This article can be cited as R.-E. Precup and S. Preitl, Popov-type stability analysis method for fuzzy control systems, Proceedings of Fifth European Congress on Intelligent Technologies and Soft Computing (EUFIT'97), Aachen, Germany, vol. 2, pp. 1306-1310, 1997.

POPOV-TYPE STABILITY ANALYSIS METHOD FOR FUZZY CONTROL SYSTEMS

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Abstract: The paper presents a stability analysis method meant for fuzzy control systems containing fuzzy controllers with dynamics. The proposed method is based on the theory of hyperstability after Popov and on a discrete time state space single input-single output linear time invariant mathematical model of the controlled plant.

1. Introduction

The stability analysis of a fuzzy control system (briefly, FCS) is necessary because only a stable FCS can: ensure disturbance rejection, guarantee desired steady states, and reduce the risk of implementing the fuzzy controller (FC).

The FC without dynamics represents a nonlinear element [1], [2] ensuring a nonlinear input-output (generalized) static map due to the nonlinearities in: the shapes of membership functions, the rule base, and the defuzzification method.

The introduction of dynamics (i.e., of integral and/or derivative actions) in the structure of a FC can be done on either the inputs or the outputs of the FC [3]. The paper deals with introducing the integral action resulting in PI-type fuzzy controllers.

Several methods for the stability analysis of a FCS are well-known [4], [5]. The paper presents a stability analysis method based on the theory of hyperstability [6] based on considering a discrete time state space mathematical model of a single input-single output linear time invariant (SISO-LTI) controlled plant (CP). An example is presented as part of the paper concerning the application of the proposed method to the design of a PI-fuzzy controller for regulation and tracking of a class of nonminimum-phased systems.

2. Mathematical models of controlled plant extended with the linear part of fuzzy controller [7]

The CP is supposed to have the following n-th order discrete time SISO-LTI state space mathematical model including the zero-order hold:

$$\underline{x}_{k+1} = \underline{A}\, \underline{x}_k + \underline{b}\, u_k, \tag{1}$$

$$y_k = \underline{c}^T \underline{x}_k, \tag{2}$$

where: u_k - the control signal; y_k - the controlled output; $\underline{x}_k$ - the state vector; $\underline{A}$, $\underline{b}$, $\underline{c}^T$ - matrices with the dimensions: dim $\underline{A}$ = (n, n), dim $\underline{b}$ = (n, 1), dim $\underline{c}^T$ = (1, n); T - upper index used to express transposition; k - lower integer index expressing the number of the current sampling period.

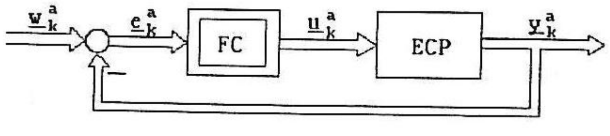

Fig.1. Block diagram of a FCS.

The block diagram of a FCS containing a FC with its dynamic transferred to the CP can be transformed as in Fig.1 for a relatively simple stability analysis.

The elements from Fig.1 have the following significance: a ∈ {i, o} - upper index corresponding to the type of integration: a = i for integration on the input of FC, a = o for integration on the output of FC; $\underline{w}_k^a$ - the reference input vector:

$$\underline{w}_k^i = \begin{bmatrix} w_k \\ w_{ik} \end{bmatrix}, \quad \underline{w}_k^o = \begin{bmatrix} w_k \\ \Delta w_k \end{bmatrix}, \tag{3}$$

with: w_k - the reference input, w_{ik} - the integral of reference input, Δw_k - the increment of reference input; $\underline{e}_k^a$ - the control error vector:

$$\underline{e}_k^{\,i} = \begin{bmatrix} e_k \\ e_{ik} \end{bmatrix}, \qquad \underline{e}_k^{\,o} = \begin{bmatrix} e_k \\ \Delta e_k \end{bmatrix}, \tag{4}$$

with: e_k - the control error, e_{ik} - the integral of control error, Δe_k - the increment of control error; $\underline{u}_k^{\,a}$, $\underline{y}_k^{\,a}$ - the control signal vector and the controlled output vector, respectively, to be presented in the sequel; ECP - the extended controlled plant (with integral action). The absence of disturbance input from the block diagram is fully justified for the sake of Popov-type stability analysis [8].

Note that the extension of controlled plant appears in terms of the state space mathematical model (1), (2), and it is caused by the existing zero-order hold.

