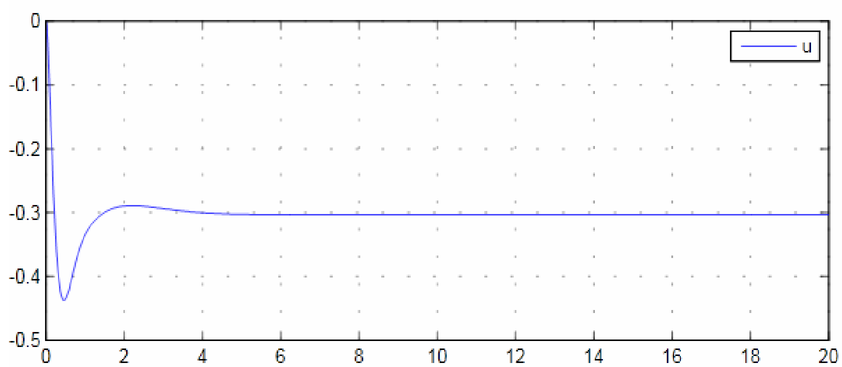

Figure 2. Control signal u (N) versus time (t(s)).

5. CONCLUSIONS

The paper has presented the stable design of Takagi-Sugeno fuzzy control for process models using Takagi-Sugeno fuzzy systems with uncertainties. The optimal tuning of the state feedback gain matrices has been ensured by three swarm intelligence algoritms – PSO, SA and GSA – that solve optimization problems. The LMIs specific to parallel distributed compensation where used not in the control design but as constraints in the optimization problems. Some simulation results have been given.

Future research will deal with the extension of these results to other linear and nonliner controllers as those presented in (Angelov, 2004), (Baranyi, 2004), (Precup et al., 2007), (Sánchez Boza et al., 2011), (Vrkalovic, 2015). Other swarm intelligence algorithms will be used including charged system search and grey wolf optimizers.

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