According to Fig.1, the FC is characterized by the following nonlinear input-output static map described by the following function:

$$\underline{F}: \mathbb{R}^2 \text{-->} \mathbb{R}^2, \tag{5}$$

$$\underline{F}(\underline{e}_k^{\,a}) = \begin{bmatrix} f(\underline{e}_k^{\,a}) \\ 0 \end{bmatrix}. \tag{6}$$

The mathematical model of ECP can be derived as follows by taking into account [9] for the introduction of additional state variables.

A) The case of integration on fuzzy controller input.

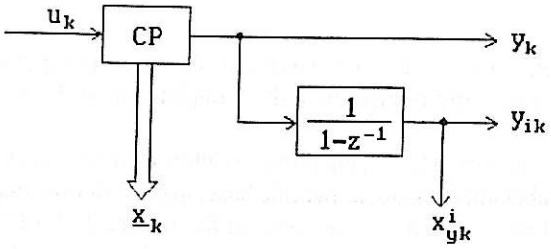

Fig.2. Block diagram of ECP in the case of integration on FC input.

The block diagram of ECP is presented in Fig.2, and it points out the additional state variable $x_{yk}^{\,i}$. The controlled output vector $\underline{y}_k^{\,i}$ can be expressed as:

$$\underline{y}_k^{\,i} = \begin{bmatrix} y_k \\ y_{ik} \end{bmatrix}, \tag{7}$$

where: y_k - the controlled output; y_{ik} - the integral of controlled output.

By the introduction of a fictituous control signal u_{fk} for having an equal number of inputs and outputs as required by the hyperstability theory in the multivariable case [8], the control signal vector becomes:

$$\underline{u}_k^{\,i} = \begin{bmatrix} u_k \\ u_{fk} \end{bmatrix}. \tag{8}$$

So, the (n+1)-th order discrete time state space mathematical model of ECP can be arranged as:

$$\underline{x}_{k+1}^{\,i} = \underline{A}^i\, \underline{x}_k^{\,i} + \underline{B}^i\, \underline{u}_k^{\,i}, \tag{9}$$

$$\underline{y}_k^{\,i} = \underline{C}^i\, \underline{x}_k^{\,i}, \tag{10}$$

where $\underline{x}_k^{\,i}$ represents the extended state vector:

$$\underline{x}_k^{\,i} = \begin{bmatrix} \underline{x}_k \\ x_{yk}^{\,i} \end{bmatrix}, \tag{11}$$

and the matrices are:

$$\underline{A}^i = \begin{bmatrix} \underline{A} & \underline{0} \\ \underline{c}^T\, \underline{A} & 1 \end{bmatrix}, \quad \dim \underline{A}^i = (n+1,\ n+1), \tag{12}$$

$$\underline{B}^i = \begin{bmatrix} \underline{b} & \underline{1} \\ \underline{c}^T\, \underline{b} & 1 \end{bmatrix}, \quad \dim \underline{B}^i = (n+1,\ 2), \tag{13}$$

$$\underline{C}^i = \begin{bmatrix} \underline{c}^T & 0 \\ \underline{0}^T & 1 \end{bmatrix}, \quad \dim \underline{C}^i = (2,\ n+1). \tag{14}$$

B) The case of integration on fuzzy controller output.

The block diagram of ECP pointing out the additional state variables $\{x_{uk},\ x_{yk}^{\,o}\}$ is presented in Fig.3. The extended state vector can be expressed as:

$$\underline{x}_k^{\,o} = \begin{bmatrix} \underline{x}_k \\ x_{uk} \\ x_{yk}^{\,o} \end{bmatrix}. \tag{15}$$

The controlled output vector and the control input vector are:

$$\underline{y}_k^{\,o} = \begin{bmatrix} y_k \\ \Delta y_k \end{bmatrix}, \qquad \underline{u}_k^{\,o} = \begin{bmatrix} \Delta u_k \\ \Delta u_{fk} \end{bmatrix}, \tag{16}$$

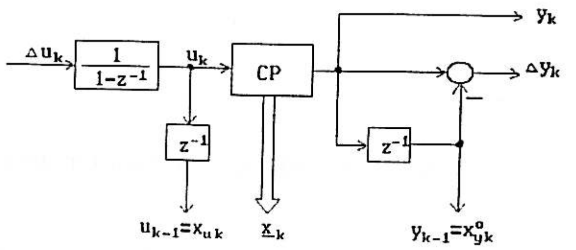

Fig.3. Block diagram of ECP in the case of integration on FC output.

where: $\Delta y_k = y_k - y_{k-1}$ - the increment of controlled output; Δu_k - the increment of control signal; Δu_{fk} - the fictitious increment of control signal introduced for the same reason as in the previous case.

The (n+2)-th order discrete time state space mathematical model of ECP in this case is as follows:

$$\underline{x}_{k+1}{}^{o} = \underline{A}^{o}\,\underline{x}_k{}^{o} + \underline{B}^{o}\,\underline{u}_k{}^{o}, \qquad (17)$$

$$\underline{y}_k{}^{o} = \underline{C}^{o}\,\underline{x}_k{}^{o}, \qquad (18)$$

with the corresponding matrices:

$$\underline{A}^{o} = \begin{bmatrix} \underline{A} & \underline{b} & 0 \\ \underline{0}^{T} & 1 & 0 \\ \underline{c}^{T} & 0 & 0 \end{bmatrix}, \quad \dim \underline{A}^{o} = (n+2,\ n+2), \qquad (19)$$

$$\underline{B}^{o} = \begin{bmatrix} \underline{b} & \underline{1} \\ 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \dim \underline{B}^{o} = (n+2,\ 2), \qquad (20)$$

$$\underline{C}^{o} = \begin{bmatrix} \underline{c}^{T} & 0 & 0 \\ \underline{c}^{T} & 0 & -1 \end{bmatrix}, \quad \dim \underline{C}^{o} = (2,\ n+2). \qquad (21)$$

The state space mathematical models from (9), (10) and (17), (18) can be written down together in the following form:

$$\underline{x}_{k+1}{}^{a} = \underline{A}^{a}\,\underline{x}_k{}^{a} + \underline{B}^{a}\,\underline{u}_k{}^{a}, \qquad (22)$$

$$\underline{y}_k{}^{a} = \underline{C}^{a}\,\underline{x}_k{}^{a}, \qquad (23)$$

where: $a \in \{i, o\}$; $\dim \underline{A}^{a} = (n^{a}, n^{a})$, $n^{i} = n+1$, $n^{o} = n+2$; $\dim \underline{B}^{a} = (n^{a}, 2)$; $\dim \underline{C}^{a} = (2, n^{a})$.

Note that the last column of $\underline{B}^{a}$ is full of ones in order to ensure that the above mentioned state space mathematical models are minimum realizations. The last column of $\underline{B}^{a}$ could take any values because it is multiplied with the fictitious controls $\{u_{fk}, \Delta u_{fk}\}$ that have no influence on control system behaviour (the second component of $\underline{F}$ is zero, relation (6)).

3. Stability analysis method

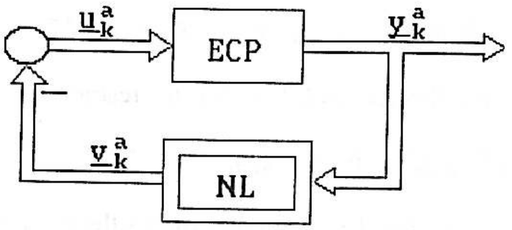

Fig.4. Block diagram of a nonlinear control system.

Generally speaking, the block diagram envolved in the stability analysis of a nonlinear control system is shown in Fig.4. The block NL from Fig.4 represents a static nonlinearity due to the nonlinear (static) part of the FC.

The relations between the block diagrams from Fig.1 and Fig.4 are [10]:

$\underline{y}_k{}^{a}$ from Fig.4 $= -\underline{e}_k{}^{a}$ from Fig.1; (24)

$\underline{u}_k{}^{a} = -\underline{v}_k{}^{a}$ from Fig.4 $= \underline{F}(\underline{e}_k{}^{a})$ from Fig.1. (25)

The second component of $\underline{F}$ is always zero (see the relation (6)) for neglecting the effect of fictitious control signals (u_{fk} and Δu_{fk}).

By taking into account the relation (24), the relation (23) becomes (26):

$$\underline{e}_k = -\,\underline{C}^{a}\,\underline{x}_k, \qquad (26)$$

and it can be written down as:

$$\underline{x}_k = \underline{C}^{c}\,\underline{e}_k, \qquad (27)$$

with the matrix $\underline{C}^{b}$ ($\dim \underline{C}^{b} = (n^{a}, 2)$) that can be easily obtained as function of $\underline{C}^{a}$.

The proposed stability analysis method can be stated in terms of the following theorem:

Theorem. The nonlinear system from Fig.4 with the mathematical model of the linear part (22), (23) is globally assymptotically stable if the three matrices $\underline{P}$ (positive definite, $\dim \underline{P} = (n^{a}, n^{a})$), $\underline{L}$ (regular, $\dim \underline{L} = (n^{a}, n^{a})$), $\underline{V}$ (any, $\dim \underline{V} = (n^{a}, 2)$) fulfil the following requirements:

I. $\underline{A}^{aT}\,\underline{P}\,\underline{A}^{a} - \underline{P} = -\,\underline{L}\,\underline{L}^{T}$, (28)

$\underline{C}^{a} - \underline{B}^{aT}\,\underline{P}\,\underline{A}^{a} = \underline{V}^{T}\,\underline{L}^{T}$, (29)

$$- \underline{B}^{aT} \underline{P} \underline{B}^{a} = \underline{V}^{T} \underline{V}; \quad (30)$$

II. by introducing the following matrices:

$$\underline{M} = \underline{C}^{bT} (\underline{L} \underline{L}^{T} - \underline{P}) \underline{C}^{b}, \dim \underline{M} = (2, 2), \quad (31)$$

$$\underline{N} = \underline{C}^{bT} (\underline{L} \underline{V} - \underline{A}^{aT} \underline{P} \underline{B}^{a} - 2 \underline{C}^{aT}), \dim \underline{N} = (2, 2), \quad (32)$$

$$\underline{R} = \underline{V}^{T} \underline{V}, \dim \underline{R} = (2, 2), \quad (33)$$

there exists the positive definite matrix $\underline{S}$ (dim $\underline{S}$ = (2, 2)) that makes the inequality (34) hold for any value of e_k:

$$f(\underline{e}_k) \underline{n}^{T} \underline{e}_k^{a} \geq \underline{e}_k^{aT} (\underline{S} - \underline{M}) \underline{e}_k^{a}, \quad (34)$$

where $\underline{n}$ represents the first column of $\underline{N}$.

Proof. The condition I is immediately fulfilled because it represents the first equation from the Kalman-Szegö lemma [11].

The Popov inequality - that ensures the global asymptotic stability of the nonlinear control system with the block diagram from Fig.4 - is reminded for the fulfilment of condition II:

$$S(k_1) = \sum_{k=0}^{k_1} \underline{v}_k^{aT} \underline{y}_k^{a} \geq - \beta_0^2, \forall k_1 \in N^{*}, \quad (35)$$

for any positive constant β_0.

By taking into account the correspondences (24) and (25) the Popov sum $S(k_1)$ from (35) becomes:

$$S(k_1) = - \sum_{k=0}^{k_1} \underline{u}_k^{aT} \underline{y}_k^{a}, \forall k_1 \in N^{*}. \quad (36)$$

The substitution of $\underline{y}_k^{f}$ from (23) in (36) followed by adding and subtracting the term $\underline{x}_{k+1}^{aT}\underline{P}\underline{x}_{k+1}^{a}$ yields:

$$S(k_1) = - \sum_{k=0}^{k_1} (\underline{x}_k^{aT} \underline{C}^{aT} \underline{x}_k^{a} + \underline{x}_{k+1}^{aT} \underline{P} \underline{x}_{k+1}^{a} - \underline{x}_{k+1}^{aT} \underline{P} \underline{x}_{k+1}^{a}), \forall k_1 \in N^{*}. \quad (37)$$

Then, $\underline{x}_{k+1}^{a}$ is substituted from (22) in (37) resulting in:

$$S(k_1) = \sum_{k=0}^{k_1} [- \underline{x}_k^{aT} \underline{A}^{aT} \underline{P} \underline{A}^{f} \underline{x}_k^{a} - \underline{x}_k^{aT} (\underline{B}^{aT} \underline{P} \underline{A}^{a} + \underline{B}^{aT} \underline{P}^{T} \underline{A}^{a} + \underline{C}^{aT}) \underline{u}_k^{a} -$$

$$- \underline{u}_k^{aT} \underline{B}^{aT} \underline{B}^{a} \underline{u}_k^{a} + \underline{x}_{k+1}^{aT} \underline{P} \underline{x}_{k+1}^{a}], \forall k_1 \in N^{*}. \quad (38)$$

By replacing the expressions $\underline{A}^{aT} \underline{P} \underline{A}^{a}$, $\underline{B}^{aT} \underline{P} \underline{A}^{a}$ and $\underline{B}^{aT} \underline{P} \underline{B}^{a}$ from the equations (26), (29) and (30), respectively, in (38), and using the relations (25), (31) ... (33), another form of the Popov sum is obtained:

$$S(k_1) = \sum_{k=0}^{k_1} \underline{x}_{k+1}^{aT} \underline{P} \underline{x}_{k+1}^{a} + \sum_{k=0}^{k_1} [\underline{e}_k^{aT} \underline{M} \underline{e}_k^{a} + \underline{e}_k^{aT} \underline{N} \underline{F}(\underline{e}_k^{a}) + \underline{F}^{T}(\underline{e}_k^{a}) \underline{R} \underline{F}(\underline{e}_k^{a})], \forall k_1 \in N^{*}. \quad (39)$$

Finally, by pointing out the positive element r_{11} of $\underline{R}$ and the elements of $\underline{F}$ from (6), the relation (39) becomes:

$$S(k_1) = \sum_{k=0}^{k_1} [\underline{x}_{k+1}^{aT} \underline{P} \underline{x}_{k+1}^{a} + r_{11} f^{2}(\underline{e}_k^{a})] + \sum_{k=0}^{k_1} [\underline{e}_k^{aT} \underline{M} \underline{e}_k^{a} + f(\underline{e}_k^{a}) \underline{n}^{T} \underline{e}_k^{a}], \forall k_1 \in N^{*}. \quad (40)$$

It is obvious that the first sum from (40) is strictly positive. Using (34) determines the second sum from (40) to be expressed as:

$$\sum_{k=0}^{k_1} \underline{x}_k^{aT} \underline{S} \underline{x}_k^{a}, \forall k_1 \in N^{*}. \quad (41)$$

Therefore, the condition II ensures the positive value of $S(k_1)$ fulfilling the Popov inequality (35).

Finally, note that only the matrix $\underline{P}$ (instead of $\underline{P}$, $\underline{L}$ and $\underline{V}$) from the relations (28) ... (30) is important for FCS stability analysis because the matrices $\underline{M}$, $\underline{N}$ and $\underline{R}$ from (31) ... (33) can be expressed as:

$$\underline{M} = - \underline{C}^{bT} \underline{A}^{aT} \underline{P} \underline{A}^{a} \underline{C}^{b}, \quad (42)$$

$$\underline{N} = - \underline{C}^{bT} (\underline{A}^{aT} (\underline{P} + \underline{P}^{T}) \underline{B}^{a} + \underline{C}^{aT}), \quad (43)$$

$$\underline{R} = - \underline{B}^{aT} \underline{P} \underline{B}^{a}. \quad (44)$$

4. Example

For the PI-fuzzy controller with integration on FC output meant for a class of nonminimum-phased systems and developed in [12] the matrices envloved in stability analysis have the following values:

$$\underline{A}^{o}=\begin{bmatrix} 0.9927 & 0.0072 & -0.0142 & 0 \\ 0 & 0.9556 & 0.1333 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \ \underline{B}^{o}=\begin{bmatrix} -0.0142 & 1 \\ 0.1333 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}, \ \underline{C}^{o}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}; \qquad (45)$$

$$\underline{C}^{b}=\begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}, \ \underline{P}=\underline{I}_4; \qquad (46)$$

$$\underline{M}=\begin{bmatrix} -1.9854 & 0 \\ 0 & 0 \end{bmatrix}, \ \underline{N}=\begin{bmatrix} 0.9719 & 5.9853 \\ 0 & 1 \end{bmatrix}, \ \underline{R}=\begin{bmatrix} -1.0180 & -1.1192 \\ -1.1192 & -4 \end{bmatrix}, \ \underline{S}=\underline{I}_2. \qquad (47)$$

The free parameter used in FC design is B_e. For $B_e \in [0.2; 0.5]$ the relation (34) is fulfilled, and the fuzzy control system is globally assymptotically stable.

5. Conclusions

The paper outlines - by applying the theory of hyperstability - a stability analysis method for FCSs containig two possible types of fuzzy controllers with dynamics, i.e. with integrator introduced on both the input and the output of the FC.

The theorem presented as part of the paper gives sufficient conditions ensuring the stability of FCS when a discrete time SISO-LTI mathematical model of the controlled plant is taken into consideration. The relation (34) represents a geometrical condition because its left hand side describes a cone and its right hand side describes a cone.

The proposed stability analysis method is similar to the method from [10] for continuous time systems, and the stability conditions are stronger than the conditions from [13] for discrete time systems.

Digital simulation results confirm the validity of the proposed stability analysis method.

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5th European Congress on Intelligent Techniques and Soft Computing
Aachen, Germany, September 8 - 11, 1997
Proceedings
Volume 2

EUFIT '97

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4. Fuzzy Sets in Artificial Intelligence

Artificial Intelligence

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Fuzzy Sets in Artificial Intelligence

Information Engineering I: Valued Constraint Satisfaction